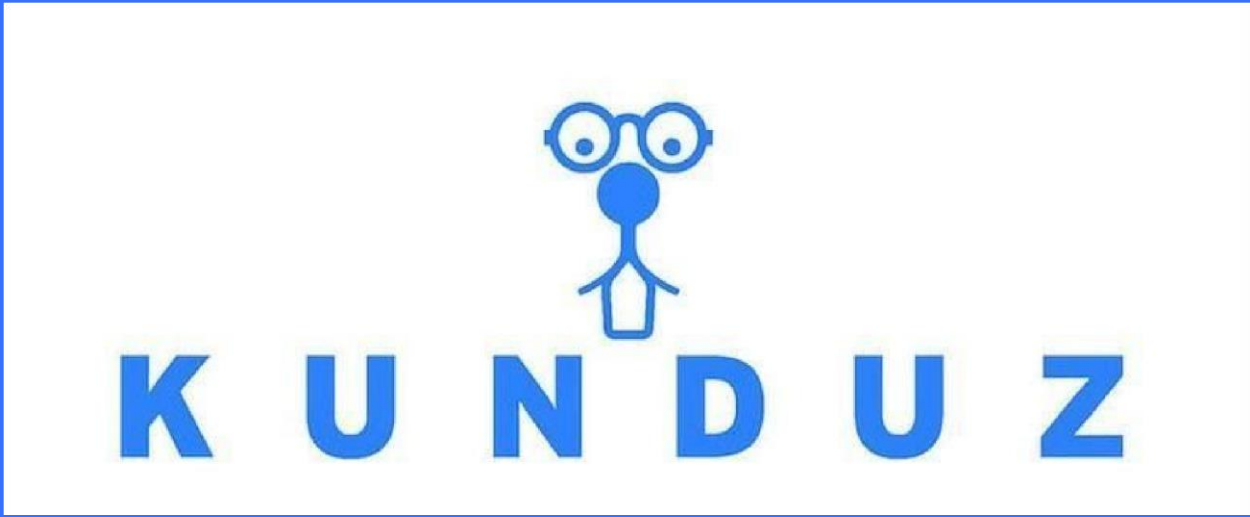


JEE Mains 2nd September 2020 shift 2 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

Sol. 01 If sum of seriese $(x+ka)+(x^2+(k-2)a)+(x^3+(k-4)a)$ $+ \cdots +$ 9 terms is $\frac{x^{10}-x-45a(x-1)}{x-1}$ then

Value of k is

Ans. $(x+ka)+(x^2+(k-2)a)+(x^3+(k-4)a)+\cdots$ + 9 terms

$= \left(x+x^2+x^3+\cdots+x^9\right)+\left(ka+(k-2)a+(k-4)a+\cdots+(k-16)a\right)$

$= \left(x+x^2+x^3+\cdots+x^9\right)+\left(ka+(ka-2a)+(ka-4a)+\cdots\right.$

$\left.+(ka-16a)\right)$

↓ G.P. with $a=x$, $r=x$

↑ A.P with $a=ka$, $d=-2a$

Sum of a G.P. $=\frac{a(r^n-1)}{r-1}$

Sum of an A.P $=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \frac{x(x^9-1)}{x-1}+\frac{9}{2}[2ka+(9-1)(-2a)]$

$\Rightarrow \frac{x^{10}-x}{x-1}+(9ka-72a)=\frac{x^{10}-x-45a(x-1)}{x-1}$

$\Rightarrow \frac{x^{10}-x+(9ka-72a)(x-1)}{(x-1)}=\frac{x^{10}-x-45a(x-1)}{(x-1)}$

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$\Rightarrow \quad 9ka - 72a = -45a$

$\Rightarrow \quad 9ka = 72a - 45a = 27a$

$\Rightarrow \quad k = \frac{27a}{9a} = \frac{27a}{9a}$

$k = 3$ ans.

2

Q. 12 Find the imaginary part of $\left(\left(3 + 2\sqrt{-54} \right)^{1/2} - \left(3 - 2\sqrt{-54} \right)^{1/2} \right)$

(1) $-\sqrt{6}$ (2) $-2\sqrt{6}$ (3) $\sqrt{6}$ (4) 6

Ans. $\{ (a \pm b)^2 = a^2 + b^2 \pm 2ab, \quad \sqrt{-1} = i, \quad i^2 = -1 \}$

$$\left(3 + 2\sqrt{-54} \right)^{1/2} = \sqrt{3 + 2 \times 3 \times \sqrt{6} \times i}$$

$$= \sqrt{9 - 6 + 2 \times 3 \times \sqrt{6}\, i}$$

$$= \sqrt{(3)^2 + (\sqrt{6}i)^2 + 2 \times 3 \times \sqrt{6}\, i}$$

$$= \sqrt{(3 + \sqrt{6}i)^2}$$

$$= \pm (3 + \sqrt{6}i)$$

$$\left(3 - 2\sqrt{-54} \right)^{1/2} = \sqrt{3 - 2 \times 3 \times \sqrt{6}\, i}$$

$$= \sqrt{9 - 6 - 2 \times 3 \times \sqrt{6}\, i}$$

$$= \sqrt{(3)^2 + (\sqrt{6}i)^2 - 2 \times 3 \times \sqrt{6}\, i}$$

$$= \sqrt{(3 - \sqrt{6}i)^2}$$

$$= \pm (3 - \sqrt{6}i)$$

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$$\left((3 + 2\sqrt{-54})^{1/2} - (3 - 2\sqrt{-54})^{1/2} \right)$$

$$= \left[\pm (3 + \sqrt{6}\, i) \right] - \left[\pm (3 - \sqrt{6}\, i) \right]$$

$$= \pm 2\sqrt{6}\, i \quad \text{and} \quad = \pm 6$$

$\therefore$ imaginary part is $\pm 2\sqrt{6}$ ans. option (2)

3

Ques 03 A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, −2, 2 and 2, 3, −1 resp. If this plane also passes through the point (α, −3, 5), then α is equal to

