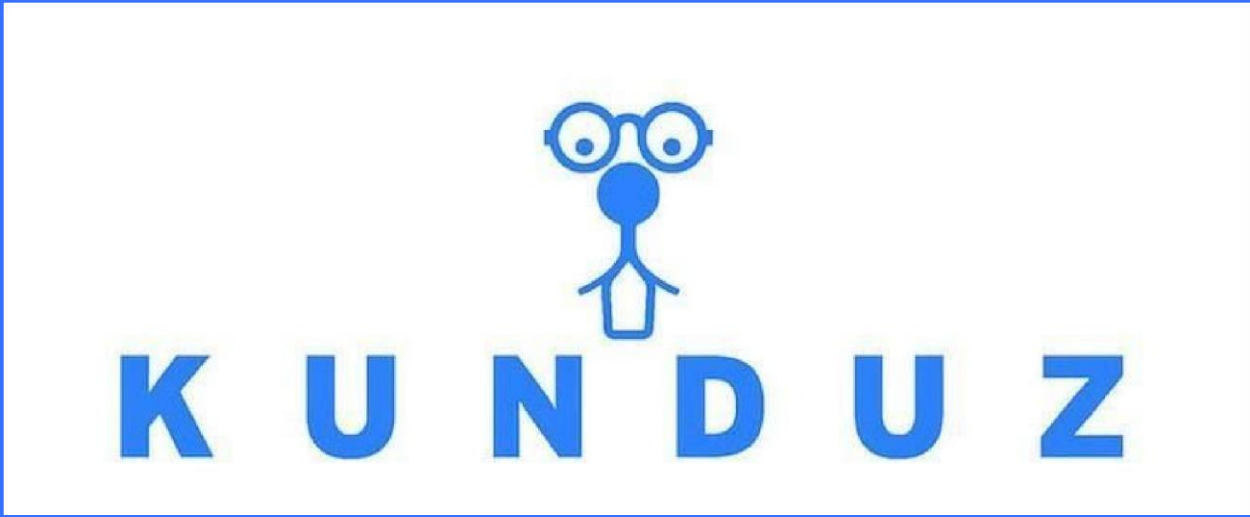


JEE Mains 2nd September 2020 shift 1 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

Q. 1. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in

(1) $(-\infty, -3] \cup [9, \infty)$ (2) $(-\infty, -9] \cup [3, \infty)$

(3) $[-3, \infty)$ (4) $(-\infty, 9]$

Ans. Let the terms are $\frac{a}{r}$, a, ar

$$\frac{a}{r} \times a \times ar = 27 \Rightarrow a^3 = 27$$

$$a = 3$$

$$S = \frac{a}{r} + a + ar = \frac{3}{r} + 3 + 3r$$

$$S = 3 + 3\left(\frac{1}{r} + r\right)$$

$3 + 3(-\infty, -2] \cup [2, \infty)$

$3 + (-\infty, -6] \cup [6, \infty)$

$(-\infty, -3] \cup [9, \infty)$ ans. option (1)

A.M. $\geq$ G.M.

$$\frac{\frac{1}{r} + r}{2} \geq \sqrt{\frac{1}{r} r}$$

$$\frac{1}{r} + r \geq 2$$

$$\frac{-\frac{1}{r} - r}{2} \geq \sqrt{\left(\frac{-1}{r}\right)(-r)}$$

$$-\frac{1}{r} - r \geq 2 \Rightarrow r + \frac{1}{r} \leq -2$$

2

Q.02 Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numered 31 to 50. A box is selected at random and a card is drawn from it. The number on on the card is found to be a non-prime number. The probability that the card was drawn from box I is:

~~Ans~~ (1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{8}{17}$ (4) $\frac{4}{17}$

Ans. A ≡ Box I is selected

B ≡ Box II is selected

total number of non prime numbered cards in Box I is 20

total number of non prime numbered cards in Box II is 15

N ≡ non prime numbered card is selected

$P(A) = \frac{1}{2} = P(B)$

$P(N) = P(A) \cdot P(N/A) + P(B)\ P(N/B)$

$$P(A/N) = \frac{P(A)\,P(N/A)}{P(A)\,P(N/A) + P(B)\,P(N/B)}$$

$$= \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}} = \frac{8}{17}$$ ans. option (3)

3

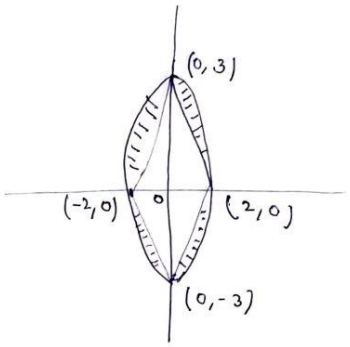
Q 03 Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is

(1) $3(\pi - 2)$ (2) $3(4 - \pi)$ (3) $6(\pi - 2)$ (4) $6(4 - \pi)$

Ans. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$\frac{|x|}{2} + \frac{|y|}{3} = 1$

$\Rightarrow 3|x| + 2|y| = 6$



area of sheded region = area of ellipse − area of rhambus

$= \pi ab - \frac{1}{2} d_1 d_2$

$= \pi(2)(3) - \frac{1}{2} \times 4 \times 6$

$= 6\pi - 12$

$= 6(\pi - 2)$ ans. option (3)

4

Ques. 4. The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$, then a is equal to:

(1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$ (3) $\frac{\sqrt{17}}{2}$ (4) $\frac{\sqrt{17}}{2}+1$

Ans. $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$

$$-1 \le \frac{|x|+5}{x^2+1} \le 1$$

$-x^2-1 \le |x|+5$ which is always true

$|x|+5 \le x^2+1$ Put $|x| = t$

$t+5 \le t^2+1$

$t^2 - t - 4 \ge 0$

$$\left(t - \frac{1+\sqrt{17}}{2}\right)\left(t - \frac{1-\sqrt{17}}{2}\right) \ge 0$$

$$t = \frac{1 \pm \sqrt{1-4(1)(-4)}}{2}$$

$$t = \frac{1 \pm \sqrt{17}}{2}$$

$t \ge \frac{1+\sqrt{17}}{2}$ $\quad \{ |x| = t \ge 0$

$|x| \ge \frac{1+\sqrt{17}}{2}$

$x \in \left(-\infty, \frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$

$\therefore a = \frac{1+\sqrt{17}}{2}$ ans. option (1)

5

Q.5 If $|x| < 1$, $|y| < 1$ and $x \neq y$ then Sum to infinity of the following series is

$$(x+y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$$

(1) $\dfrac{x+y+xy}{(1-x)(1-y)}$

(2) $\dfrac{x+y-xy}{(1-x)(1-y)}$

(3) $\dfrac{x+y-xy}{(1+-x)(1+y)}$

(4) $\dfrac{x+y+xy}{(1+x)(1+y)}$

Ans. $(x+y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$

$$= \frac{(x-y)(x+y)}{(x-y)} + \frac{(x-y)(x^2+xy+y^2)}{(x-y)} + \frac{(x-y)(x^3+x^2y+xy^2+y^3)}{(x-y)} + \ldots \infty$$

$$= \frac{x^2-y^2}{x-y} + \frac{x^3-y^3}{x-y} + \frac{x^4-y^4}{x-y} + \ldots$$

$$= \frac{1}{x-y}\left(x^2 + x^3 + x^4 + \ldots \infty\right) - \frac{1}{(x-y)}\left(y^2 + y^3 + y^3 + \ldots \infty\right)$$

$$= \frac{1}{x-y}\left(\frac{x^2}{1-x}\right) - \left(\frac{1}{x-y}\right)\left(\frac{y^2}{1-y}\right)$$

$$= \frac{1}{x-y}\left(\frac{x^2}{1-x} - \frac{y^2}{1-y}\right)$$

$$= \frac{1}{x-y}\left(\frac{x^2 - x^2y - y^2 + xy^2}{(1-x)(1-y)}\right) = \frac{x+y-xy}{(1-x)(1-y)}$$ Ans.

Option (3)

6

Ques 6 The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}} \right)^3$ is

(1) $-\frac{1}{2}(1-i\sqrt{3})$ (2) $\frac{1}{2}(1-i\sqrt{3})$

(3) $-\frac{1}{2}(\sqrt{3}-i)$ (4) $\frac{1}{2}(\sqrt{3}-i)$

Ans. Let $\frac{2\pi}{9}=\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$z_1 = 1+\sin\theta+i\cos\theta$

$z_1 = 1+\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)$

$z_1 = 2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)+i\,2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$z_1 = 2\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\left(\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$

$z_1 = 2\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)e^{i\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$

$z_2 = 1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9} = 1+\cos\left(\frac{\pi}{2}-\theta\right)-i\sin\left(\frac{\pi}{2}-\theta\right)$

$z_2 = 2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)-i\,2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$z_2 = 2\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\left(\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)-i\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$