(1) 5 (2) 10 (3) −5 (4) −10

Ans. direction ratios of lines are 1, −2, 2 and 2, 3, −1

normal to the plane

$$\vec{\eta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix}$$

$$\vec{\eta} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$$\vec{\eta} = -4\hat{i} + 5\hat{j} + 7\hat{k}$$

Plane passes through $\vec{P} = 3\hat{i} + \hat{j} + \hat{k}$

equation of the plane

$$\vec{x} \cdot \vec{\eta} = \vec{\eta} \cdot \vec{P}$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (-4\hat{i} + 5\hat{j} + 7\hat{k}) = (-4\hat{i} + 5\hat{j} + 7\hat{k}) \cdot (3\hat{i} + \hat{j} + \hat{k})$$

$$-4x + 5y + 7z = -12 + 5 + 7$$

$$4x - 5y - 7z = 0$$

$(\alpha, -3, 5)$ lies on the plane

$$4\alpha - 5(-3) - 7(5) = 0 \Rightarrow 4\alpha + 15 - 35 = 0$$

$$4\alpha = 20 \Rightarrow \alpha = 5 \quad \text{ans.}$$

4

Q 04 If $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ be a curve then find equation of normal at $x = 0$

(1.) $x + 4y = 8$ (2) $x + 4y = 2$ (3) $2x + y = 2$

(4) $2x - y = 2$

Ans.

at $x = 0$, $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$

$y = (1+0)^{2y} + \cos^2(\sin^{-1}(0))$

$y = 1^{2y} + \cos^2(0)$ $\quad \{ \sin^{-1}(0) = 0, \ \cos 0 = 1$

$y = 1 + 1$

$y = 2$

at $x = 0$, $y = 2$

$\sin^{-1}x = \theta \Rightarrow x = \sin\theta$

$\cos\theta = \frac{\sqrt{1-x^2}}{1} \Rightarrow \theta = \cos^{-1}(\sqrt{1-x^2})$



$\because \cos(\cos^{-1}p) = p$

$\therefore \cos(\cos^{-1}(\sqrt{1-x^2})) = \sqrt{1-x^2}$

$(1+x)^{2y} = e^{\ln(1+x)^{2y}}$ $\quad \{ \ln A^B = B \ln A$

$(1+x)^{2y} = e^{2y\ln(1+x)}$

$(1+x)^{2y} = e^{2y} e^{\ln(1+x)}$

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$$\cos^2(\sin^{-1}x) = \left(\cos\left(\cos^{-1}\left(\sqrt{1-x^2}\right)\right)\right)^2$$

$$\cos^2(\sin^{-1}x) = \left(\sqrt{1-x^2}\right)^2$$

$$\cos^2(\sin^{-1}x) = 1-x^2$$

$$y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$$

$$y = e^{2y\ln(1+x)} + 1-x^2$$

differentiate both the sides $\left\{\ln 1 = 0\right.$

$$\frac{dy}{dx} = e^{2y\ln(1+x)}\cdot\left(\frac{2y}{1+x} + \ln(1+x)\cdot 2\frac{dy}{dx}\right) - 2x$$

at $x=0,\ y=2,\ \frac{dy}{dx} = ?$

$$\frac{dy}{dx} = e^{2\cdot 2\ln(1+0)}\left(\frac{2\cdot 2}{1+0} + \ln(1+0)\cdot 2\frac{dy}{dx}\right) - 2(0)$$

$$\frac{dy}{dx} = e^{0}\left(4 + 0\cdot 2\cdot\frac{dy}{dx}\right) - 0$$

$$\frac{dy}{dx} = 4 \qquad \text{slope of tangent}$$

slope of normal is given by $-\frac{1}{dy/dx} = -\frac{1}{4}$

equation of normal at $x=0,\ y=2$

$$y-2 = -\frac{1}{4}(x-0) \Rightarrow 4y-8 = -x$$

$$\Rightarrow x+4y = 8 \quad \text{ans. option (1)}$$

5

Q. 05 $\lim_{x \to 0} \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x} = ?$

(1) e (2) e^2 (3) e^4 (4) $\frac{1}{e}$

Ans. Let $y = \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x}$

$$\ln y = \ln\left(\left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x}\right)$$

$$\ln y = \frac{1}{x} \ln\left(\tan\left(\frac{\pi}{4} + x\right)\right)$$

$$\ln y = \lim_{x \to 0} \frac{\ln\left(\tan\left(\frac{\pi}{4} + x\right)\right)}{x}$$

when we put $x = 0$, it is $\frac{0}{0}$ form

then differentiate numerator and denominator

$$\ln y = \lim_{x \to 0} \frac{\frac{1}{\tan\left(\frac{\pi}{4} + x\right)} \cdot \sec^2\left(\frac{\pi}{4} + x\right)}{1}$$

$$\ln y = \lim_{x \to 0} \frac{\sec^2\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} + x\right)}$$

now put $x = 0$

$$\ln y = \frac{\sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\pi/4\right)} = \frac{\left(\sqrt{2}\right)^2}{1} = 2$$

$y = e^2$ ans. option (2)

6

Q.6 If $x^2 - y^2\sec^2\theta = 10$ be a hyperbola and $x^2\sec^2\theta + y^2 = 5$ be an ellipse such that the eccentricity of hyperbola $= \sqrt{5}$ eccentricity of ellipse then find the length of latus rectum of ellipse

(1) $\frac{4\sqrt{5}}{3}$ (2) $\frac{4}{3\sqrt{5}}$ (3) $\frac{20\sqrt{5}}{3}$ (4) $\sqrt{30}$

Ans If equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

then eccentricity $e_H = \sqrt{\frac{a^2+b^2}{a^2}}$

If equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

then eccentricity $e_E = \sqrt{\frac{a^2-b^2}{a^2}}$ for $a > b$

hyperbola $x^2 - y^2\sec^2\theta = 10$

$\Rightarrow \frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$

$e_H = \sqrt{\frac{(10)^2 + (10\cos^2\theta)}{10}}$

$e_H = \sqrt{1 + \cos^2\theta}$

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Ellipse $x^2 \sec^2\theta + y^2 = 5$