$z_2 = 2\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)e^{-i\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$

$\left(\frac{z_1}{z_2}\right)^3 = \left(\frac{e^{i\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}}{e^{-i\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}}\right)^3 = \left(e^{i\left(\frac{\pi}{2}-\theta\right)}\right)^3$

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$$\left(\frac{z_1}{z_2}\right)^3 = e^{i\left(\frac{3\pi}{2} - 3\theta\right)}$$

$$= e^{i\left(270^\circ - 3\times\frac{2\pi}{9}\right)}$$

$$= e^{i(270^\circ - 120^\circ)}$$

$$= e^{i(150^\circ)}$$

$$= \cos 150^\circ + i \sin 150^\circ$$

$$= \cos(90+60^\circ) + i\sin(90+60^\circ)$$

$$= -\sin 60^\circ + i\cos 60^\circ$$

$$= -\frac{\sqrt{3}}{2} + i\frac{1}{2}$$

$$= -\frac{1}{2}(\sqrt{3} - i) \quad \text{ans.} \quad \text{option(1)}$$

7

Ques 7 Let $y = y(x)$ be the solution of the differential equations

$$\frac{2+\sin x}{y+1} \cdot \frac{dy}{dx} = -\cos x, \quad y > 0, \quad y(0) = 1. \text{ If } y(\pi) = a$$

and $\frac{dy}{dx}$ at $x = \pi$ is b, then ordered pair (a, b) is equal to:

(1) $\left(2, \frac{3}{2}\right)$ (2) $(1, -1)$ (3) $(2, 1)$ (4) $(1, 1)$

Ans.

$$\frac{2+\sin x}{y+1} \times \frac{dy}{dx} = -\cos x$$

$$\Rightarrow \int \frac{1}{y+1}\, dy = -\int \frac{\cos x}{2+\sin x}\, dx$$

$$\begin{cases} 2+\sin x = t \\ \cos x\, dx = dt \end{cases}$$

$$\Rightarrow \int \frac{1}{y+1}\, dy = -\int \frac{dt}{t}$$

$$\Rightarrow \ln(y+1) = -\ln t + \ln c$$

$$\Rightarrow \ln(y+1) + \ln(2+\sin x) = \ln c$$

$$\Rightarrow (y+1)(2+\sin x) = c$$

$$\begin{cases} \sin 0 = 0 \\ \sin \pi = 0 \end{cases}$$

Put $x = 0$ and $y = 1$

$(1+1)(2+\sin 0) = c \Rightarrow c = 4$

$(y+1)(2+\sin x) = 4$

Put $x = \pi$

$(y+1)(2+\sin \pi) = 4$

$(y+1) = 2 \Rightarrow y = 1 = a$

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$$\frac{dy}{dx} = \frac{-\cos x}{(2+\sin x)} \quad (y+1)$$

Put $x = \pi$, $y = 1$

$\left.\begin{array}{l} \\ \\ \end{array}\right\}$ $\cos\pi = -1$

$$b = \frac{-\cos(\pi)}{2+\sin\pi} \quad (1+1)$$

$$b = \frac{1}{2} \times 2$$

$$b = 1$$

$(a, b) \equiv (1, 1)$ ans. option (4)

8

Ques: The plane passing through the points $(1,2,1)$, $(2,1,2)$ and parallel to the line, $2x = 3y$, $z = 1$ also passes through the point :

(1) $(-2,0,1)$ (2) $(0,6,-2)$ (3) $(2,0,-1)$ (4) $(0,-6,2)$

Ans. $P(1,2,1)$ $Q(2,1,2)$

$\vec{P} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{Q} = 2\hat{i} + \hat{j} + 2\hat{k}$

$2x = 3y$, $z = 1$ → line

$\Rightarrow \quad \frac{x}{3} = \frac{y}{2} = \frac{z-1}{0}$

$\vec{m}$ = direction vector of the line = $(3,2,0)$

$\vec{PQ} = (2-1)\hat{i} + (1-2)\hat{j} + (2-1)\hat{k}$

$\vec{PQ} = \hat{i} - \hat{j} + \hat{k}$

$\vec{\eta} = \vec{PQ} \times \vec{m}$

$$\vec{\eta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(2-(0)) - \hat{j}(3-(0)) + \hat{k}(-3-2)$$

$\vec{\eta} = 2\hat{i} - 3\hat{j} - 5\hat{k}$

equation of plane $\vec{x} \cdot \vec{\eta} = \vec{\eta} \cdot \vec{P}$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - 3\hat{j} - 5\hat{k}) = (2\hat{i} - 3\hat{j} - 5\hat{k}) \cdot (\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow 2x - 3y - 5z = 2 - 6 - 5$

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$\Rightarrow \quad 2x - 3y - 5z + 9 = 0$

Check each option, which point passes through plane

$(-2, 0, 1)$ passes through the plane

ans. option (1)

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9

Ques 9 Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^3 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{1/9} + \beta x^{-1/6}\right)^{10}$ is $10k$, then k is equal to :

(1) 176 (2) 336 (3) 352 (4) 84

Ans. In $\left(\alpha x^{1/9} + \beta x^{-1/9}\right)^{10}$

$$T_{r+1} = {}^{10}C_r \left(\alpha x^{1/9}\right)^{10-r} \left(\beta x^{-1/6}\right)^r$$

$$= {}^{10}C_r \, \alpha^{10-r} \beta^r \, x^{\left(\frac{10-r}{9} - \frac{r}{6}\right)}$$

$\because$ this term is independent of x

$$\therefore \quad \frac{10-r}{9} - \frac{r}{6} = 0$$

$$\Rightarrow \quad \frac{10}{9} = \frac{r}{6} + \frac{r}{9} = \frac{5r}{18}$$

$$\Rightarrow \quad r = 4$$

$$T_{4+1} = {}^{10}C_4 \, \alpha^6 \beta^4 = 210 \alpha^6 \beta^4$$

$$\alpha^3 + \beta^2 = 4 \quad \ldots\ldots \text{(given)}$$

then by AM $\geq$ GM

$$\frac{\frac{\alpha^3}{2} + \frac{\alpha^3}{2} + \frac{\beta^2}{2} + \frac{\beta^2}{2}}{4} \geq \left(\frac{\alpha^6 \beta^4}{16}\right)^{1/4}$$

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$$\Rightarrow \frac{\frac{2(\alpha^3+\beta^2)}{2}}{4} \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{1/4}$$

$$\Rightarrow \frac{\frac{2(4)}{2}}{4} \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{1/4}$$

$$\Rightarrow 1 \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{1/4} \Rightarrow (1)^4 \geq \frac{\alpha^6\beta^4}{16}$$

$$\Rightarrow \alpha^6\beta^4 \leq 16$$

$$\therefore T_5)_{max} = 210 \times 16 = 3360 = 10k$$

$$\therefore k = 336 \quad \underline{\text{ans.}} \quad \text{option (2)}$$

10

Q 10 If $R = \{(x, y) : x, y \in Z,\ x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is;

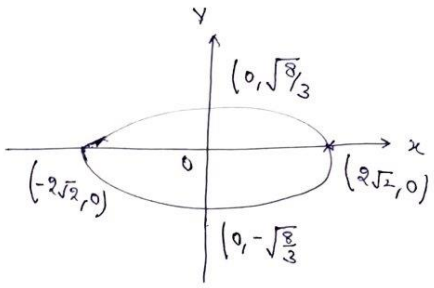
(1) $\{-1, 0, 1\}$ (2) $\{-2, -1, 1, 2\}$

(3) $\{0, 1\}$ (4) $\{-2, -1, 0, 1, 2\}$

Ans.

$$x^2 + 3y^2 \leq 8$$

$$\frac{x^2}{8} + \frac{y^2}{8/3} \leq 1$$



Domain of R^{-1} = Range of R

Range of R will contain all possible value of Y

Range of $R = \{0, 1, -1\}$

$\therefore$ Domain of $R^{-1} = \{0, 1, -1\}$ ans. option (1)

11

Ques 11 If $P(x)$ be a polynomial of degree three that has a local maximum value 8 at $x = 1$ and a local minimum value 4 at $x = 2$, then $P(0)$ is equal to

Ans. $\because$ local max. value $= 8$ at $x = 1$

and loc min. value $= 4$ at $x = 2$

$\therefore \quad P'(x) = \lambda (x-1)(x-2)$

$P'(x) = \lambda (x^2 - 3x + 2)$

$P(x) = \lambda \left(\frac{x^3}{3} - \frac{3}{2}x^2 + 2x \right) + C$

$P(1) = 8 \quad \text{---} \quad (\text{given})$

$P(1) = \lambda \left(\frac{1}{3} - \frac{3}{2} + 2 \right) + C = 8$

$\frac{5\lambda}{6} + C = 8 \quad \text{---} \quad (1)$

$P(2) = 4 \quad \text{---} \quad (\text{given})$

$P(2) = \lambda \left(\frac{8}{3} - \frac{3}{2} \times 4 + 4 \right) + C = 4$

$\frac{2\lambda}{3} + C = 4 \quad \text{---} \quad (2)$

from equation (1) and (2)

$\lambda = 24 \quad \text{and} \quad C = -12$

$P(x) = 24 \left(\frac{x^3}{3} - \frac{3}{2}x^2 + 2x \right) - 12$

$P(0) = -12$ ans.