$\Rightarrow \dfrac{x^2}{5\cos^2\theta} + \dfrac{y^2}{5} = 1$

$\because 5 \geq 5\cos^2\theta$

$\therefore a^2 = 5$

$\therefore b^2 = 5\cos^2\theta$

$e_E = \sqrt{\dfrac{5 - 5\cos^2\theta}{5}} = \sqrt{1 - \cos^2\theta}$

$e_E = \sin\theta$

$e_H = \sqrt{5}\, e_E$

$\Rightarrow \sqrt{1 + \cos^2\theta} = \sqrt{5}\sin\theta$

$\Rightarrow 1 + \cos^2\theta = 5\sin^2\theta \Rightarrow 1 + \cos^2\theta = 5 - 5\cos^2\theta$

$\Rightarrow 6\cos^2\theta = 4 \Rightarrow \cos^2\theta = 4/6$

$\cos^2\theta = 2/3$

Length of latus rectum of an Ellipse is given by $= \dfrac{2b^2}{a}$

$\{ b^2 = 5\cos^2\theta , \quad a = \sqrt{5} \}$

Length of latus rectum $= \dfrac{2 \times 5\cos^2\theta}{\sqrt{5}} = \dfrac{10 \times 2/3}{\sqrt{5}}$

$= \dfrac{4\sqrt{5}}{3}$ ans. option (1)

7

Ques 07 Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \to q$ (2) $(q \to p) \vee \sim(p \to q)$

(3) $(\sim q) \vee (p \wedge q) \to q$ (4) $(p \to q) \wedge (q \to p)$

Ans. Option (1) ans.

(1) $\sim p \wedge (p \vee q) \to q$

$= (\sim p \wedge p) \vee (\sim p \wedge q) \to q$

$= c \vee (\sim p \wedge q) \to q$

$= (\sim p \wedge q) \to q$

$= \sim(\sim p \wedge q) \vee q$

$= (p \vee \sim q) \vee q \quad = p \vee t \quad = t$

(3) $(\sim q) \vee (p \wedge q) \to q$

$= (\sim q \vee p) \wedge (\sim q \vee q) \to q$

$= (\sim q \vee p) \to q$

$= (q \wedge \sim p) \vee q$

$= q$

use $\sim(p \to q)$

$= p \wedge \sim q$

$\Rightarrow p \to q = \sim p \vee q$

for option (2) and (4)

p	q	$p \to q$	$q \to p$	$\sim(p \to q)$	$(p \to q) \wedge (q \to p)$	$(q \to p) \vee \sim(p \to q)$
T	T	T	T	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

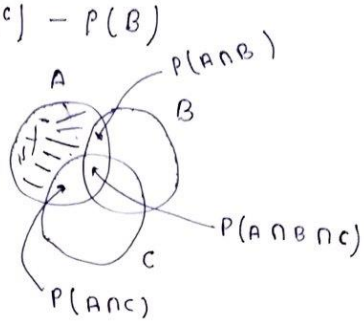
8

Sol 08 If A, B, C are three pairwise independent events such that $P(A \cap B \cap C) = 0$ then $P((B^c \cap C^c)/A)$ is equal to –

(1) $P(C) + P(B)$ (2) $P(C^c) + P(B)$

(3) $P(B^c) = P(C)$ (4) $P(C^c) - P(B)$

Ans. {shaded part is $P(A \cap (B^c \cap C^c))$}



$$P\left(\frac{B^c \cap C^c}{A}\right) = \frac{P(A \cap (B^c \cap C^c))}{P(A)}$$

$$P\left(\frac{B^c \cap C^c}{A}\right) = \frac{P(A) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)]}{P(A)}$$

$P(A \cap B \cap C) = 0$ -- given

$P(A \cap B) = P(A) \cdot P(B)$ -- Pairwise independent

$P(A \cap C) = P(A) \cdot P(C)$ -- Pairwise independent

$$P\left(\frac{B^c \cap C^c}{A}\right) = \frac{P(A) - [P(A) \cdot P(B) + P(A) \cdot P(C) - 0]}{P(A)}$$

$$= 1 - P(B) - P(C)$$

$$= P(C^c) - P(B)$$

$1 = P(B) + P(B^c)$

$1 = P(C) + P(C^c)$

ans. option (4)

9

Ques. 09 Let $A = \{ x = (x, y, z)^T : Px = 0 \text{ and } x^2 + y^2 + z^2 = 1 \}$, where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A :

(1) is a Singleton (2) Contains more than two elements

(3) Contains exactly two elements (4) is an empty set.

Ans. $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$

$|P| = 1(-3+36) - 2(2+4) + 1(-18-3)$

$|P| = 33 - 12 - 21 = 0$

$\therefore Px = 0$ has infinite solution

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 2y + z = 0$

$-2x + 3y - 4z = 0$

$x + 9y - z = 0$

Put $z = \lambda$

$x + 2y = -\lambda$

$-x + 9y = -\lambda$

$-7y = -2\lambda \Rightarrow y = \frac{2\lambda}{7}$

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$x + 2y = -\lambda$

$x + \frac{4\lambda}{7} = -\lambda \Rightarrow x = -\frac{11}{7}\lambda$

$(x, y, z) \equiv \left(\frac{-11\lambda}{7}, \frac{2\lambda}{7}, \lambda\right)$

$x^2 + y^2 + z^2 = 1$ ---- given

$\Rightarrow \lambda^2\left(\frac{121}{49} + \frac{4}{49} + 1\right) = 1$

$\Rightarrow \lambda^2\left(\frac{174}{49}\right) = 1$

$\Rightarrow \lambda^2 = \frac{49}{174}$

$\lambda = \pm \frac{7}{\sqrt{174}}$

$\therefore$ two ~~sets~~ solutions only <u>as</u> option (3)

10

Q10. If a curve $y = f(x)$ satisfy the differential equation $2x^2\,dy = (2xy + y^2)\,dx$ and passes $(1, 1)$ then find $f(1/2)$

(1) $\frac{1}{1+\ln 2}$ (2) $\frac{1}{1-\ln 2}$ (3) $\frac{2}{1-\ln 2}$ (4) $\frac{2}{1+\ln 2}$