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12

Ques. 12 A line parallel to the straight line $2x - y = 0$ is tangent to the ~~parabola~~ hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1), then $x_1^2 + 5y_1^2$ is equal to:

(1) 10 (2) 5 (3) 8 (4) 6

Ans. hyperbola : $\frac{x^2}{4} - \frac{y^2}{2} = 1$

$P(x_1, y_1) \equiv P(2\sec\theta, \sqrt{2}\tan\theta)$

equation of tangent at P

$$\frac{x(2\sec\theta)}{4} - \frac{y(\sqrt{2}\tan\theta)}{2} = 1$$

$$\frac{y\tan\theta}{\sqrt{2}} = \frac{x\sec\theta}{2} - 1$$

$$l_1 : \quad y = \frac{1}{\sqrt{2}\sin\theta}x - \frac{\sqrt{2}}{\tan\theta}.$$

↳ this line is parallel to $l_2 : 2x - y = 0$

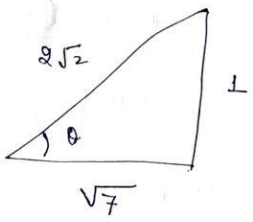
$l_2 : \quad y = 2x$

~~Solo~~ Slope of l_1 = Slope of l_2

$$\frac{1}{\sqrt{2}\sin\theta} = 2 \Rightarrow \sin\theta = \frac{1}{2\sqrt{2}}$$

$$x_1 = 2\sec\theta = 2 \times \frac{2\sqrt{2}}{\sqrt{7}}$$

$$x_1 = \frac{4\sqrt{2}}{\sqrt{7}}$$



$$x_2 = \sqrt{2}\tan\theta = \sqrt{2} \times \frac{1}{\sqrt{7}}$$

$$x_2 = \frac{\sqrt{2}}{\sqrt{7}}$$

$$x_1^2 + 5x_2^2 = \left(\frac{4\sqrt{2}}{\sqrt{7}}\right)^2 + 5 \times \left(\frac{\sqrt{2}}{7}\right)^2$$

$$= \frac{32}{7} + \frac{10}{7}$$

$$= \frac{42}{7}$$

$$x_1^2 + 5x_2^2 = 6 \text{ ans. option (4)}$$

13

Ques 13. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is:

(1) If I will catch the train, then I reach the station in time.

(2) If do not reach the station in time, then I will not catch the train.

(3) If I do not reach the station in time, then I will catch the train.

(4) If I will not catch the train, then I do not reach the station in time.

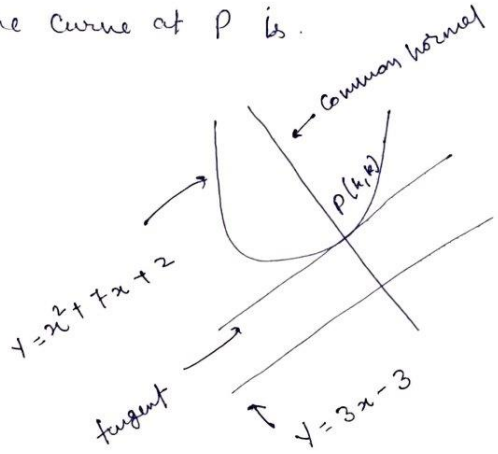
Ans. Contrapositive of $p \to q$ is $\sim q \to \sim p$

∴ option (4) ans.

14

Que 14 Let $P(h,k)$ be a point on the curve $Y = x^2 + 7x + 12$, nearest to the line, $Y = 3x - 3$. Then the equation of the normal to the curve at P is.

Ans. $Y = x^2 + 7x + 2$



tangent at P will be Parallel to $Y = 3x - 3$

$$\therefore \left(\frac{dy}{dx}\right)_P = 3$$

$\Rightarrow$ Slope of normal at $P = -1/3$

$$Y = x^2 + 7x + 2$$

$$\Rightarrow \frac{dy}{dx} = 2x + 7 = 3 \quad \Rightarrow x = -2$$

$$Y = x^2 + 7x + 2$$

$$Y(-2) = (-2)^2 + 7(-2) + 2 = -8$$

$$P(h,k) \equiv P(-2, -8)$$

Slope of ~~P at~~ normal at $P = -1/3$

equation of normal at P

$$Y - (-8) = -\frac{1}{3}(x - (-2))$$

$$\Rightarrow 3Y + 24 = -x - 2$$

$$\Rightarrow x + 3Y + 26 = 0 \quad \text{ans.}$$

15

Ques 15 Let A be a 2×2 real matrix with entries from $\{0,1\}$ and $|A| \neq 0$. Consider the following statements;

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$

where I_2 denotes 2×2 identity matrix and $tr(A)$ denotes the sum of the diagonal entries of A. Then:

(1) (P) is true and Q is false (2) Both (P) and (Q) are true

(3) Both (P) and (Q) are false (4) (P) is false and (Q) is true

Sol. Let.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = ad - bc$$

Statement P

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow |A| \neq -1$$

$\therefore$ Statement P is false

Statement Q

$$|A| = 1 \Rightarrow ad = 1 \Rightarrow a = 1 \& d = 1 \& bc = 0$$

$$tr(A) = a + d = 2$$

Statement (Q) is true

Ans. option (4)

16

Ques. 16 Let S be the set of all $\lambda \in R$ for which the system of linear equations

$$2x - y + 2z = 2$$
$$x - 2y + \lambda z = -4$$
$$x + \lambda y + z = 4$$

has no solution. Then the set S

(1) is a singleton (2) contains exactly two elements

(3) contains more than two elements (4) is an empty set

Ans. For no solution $\Delta = 0$ and at least one of $\Delta_1, \Delta_2, \Delta_3$ is non-zero

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$$

$$\Rightarrow -4 - 2\lambda^2 + 1 - \lambda + 2\lambda + 4 = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0 \Rightarrow (\lambda - 1)(2\lambda + 1) = 0$$

$$\lambda = 1 \quad \text{or} \quad \lambda = -1/2$$

If $\lambda = 1$

$$2x - y + 2z = 9$$
$$x - 2y + z = -4$$
$$x + y + z = 4$$

D_x $\Delta_x \neq 0$ $\Rightarrow$ No solution

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If $\lambda = -1/2$

$$2x - y + 2z = 2$$

$$2x - 4y - z = -8$$

$$2x - y + 2z = 8$$

$\Rightarrow$ No solution

$\Rightarrow S = \{1, -1/2\}$

17

Ques. 17 If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then

(1) $|b - a| = 1$ (2) $b = \frac{\pi}{2} + a$ (3) $|a + b| = 1$ (4) $b = a$

Ans. Slope of line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$

$$= \frac{2 - 3/2}{1/2 - 0} = \frac{1/2}{1/2} = 1$$

$$y = x + \sin y \Rightarrow \frac{dy}{dx} = 1 + \cos y \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1 - \cos y} = 1$$

$$\Rightarrow 1 - \cos y = 1 \Rightarrow \cos y = 0$$

$$\left(\frac{dy}{dx}\right)_{(a,b)} = \left(\frac{1}{1 - \cos y}\right)_{(a,b)} = 1$$

$$\Rightarrow \frac{1}{1 - \cos b} = 1 \Rightarrow \cos b = 0$$

$$\therefore \sin b = 1$$

$y = x + \sin y$

at (a, b)

$$\Rightarrow b = a + \sin b$$

$$\Rightarrow b - a = \sin b$$

$$\Rightarrow |b - a| = |\sin b| \Rightarrow |b - a| = 1$$ ans. option 1

18

Ques: 18 If a function $f(x)$ defined by

$$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$$

be continuous for some $a, b, c \in R$ and $f'(0) + f'(2) = e$

then value of a is :

(1) $\dfrac{1}{e^2 - 3e + 13}$

(2) $\dfrac{e}{e^2 - 3e + 13}$

(3) $\dfrac{e}{e^2 - 3e - 13}$

(4) $\dfrac{e}{e^2 + 3e + 13}$

Ans: $$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$$

Continuous at $x = 1$

$$ae + \frac{b}{e} = c \quad \text{---} \quad (1)$$

Continuous at $x = 3$

$$9c = 9a + 6c \Rightarrow c = 3a \quad \text{---} \quad (2)$$

$$\therefore b = ae(3 - e) \quad \text{---} \quad (3)$$

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$$f'(x) = \begin{cases} ae^{x} - be^{-x} & -1 < x < 1 \\ 2cx & 1 < x < 3 \\ 2ax + 2c & 3 < x < 4 \end{cases}$$