Ans. $2x^2\,dy = (2xy + y^2)\,dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} = \frac{y}{x} + \frac{y^2}{2x^2}$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{1}{y}\frac{1}{x} + \frac{1}{2x^2}$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\frac{1}{x} = \frac{1}{2x^2}$$

Put $-\frac{1}{y} = t$ differentiate both the sides with respect to x

$$\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + t\cdot\frac{1}{x} = \frac{1}{2x^2}$$

$\uparrow P(x)$ $\uparrow Q(x)$

this is a linear differential equation

Integrating Factor $= e^{\int P(x)\,dx}$

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$\Rightarrow$ I.F $= e^{\int \frac{1}{x} dx} = e^{\ln x}$

$\Rightarrow$ I.F. $= x$

$\Rightarrow t\,(I.F.) = \int (I.F.)\, Q(x)\, dx + c$

$\Rightarrow t\,(x) = \int x \cdot \frac{1}{2x^2}\, dx + c$

$\Rightarrow t \cdot x = \int \frac{1}{2} \cdot \frac{1}{x}\, dx + c$

$\Rightarrow t \cdot x = \frac{1}{2} \cdot \ln x + c$

But $t = -1/y$

$\Rightarrow -\frac{x}{y} = \frac{\ln x}{2} + c$

the curve passes through $(1, 2)$

But $x = 1$ and $y = 2$

$-\frac{1}{2} = \frac{\ln 1}{2} + c$ $\quad \{ \ln 1 = 0$

$-\frac{1}{2} = c$

$\Rightarrow -\frac{x}{y} = \frac{\ln x}{2} - \frac{1}{2}$

$\Rightarrow -\frac{x}{y} = \frac{\ln x - 1}{2} \Rightarrow y = \frac{-2x}{\ln x - 1}$

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$$Y = f(x) = \frac{-2x}{\ln x - 1} = \frac{2x}{1 - \ln x}$$

$$f\left(\frac{1}{2}\right) = \frac{2\left(\frac{1}{2}\right)}{1 - \ln \frac{1}{2}}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{1 - (\ln 1 - \ln 2)} \qquad \left\{ \ln 1 = 0 \right.$$

$$f\left(\frac{1}{2}\right) = \frac{1}{1 + \ln 2}$$ ans. option (1)

11

Q.11 If $a, b, c \in R$ such that $a^3+b^3+c^3 = 2$ and

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$. then find abc

(1) $\frac{2}{3}$ (2) $-\frac{2}{3}$ (3) $\frac{1}{3}$ (4) 1

Ans.

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$\Rightarrow a(cb-a^2) - b(b^2-ca) + c(ab-c^2) = 0$

$\Rightarrow abc - a^3 - b^3 + abc + abc - c^3 = 0$

$\Rightarrow 3abc = a^3+b^3+c^3$

$\Rightarrow abc = \frac{a^3+b^3+c^3}{3}$ $\quad \left\{ a^3+b^3+c^3 = 2 \text{ --- given} \right.$

$\Rightarrow abc = \frac{2}{3}$ ans. option (1)

12

Q 12 If $\sin^4\theta + \cos^4\theta + \lambda = 0$ has a real solution then range of λ is

(1) $[-1, 1]$ (2) $[-1, -1/2]$ (3) $[1/2, 1]$ (4) $[-\frac{1}{2}, \frac{1}{2}]$

Ans. $\sin^4\theta + \cos^4\theta + \lambda = 0$

$\Rightarrow (\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta + \lambda = 0$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^2 - \frac{2}{2} \cdot 2\sin^2\theta\cos^2\theta + \lambda = 0$

$\Rightarrow 1 - \frac{4\sin^2\theta\cos^2\theta}{2} + \lambda = 0$

$\Rightarrow 1 - \frac{(2\sin\theta\cos\theta)^2}{2} + \lambda = 0$

$\Rightarrow 1 - \frac{(\sin 2\theta)^2}{2} + \lambda = 0$

$\Rightarrow \lambda = \frac{(\sin 2\theta)^2}{2} - 1$

$-1 \le \sin 2\theta \le 1$

$\Rightarrow 0 \le (\sin 2\theta)^2 \le 1$

$\Rightarrow \frac{0}{2} \le \frac{(\sin 2\theta)^2}{2} \le \frac{1}{2}$

$\Rightarrow 0 - 1 \le \frac{(\sin 2\theta)^2}{2} - 1 \le \frac{1}{2} - 1$

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$$\Rightarrow \quad -1 \leq \frac{(\sin 20)^2}{2} - 1 \leq -1/2$$

$$\Rightarrow \quad -1 \leq \lambda \leq -1/2$$

$$\therefore \quad \lambda \in [-1, -1/2] \quad \underline{\text{ans}}. \text{ option (2)}$$

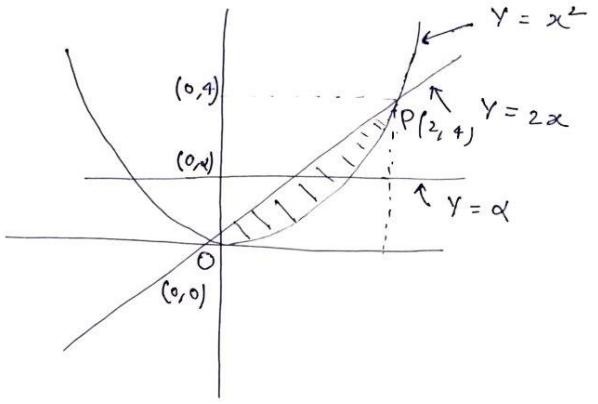
13

Q 13 If $Y=\alpha$ bisects the area bounded by region given by $x^2 \leq Y \leq 2x$ then :

(1) $8\alpha^{3/2} - 3\alpha^2 = 8$ (2) $8\alpha^{3/2} - 3\alpha^2 = 4$

(3) $4\alpha^{3/2} - 3\alpha^2 = 8$ (4) $4\alpha^{3/2} + 3\alpha^2 = 8$

Ans.