$f'(0) = a - b$ as $-1 < 0 < 1$

$f'(2) = 4c$ as $1 < 2 < 3$

$f'(0) + f'(2) = e$ --- (given)

$a - b + 4c = e$

Put values of b and c $\begin{cases} b = ae(3-e) \\ c = 3a \end{cases}$

$\Rightarrow a - ae(3-e) + 4(3a) = e$

$\Rightarrow 13a - a(3e - e^2) = e$

$\Rightarrow a(13 - 3e + e^2) = e$

$\Rightarrow a = \dfrac{e}{e^2 - 3e + 13}$ ans option (2)

19

Ques 19 Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$

If $S_n = \alpha^n + \beta^n$, $n = 1, 2, 3 \ldots\ldots$, then

(1) $6S_6 + 5S_5 = 2S_4$ (2) $5S_6 + 6S_5 = 2S_4$

(3) $5S_6 + 6S_5 + 2S_4 = 0$ (4) $6S_6 + 5S_5 + 2S_4 = 0$

Ans. α, β are roots of $5x^2 + 6x - 2 = 0$

$5\alpha^2 + 6\alpha - 2 = 0$ $\qquad$ $5\beta^2 + 6\beta - 2 = 0$

$\alpha^2 = \dfrac{2 - 6\alpha}{5}$ $\qquad$ $\beta^2 = \dfrac{2 - 6\beta}{5}$

$S_n = \alpha^n + \beta^n$

$S_n = \alpha^{n-2}(\alpha^2) + \beta^{n-2}(\beta^2)$

$S_n = \alpha^{n-2}\left(\dfrac{2 - 6\alpha}{5}\right) + \beta^{n-2}\left(\dfrac{2 - 6\beta}{5}\right)$

$5S_n = 2\alpha^{n-2} - 6\alpha^{n-1} + 2\beta^{n-2} - 6\beta^{n-1}$

$5S_n = -6\left(\alpha^{n-1} + \beta^{n-1}\right) + 2\left(\alpha^{n-2} + \beta^{n-2}\right)$

$5S_n = -6S_{n-1} + 2S_{n-2}$

$5S_n + 6S_{n-1} = 2S_{n-2}$ $\qquad$ { Put $n = 6$

$5S_6 + 6S_5 = 2S_4$ ans. option (2)

20

Ques. 20 Let $X = \{x \in N : 1 \le x \le 17\}$ and $Y = \{ax+b : x \in X$ and $a, b \in R, a > 0\}$. If mean and Variance of elements of Y are 17 and 216 respectively then $a+b$ is equal to:

(1) 7 (2) 9 (3) -7 (4) -27

Ans. $X = \{1, 2, 3, \ldots, 17\}$

$$\text{Mean} = \bar{X} = \frac{1+2+3+\cdots+17}{17} = 9$$

$$\text{Variance} = \frac{1}{n}\sum x_i^2 - (\bar{X})^2 = \sigma_x^2$$

$$\sigma_x^2 = \frac{1^2+2^2+3^2+\cdots+17^2}{17} - (9)^2$$

$$\sigma_x^2 = |105 - 81| = 24$$

$$Y = \{ax+b : x \in X \text{ and } a, b \in R, a > 0\}$$

$$\text{Mean} = \bar{Y} = \frac{(a(1)+b)+(a(2)+b)+\cdots+(a(17)+b)}{17}$$

Given $\cdots$ $$17 = \frac{a(1+2+\cdots+17)}{17} + \frac{17b}{17}$$

$$\Rightarrow 17 = a\bar{X} + b$$

$$\Rightarrow 9a + b = 17$$

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Variance $= \sigma_y^2 = \frac{1}{n}\sum y_i^2 - (\bar{y})^2 = 216$

$\sigma_y^2 = a^2 \sigma_x^2 = 216$

$\Rightarrow a^2 \times 24 = 216$

$\Rightarrow a^2 = \frac{216}{24} = 9$

$a = \pm 3$

$a = 3$ as $a > 0$

$9a + b = 17$

$\Rightarrow 9 \times 3 + b = 17$

$\Rightarrow b = -10$

$a + b = 3 - 10 = -7$ ans. option (3)

21

Ques. 21 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vector such that $|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8$. Then $|\vec{a}+\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$ is equal to ___

Ans. $|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8$

$\vec{a}\cdot\vec{a} = 1$, $\vec{b}\cdot\vec{b} = 1$, $\vec{c}\cdot\vec{c} = 1$ unit vectors

$\Rightarrow (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) + (\vec{a}-\vec{c})\cdot(\vec{a}-\vec{c}) = 8$

$\Rightarrow \vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{b} - 2\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{a} + \vec{c}\cdot\vec{c} - 2\vec{a}\cdot\vec{c} = 8$

$\Rightarrow 1+1-2\vec{a}\cdot\vec{b}+1+1-2\vec{a}\cdot\vec{c} = 8$

$\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = -2$

$|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$.