$x^2 \leq Y \leq 2x$

Shaded area

$Y = x^2$

$Y = 2x$

for Point of intersection

$x^2 = 2x \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0$

$x = 0$

$Y = 0$

$O\ (0,0)$

$x = 2$

$Y = x^2 = (2)^2 = 4$

$Y = 4$

$P\ (2,4)$

$$\int_0^{\alpha} \left(\sqrt{Y} - \frac{Y}{2}\right) dy = \int_{\alpha}^{4} \left(\sqrt{Y} - \frac{Y}{2}\right) dy$$ Given that $Y=\alpha$ bisect the shaded region

$$\left\{ \begin{array}{lll} Y = x^2 & \Rightarrow & x = \sqrt{Y} \\ Y = 2x & \Rightarrow & x = Y/2 \end{array} \right\}$$

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$$\Rightarrow \int_0^{\alpha} \left(\sqrt{y} - \frac{y}{2}\right) dy = \int_{\alpha}^{4} \left(\sqrt{y} - \frac{y}{2}\right) dy$$

$$\Rightarrow \left(\frac{2}{3} y^{3/2} - \frac{y^2}{4}\right)_0^{\alpha} = \left(\frac{2}{3} y^{3/2} - \frac{y^2}{4}\right)_{\alpha}^{4}$$

$$\Rightarrow \left[\left(\frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4}\right) - \left(\frac{2}{3}(0)^{3/2} - \frac{(0)^2}{4}\right)\right]$$

$$= \left[\left(\frac{2}{3}(4)^{3/2} - \frac{(4)^2}{4}\right) - \left(\frac{2}{3}(\alpha)^{3/2} - \frac{\alpha^2}{4}\right)\right]$$

$$\Rightarrow \frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4} = \frac{16}{3} - 4 - \frac{2}{3}\alpha^{3/2} + \frac{\alpha^2}{4}$$

$$\Rightarrow \frac{4}{3}\alpha^{3/2} - \frac{\alpha^2}{2} = \frac{4}{3}$$

$$\Rightarrow 4\alpha^{3/2} - \frac{3}{2}\alpha^2 = 4$$

$$\Rightarrow 8\alpha^{3/2} - 3\alpha^2 = 8 \quad \underline{\text{ans.}} \quad \text{option (1)}$$

14

Ques. 14 The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is

(1) $\left(0, \frac{\pi}{2}\right)$ (2) $\left(0, \frac{\pi}{4}\right)$ (3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{3\pi}{4}\right)$

Ans. $x + y - 1 = 0$

Sign of $(1, 2)$ wrt this line

$1 + 2 - 1 > 0$

Sign of $(\sin\theta, \cos\theta)$ will also be positive

$\sin\theta + \cos\theta - 1 > 0$

$\therefore \quad \sin\theta + \cos\theta > +1$

$$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right) > 1$$

$$\Rightarrow \left(\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$$

$$0 < \theta < \frac{\pi}{2}$$

$\theta \in \left(0, \frac{\pi}{2}\right)$ ans. option (1)

15

Q 15 / 15 If $f(x)$ be a quadratic polynomial such that $f(x) = 0$ has a root 3 and $f(2) + f(-1) = 0$ then other root lies in

(1) $(-1, 0)$ (2) $(0, 1)$ (3) $(-2, 1)$ (4) $(1, 2)$

Ans. let the roots of $f(x)$ be α and β

$$\alpha = 3$$

$$f(x) \equiv x^2 - (\alpha + \beta)x + \alpha\beta$$

$f(x) = 0$, has one root 3

$$f(x) = x^2 - (3 + \beta)x + 3\beta = 0$$

$$f(2) + f(-1) = 0$$

$$(2)^2 - (3 + \beta)(2) + 3\beta + (-1)^2 - (3 + \beta)(-1) + 3\beta = 0$$

$$\Rightarrow \quad 4 - 6 - 2\beta + 3\beta + 1 + 3 + \beta + 3\beta = 0$$

$$\Rightarrow \quad 5\beta + 2 = 0$$

$$\beta = -\frac{2}{5} = -0.4$$

$\therefore \quad \beta \in (-1, 0)$ ans. option (1)

16

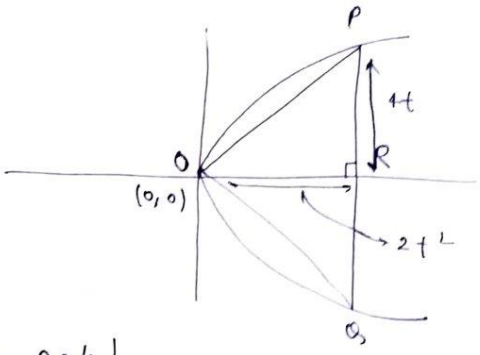
Q.16 An equilateral triangle is ~~inscribed~~ inscribed in a Parabola $Y^2 = 8x$ whose one vertex coincide with vertex of Parabola. Find the area of triangle.

(1) $196\sqrt{3}$ (2) $194\sqrt{3}$ (3) $192\sqrt{3}$ (4) $190\sqrt{3}$

Ans. $Y^2 = 4ax$

$Y^2 = 8x$

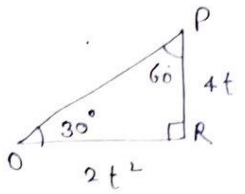
$4a = 8 \Rightarrow a = 2$



any point of parabola is given by $P(at^2, 2at)$

$P(2t^2, 4t)$

$\triangle OPQ$ is an equilateral triangle



$\therefore \angle POQ = 60^\circ$

$\therefore \angle OPQ = 60^\circ$

$\tan 30^\circ = \frac{4t}{2t^2}$

$\frac{1}{\sqrt{3}} = \frac{2}{t} \Rightarrow t = 2\sqrt{3}$

$OR = 2t^2 = 2(2\sqrt{3})^2 = 24$

$PR = 4t = 4(2\sqrt{3}) = 8\sqrt{3}$

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area of $\triangle OPQ$ $= 2 \times$ area of $\triangle OPR$

$= 2 \times \frac{1}{2} \times OR \times PR$

$= 2 \times \frac{1}{2} \times 24 \times 8\sqrt{3}$

$= 192\sqrt{3}$ ans. option (3)

area of $\triangle OPQ$

17

Ques 17 Let $f: R \to R$ be a function which satisfies $f(x+y) = f(x) + f(y)$ $\forall\ x, y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{n-1} f(k)$, $n \in N$, then the value of n for which $g(n) = 20$, is

(1) 9 (2) 20 (3) 5 (4) 4

Ans. $f(x+y) = f(x) + f(y)$

$f(x) = \lambda x$, $f(1) = 2$ ... given

$f(1) = \lambda = 2$

$f(x) = 2x$ $\qquad f(k) = 2k$

$$g(n) = \sum_{k=1}^{n-1} f(k) = \sum_{k=1}^{n-1} 2k = 2\sum_{k=1}^{n-1} k$$

$$g(n) = 2\left(1 + 2 + 3 + \dots + n-1\right)$$

$$g(n) = 2 \times \frac{(n-1)(n-1+1)}{2}$$

$g(n) = n(n-1)$, ... $g(n) = 20$ ... given

$n(n-1) = 20$

$\Rightarrow n^2 - n - 20 = 0$

$\Rightarrow n^2 - 5n + 4n - 20 = 0$

$(n-5)(n+4) = 0$

$n = 5, -4$

$n = 5$ as $n \in N$ ans: option (3)

18

Ques. 18 If the sum of first 11 terms of an A.P. $a_1, a_2, a_3 \ldots$ is 0 $(a_1 \neq 0)$, then the sum of A.P., $a_1, a_3, a_5, \ldots a_{23}$ is ka_1, where k is equal to :

(1) $\frac{121}{10}$ (2) $-\frac{121}{10}$ (3) $-\frac{72}{5}$ (4) $\frac{72}{5}$

Ans. let the first term $a_1 = a$ and common diff $= d$

$$S_{11} = 0 \Rightarrow \frac{11}{2}[2a + 10d] = 0$$

$$a = -5d$$

$a_1, a_3, a_5 \ldots, a_{23}$ form another A.P.

number of terms $= 12$

first term $a_1 = a$

Common difference $= 2d$

$$S_{12} = \frac{12}{2}(2a + 11(2d)) = ka_1 \quad \{a = -5d, \; a_1 = a\}$$

$$\Rightarrow \frac{12}{2}\left(2a + 22\left(-\frac{9}{5}\right)\right) = ka$$

$$\Rightarrow 12a - \frac{132}{5}a = ka$$

$$\Rightarrow a\left(\frac{60 - 132}{5}\right) = ka$$

$$\Rightarrow k = -\frac{72}{5}$$ ans option (3)

19

Q 19 / 19 There are n stations in a circular path. Two consecutive stations are connected by blue line and two non-consecutive stations are connected by red line. If no. of red lines is equal to 99 times number of blue line then value of n is.

(1) 201 (2) 200 (3) 199 (4) 202

Ans total number of stations = n

∴ then two consecutive stations = n

two non consecutive stations = ${}^{n}C_{2} - n$

$${}^{n}C_{2} - n = 99n$$

$$\Rightarrow \frac{n.(n-1)}{1.2} - n = 99n$$

$$\Rightarrow \frac{n^2 - n}{2} = 100n$$

$$\Rightarrow n^2 - n = 200n$$

$$\Rightarrow n^2 - 201n = 0$$

$$\Rightarrow n(n - 201) = 0$$

$$n - 201 = 0$$

$$n = 201 \quad \text{ans.} \quad \text{option (1)}$$

20

Q20 If $f(x) = \frac{\ln(1+x)}{x}$, $x \in (-1, \infty)$ and $f(0) = 1$

then $f(x)$ is

(1) decreasing in $(-1, 0)$ and increasing in $(0, \infty)$

(2) always increasing

(3) always decreasing

(4) increasing in $(-1, 0)$ and decreasing in $(0, \infty)$

Ans. If a function f is continuous and differentiable in an interval (a, b) then

If $f'(x) > 0$, then $f(x)$ is increasing

If $f'(x) < 0$, then $f(x)$ is decreasing

$$f(x) = \frac{\ln(1+x)}{x}, \quad x \in (-1, \infty)$$

$$f'(x) = \frac{\frac{x}{1+x} - \ln(1+x)}{x^2}$$

$$f'(x) = \frac{x - (1+x)\ln(1+x)}{x^2(1+x)} \quad \begin{matrix} \leftarrow P(x) \\ \leftarrow Q(x) \end{matrix}$$

$Q(x) = x^2(1+x)$ is always positive for all values of x as $x \in (-1, \infty)$

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$$P(x) = x - (1+x)\ln(1+x)$$

$$P'(x) = 1 - \ln(1+x) - \frac{1+x}{1+x}$$

$$P'(x) = -\ln(1+x)$$

$$P''(x) = -\frac{1}{1+x} < 0 \quad \text{for} \quad x \in (-1, \infty)$$

$\therefore$ $P(x)$ will have a maximum values

$\therefore$ $P'(x) = 0 \Rightarrow P'(x) = -\ln(1+x) = 0$

$\Rightarrow x = 0$

at $x = 0$, $P(x)$ will have max. value

$$P(x) = x - (1+x)\ln(1+x)$$

$$P(0)_{max} = 0 - (1+0)\ln(1+0) \qquad \{\ln 1 = 0\}$$

$$P(0)_{max} = 0$$

$\therefore$ $P(x) < 0$ for $x \in (-1, \infty)$

$\therefore$ $f'(x) = \frac{P(x)}{q(x)} < 0$ as $P(x) < 0$, $q(x) > 0$

$\therefore$ $f'(x)$ is always decreasing for $x \in (-1, \infty)$

ans. option (3)

21

Q#21] Point P divides line joining $A(\hat{i}+\hat{j}+\hat{k})$ and $B(2\hat{i}+\hat{j}+3\hat{k})$ in the ratio $\lambda:1$ such that

$$\vec{OB}\cdot\vec{OP} - 3|\vec{OA}\times\vec{OP}|^2 = 6.$$ Find λ

Ans.