$= (\vec{a}+2\vec{b})\cdot(\vec{a}+2\vec{b}) + (\vec{a}+2\vec{c})\cdot(\vec{a}+2\vec{c})$

$= \vec{a}\cdot\vec{a} + 4\vec{b}\cdot\vec{b} + 4\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{a} + 4\vec{c}\cdot\vec{c} + 4\vec{a}\cdot\vec{c}$.

$= 1 + 4 + 4\vec{a}\cdot\vec{b} + 1 + 4 + 4\vec{a}\cdot\vec{c}$

$= 10 + 4(\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c})$

$= 10 + 4(-2)$

$= 10 - 8 = 2$ ans.

22

Ques: 22. If $\lim_{x \to 1} \frac{x^n + x^2 + x^3 + \dots + x^n - n}{x-1} = 820$, $(n \in N)$

then the value of n is equal to ——

Ans. $\lim_{x \to 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x-1}$

$\because$ it is $\frac{0}{0}$ when $x = 1$

$\therefore$ using L' Hopital's Rule

$$\lim_{x \to 1} \frac{1 + 2x + 3x + \dots n x^{n-1}}{1} = 820$$

$$\Rightarrow 1 + 2 + 3 + \dots + n = 820$$

$$\Rightarrow \frac{n(n+1)}{2} = 820$$

$$\Rightarrow n^2 + n - 1640 = 0$$

$$\Rightarrow n^2 + 41n - 40n - 1640 = 0$$

$$(n + 41)(n - 40) = 0$$

$n = 40$ as $n \in N$

$\therefore$ ans.

23

Ques. 23 The number of integral values of k for which the line $3x + 4y = k$ intersects the circle $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ——

Ans. $x^2 + y^2 - 2x - 4y + 4 = 0$

$(x-1)^2 + (y-2)^2 = 1$

Center $(1, 2)$, radius $= 1$

If a line intersects the circle at two distinct points then its distance from the center will be less than radius

line : $3x + 4y - k = 0$

$$\left| \frac{3(1) + 4(2) - k}{\sqrt{3^2 + 4^2}} \right| < 1$$

$$\frac{|11 - k|}{5} < 1$$

$$-5 < k - 11 < 5$$

$$6 < k < 16$$

$k = 7, 8, 9, 10, 11, 12, 13, 14, 15$

number of integral values of k is 9 ans.

24

Ques. 24 If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without the meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ____

Ans. Rank of MOTHER

E	H	M	O	R	T
1	2	3	4	5	6

first will come the words starting with letter 'E' then 'H' and then starting with 'M'

E _ _ _ _ _ = $5!_0$ = 120

H _ _ _ _ _ = $5!_0$ = 120

M E _ _ _ _ = $4!_0$ = 24

M H _ _ _ _ = $4!_0$ = 24

M O E _ _ _ = $3!_0$ = 6

M O H _ _ _ = $3!_0$ = 6

M O R _ _ _ = $3!_0$ = 6

M O T E _ _ = $2!_0$ = 2

M O T H E R = 1 = 1

309

Sum = 309

∴ position of the word 'MOTHER' is 309 Ans.

25

Ques 25 The integral $\int_0^2 ||x-1|-x|\,dx$ is equal to —

Ans. $\int_0^2 ||x-1|-x|\,dx$

$= \int_0^1 |1-x-x|\,dx + \int_1^2 |x-1-x|\,dx$

$= \int_0^{1/2} (1-2x)\,dx + \int_{1/2}^1 (2x-1)\,dx + \int_1^2 dx$

$= \left(x - x^2\right)_0^{1/2} + \left(x^2 - x\right)_{1/2}^1 + \left(x\right)_1^2$

$= \left(\frac{1}{2} - \frac{1}{4}\right) + \left(1 - 1 - \left(\frac{1}{4} - \frac{1}{2}\right)\right) + (2-1)$

$= \frac{1}{4} + \frac{1}{4} + 1$

$= \frac{3}{2}$ ans.

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