$$\vec{OA} = \vec{a} = \hat{i}+\hat{j}+\hat{k}$$

$$\vec{OB} = \vec{b} = 2\hat{i}+\hat{j}+3\hat{k}$$

$$\vec{a}\cdot\vec{b} = (\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}+\hat{j}+3\hat{k})$$

$$\vec{a}\cdot\vec{b} = \hat{i}\cdot 2\hat{i} + \hat{j}\cdot\hat{j} + \hat{k}\cdot 3\hat{k}$$

$$\vec{a}\cdot\vec{b} = 2+1+3 = 6$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

$$\vec{a}\times\vec{b} = \hat{i}(3-1) - \hat{j}(3-2) + \hat{k}(1-2)$$

$$\vec{a}\times\vec{b} = 2\hat{i}-\hat{j}-\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{(2)^2+(-1)^2+(-1)^2} = \sqrt{4+1+1}$$

$$|\vec{a}\times\vec{b}| = \sqrt{6}$$

$$|\vec{a}\times\vec{b}|^2 = 6$$

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$\vec{OP} = \vec{p}$, Point P divides line joining A and B in $\lambda : 1$

then $\vec{OP} = \vec{p} = \dfrac{\vec{a} + \lambda \vec{b}}{\lambda + 1}$

$$\vec{OB} \cdot \vec{OP} - 3 \left| \vec{OA} \times \vec{OP} \right|^2 = 6$$

$$\vec{b} \cdot \left(\frac{\vec{a} + \lambda \vec{b}}{\lambda + 1} \right) - 3 \left| \vec{a} \times \left(\frac{\vec{a} + \lambda \vec{b}}{\lambda + 1} \right) \right|^2 = 6$$

$$\left\{ \begin{array}{l} \vec{b} \cdot \vec{b} = (2\hat{i} + \hat{j} + 3\hat{k}) \cdot (2\hat{i} + \hat{j} + 3\hat{k}) \\ \vec{b} \cdot \vec{b} = 4 + 1 + 9 = 14 \\ \vec{a} \times \vec{a} = 0 \ , \ \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \end{array} \right\}$$

$$\Rightarrow \frac{\vec{b} \cdot \vec{a} + \lambda \vec{b} \cdot \vec{b}}{\lambda + 1} - 3 \left| \frac{\vec{a} \times \vec{a} + \lambda \vec{a} \times \vec{b}}{\lambda + 1} \right|^2 = 6$$

$$\Rightarrow \frac{6 + \lambda \cdot 14}{\lambda + 1} - 3 \cdot \left| \frac{0 + \lambda [\vec{a} \times \vec{b}]}{\lambda + 1} \right|^2 = 6$$

$$\Rightarrow \frac{6 + 14\lambda}{\lambda + 1} - 3 \cdot \frac{\lambda^2}{(\lambda+1)^2} |\vec{a} \times \vec{b}|^2 = 6$$

$$\Rightarrow \frac{6 + 14\lambda}{\lambda + 1} - 3 \cdot \frac{\lambda^2}{(\lambda+1)^2} (6) = 6$$

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$$\Rightarrow (6+14\lambda)(\lambda+1) - 18\lambda^2 = 6(\lambda+1)^2$$

$$\Rightarrow 6\lambda + 6 + 14\lambda^2 + 14\lambda - 18\lambda^2 = 6\lambda^2 + 6 + 12\lambda$$

$$\Rightarrow -4\lambda^2 + 20\lambda + 6 = 6\lambda^2 + 6 + 12\lambda$$

$$\Rightarrow 10\lambda^2 - 8\lambda = 0$$

$$\Rightarrow \lambda(10\lambda - 8) = 0$$

$$10\lambda - 8 = 0$$

$$\lambda = \frac{8}{10}$$

$$\lambda = 0.8 \quad \text{ans.}$$

22

Q22 $\int_1^2 |2x - [3x]|\, dx = ?$ (where $[\cdot]$ denotes greatest integer function)

Ans. for any real number $n = [n] + \{n\}$

$\{x\}$ fractional function

$$I = \int_1^2 |2x - [3x]|\, dx$$

$$I = \int_1^2 |3x - x - [3x]|\, dx$$

$$I = \int_1^2 |3x - [3x] - x|\, dx$$

$$I = \int_1^2 |\{3x\} - x|\, dx$$

$$I = \int_1^2 (x - \{3x\})\, dx$$

$$I = \int_1^2 x\, dx - \int_1^2 \{3x\}\, dx$$

$\downarrow$ I_1 $\qquad$ $\downarrow$ I_2

$$I_1 = \int_1^2 x\, dx = \left[\frac{x^2}{2}\right]_1^2 = \left[\frac{2^2}{2} - \frac{1^2}{2}\right]$$

$$I_1 = \frac{3}{2}$$

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$$I_2 = \int_1^2 \{3x\}\, dx$$

$3x = t$ at $x = 1$, $t = 3$

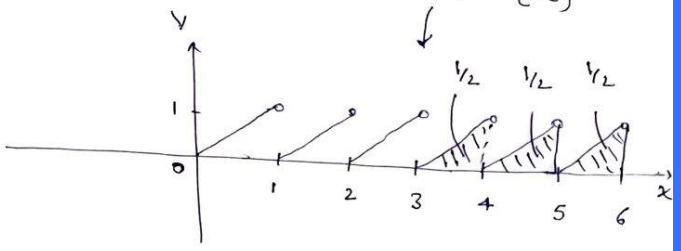
at $x = 2$, $t = 6$

$3\,dx = dt$

$dx = \frac{1}{3}\,dt$

$$I_2 = \frac{1}{3}\int_3^6 \{t\}\, dt$$

$Y = \{x\}$



$$I_2 = \frac{1}{3}\left[\int_3^4 \{t\}\, dt + \int_4^5 \{t\}\, dt + \int_5^6 \{t\}\, dt\right]$$

$$I_2 = \frac{1}{3}\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right] = \frac{1}{3}\left[\frac{3}{2}\right]$$

$$I_2 = \frac{1}{2}$$

$$I = I_1 - I_2$$

$$I = \frac{3}{2} - \frac{1}{2}$$

$I = 1$ ans.

$I = 1$ ans.

23

Q23 The ratio of three consecutive binomial coefficients in the expansion of $(1+x)^n$ is $2:5:12$, find n

(1) 120 (2) 34 (3) 118 (4) 35

Ans. Let the three consecutive binomial coefficient be

$${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$$

$$\left\{ {}^nC_r = \frac{n!}{(n-r)!\,r!} \right\} \quad \left\{ n! = n\times(n-1)\times\cdots\times 3\cdot 2\cdot 1 \right.$$

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2 : 5 : 12$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5}$$

$$\Rightarrow \frac{n! / (n-(r-1))!\times(r-1)!}{n! / (n-r)!\times r!} = \frac{2}{5}$$

$$\Rightarrow \frac{(n-r)!\times r!}{(n-r+1)!\,(r-1)!} = \frac{2}{5}$$

$$\Rightarrow \frac{(n-r)!\times r\times(r-1)!}{(n-r+1)(n-r)!\times(r-1)!} = \frac{2}{5}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{2}{5} \Rightarrow 5r = 2n-2r+2$$

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$$\Rightarrow \quad k = \frac{2n+2}{7} \quad \text{---- (1)}$$

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{5}{12}$$

$$\Rightarrow \quad \frac{n! / (n-r)! \times r!}{n! / (n-(r+1))! \times (r+1)!} = \frac{5}{12}$$

$$\Rightarrow \quad \frac{(n-r-1)! \times (r+1)!}{(n-r)! \times r!} = \frac{5}{12}$$

$$\Rightarrow \quad \frac{(n-r-1)! \times (r+1) \times r!}{(n-r) \times (n-r-1)! \times r!} = \frac{5}{12}$$

$$\Rightarrow \quad \frac{r+1}{n-r} = \frac{5}{12}$$

$$\Rightarrow \quad 12r + 12 = 5n - 5r$$

$$\Rightarrow \quad 17r = 5n - 12$$

$$\Rightarrow \quad r = \frac{5n-12}{17} \quad \text{---- (2)}$$

from eq. (1) and (2)

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$$\frac{2n+2}{7} = \frac{5n-12}{17}$$

$$\Rightarrow \quad 34n + 34 = 35n - 84$$

$$\Rightarrow \quad n = 118 \quad \text{ans.} \quad \text{option (3)}$$

24

Q 15. Ques. 24 If $Y = \sum_{k=1}^{6} k \cos^{-1}\left(\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right)$ then $\frac{dy}{dx} = ?$

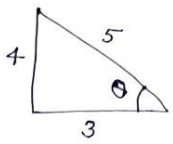
(1) 92 (2) 91 (3) 90 (4) 89

Ans. $\left\{ \cos^{-1}(\cos x) = x \right.$

$\left. \cos A \cos B - \sin A \sin B = \cos(A+B) \right\}$

Let

$\frac{3}{5} = \cos\theta$, $\frac{4}{5} = \sin\theta$



$$Y = \sum_{k=1}^{6} k \cos^{-1}\left(\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right)$$

$$Y = \sum_{k=1}^{6} k \cos^{-1}\left(\cos\theta \cos kx - \sin\theta \sin kx\right)$$

$$Y = \sum_{k=1}^{6} k \cos^{-1}\left(\cos(\theta + kx)\right)$$

$$Y = \sum_{k=1}^{6} k(\theta + kx)$$

$$Y = \sum_{k=1}^{6} (k\theta + k^2 x)$$

differentiate both the sides with respect x

$$\frac{dy}{dx} = \sum_{k=1}^{6} (0 + k^2)$$

$$\frac{dy}{dx} = \sum_{k=1}^{6} k^2$$

$$\frac{dy}{dx} = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$$

$$\frac{dy}{dx} = 1 + 4 + 9 + 16 + 25 + 36$$

$$\frac{dy}{dx} = 91 \quad \text{ans.} \quad \text{option (2)}$$

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25

Que 25 Let $a_1, a_2, a_3, \ldots a_{11}$ are in increasing A.P and if Variance of these numbers is 90 then Value of Common difference of A.P. is

Ans. Let the first term of the given A.P be a and Common difference be d

then

$a_1 = a, \quad a_2 = a+d, \quad a_3 = a+2d \ldots a_{11} = a+10d$

(mean) $\bar{X} = \dfrac{a_1 + a_2 + a_3 + \ldots + a_{11}}{11}$

$\bar{X} = \dfrac{a + a+d + a+2d + \ldots + a+10d}{11}$

$\bar{X} = \dfrac{11a + 55d}{11}$

$\bar{X} = a+5d$

Variance $(\sigma^2) = \dfrac{1}{n}\sum (x_i - \bar{x})^2 = 90$

$\Rightarrow \dfrac{1}{11}\sum_{i=1}^{11} (a_i - \bar{X})^2 = 90$

$\Rightarrow \dfrac{1}{11}\left((a_1 - \bar{X})^2 + (a_2 - \bar{X})^2 + \ldots (a_{11} - \bar{X})^2\right) = 90$

$\Rightarrow \dfrac{1}{11}\left((a - (a+5d))^2 + (a+d - (a+5d))^2 \ldots + \{a+10d - (a+5d)\}^2\right)$

$= 90$

$$\Rightarrow \frac{1}{11}\left((5d)^2 + (4d)^2 + (3d)^2 + \ldots\ldots (4d)^2 + (5d)^2 \right) = 90$$

$$\Rightarrow \frac{1}{11}\left(25d^2 + 16d^2 + 9d^2 + \ldots\ldots 16d^2 + 25d^2 \right) = 90$$

$$\Rightarrow \frac{1}{11}\left(2\left(25d^2 + 16d^2 + 9d^2 + 4d^2 + d^2 \right) \right) = 90$$

$$\Rightarrow \frac{1}{11}\left(2 \times 55 d^2 \right) = 90$$

$$\Rightarrow d^2 = \frac{90 \times 11}{2 \times 55}$$

$$d^2 = 9 \quad \Rightarrow d = \pm 3$$

$d = +3$ as A.P. is increasing.

$\downarrow$ ans. option

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