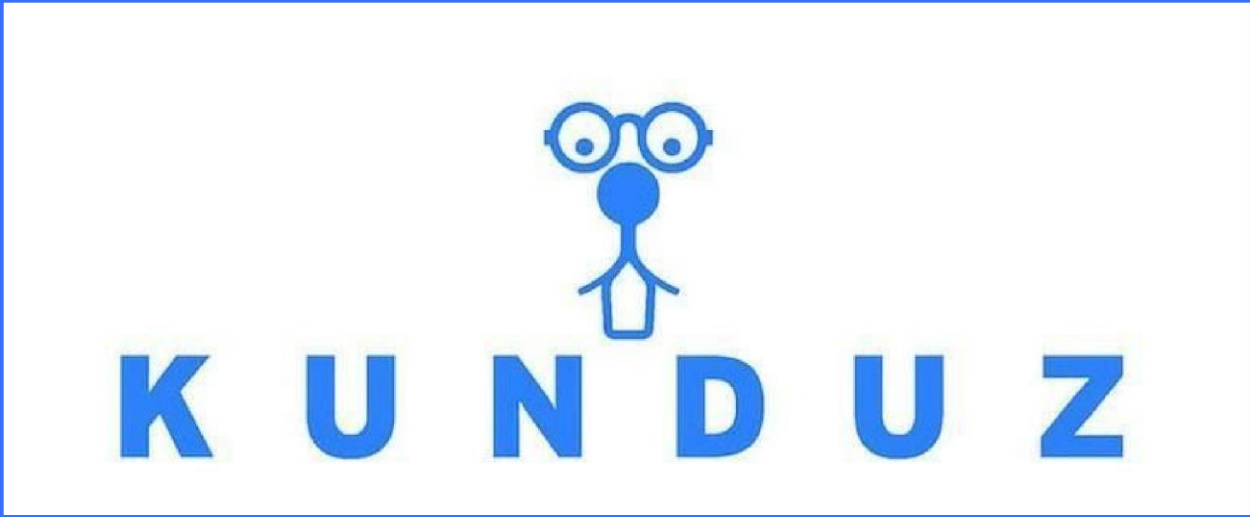


JEE Mains 3rd September 2020 shift 1 Mathematics solutions prepared by KUNDUZ tutors

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Question number

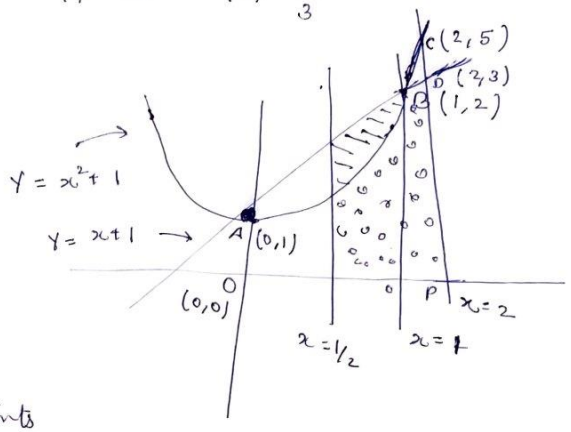
Question and Answers

1

Ques. 1 Area enclosed by $0 \le y \le x^2+1$, $0 \le y \le x+1$, $\frac{1}{2} \le x \le 2$ is

(1) $\frac{1}{12}$ (2) $\frac{1}{6}$ (3) 1 (4) $\frac{1}{3}$

Ans.



for intersection points

$y = x^2+1$ $\quad y = x+1$

$\Rightarrow x^2+1 = x+1$

$\Rightarrow x^2 - x = 0 \quad \Rightarrow x(x-1) = 0 \quad \Rightarrow x = 0, 1$

$x = 0$, $y = 1$ $\quad x = 1$, $y = 2$

A (0, 1) $\quad$ B (1, 2)

$x = 2$, $y = 5$ $\quad x = 2$, $y = 3$

C (2, 5) $\quad$ D (2, 3)

for $y = x^2+1$ and $x = 2$ $\quad$ for $y = x+1$ and $x = 2$

required area $= \int_{1/2}^{1} (x^2+1)\,dx + \int_{1}^{2} (x+1)\,dx$

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$$= \left(\frac{x^3}{3} + x \right)_{1/2}^{1} + \left(\frac{x^2}{2} + x \right)_{1}^{2}$$

$$= \left(\frac{(1)^3}{3} + 1 - \frac{(\frac{1}{2})^3}{3} - \frac{1}{2} \right) + \left(\frac{(2)^2}{2} + 2 - \frac{(1)^2}{2} - 1 \right)$$

$$= \left(\frac{1}{3} + 1 - \frac{1}{24} - \frac{1}{2} \right) + \left(2 + 2 - \frac{1}{2} - 1 \right)$$

$$= \frac{8 + 24 - 1 - 12}{24} + \frac{5}{2}$$

$$= \frac{19}{24} + \frac{5}{2} = \frac{19 + 60}{24}$$

$$= \frac{79}{24} \quad \text{ans.}$$

2

Q 2 If $T_1, T_2, T_3 \ldots$ are in A.P. such that $T_1 + T_2 + T_3 \ldots + T_{25} = T_{26} + T_{27} + \ldots + T_{40}$ and first term is 3 then value of common difference of A.P. is

Ans. Let the common difference be d

first term $a = 3$

Sum of first 25 terms $= \frac{25}{2}[2 \times 3 + (25-1)d]$

$S_{25} = \frac{25}{2}[6 + 24d]$

Sum of first 40 terms $= \frac{40}{2}[2 \times 3 + (40-1)d]$

$S_{40} = \frac{40}{2}[6 + 39d]$

$T_1 + T_2 + T_3 + \ldots + T_{25} = T_{26} + T_{27} + \ldots + T_{40}$

$S_{25} = S_{40} - S_{25}$

$\Rightarrow 2S_{25} = S_{40}$

$\Rightarrow 2 \times \frac{25}{2}(6 + 24d) = \frac{40}{2}(6 + 39d)$

$\Rightarrow 5(6 + 24d) = 4(6 + 39d)$

$\Rightarrow 30 + 120d = 24 + 156d$

$\Rightarrow 36d = 6$

$d = \frac{1}{6}$ $\Rightarrow d = \frac{1}{6}$ ans.

3

Q 3 $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{1-i}\right)^{\frac{n}{3}} = 1$, $m, n \in N$, find HCF of (m, n) for least m & n

(1) 4 (2) 8 (3) 6 (4) 9

Ans. $\begin{cases} \sqrt{-1} = i \Rightarrow -1 = i^2 \Rightarrow 1 = i^4 \\ (a-b)(a+b) = a^2 - b^2 \end{cases}$

$$\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = 1 \Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^{\frac{m}{2}} = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{(1)^2-(i)^2}\right)^{\frac{m}{2}} = 1$$

$$\Rightarrow \left(\frac{1+i^2+2i}{1-i^2}\right)^{\frac{m}{2}} = 1 \Rightarrow \left(\frac{1-1+2i}{1-(-1)}\right)^{\frac{m}{2}} = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^{\frac{m}{2}} = i^4 \quad \{ i^4 = 1 \}$$

$$\Rightarrow (i)^{\frac{m}{2}} = i^4$$

$$\Rightarrow \frac{m}{2} = 4 \Rightarrow m = 8$$

$$\left(\frac{1+i}{1-i}\right)^{\frac{n}{3}} = 1 \Rightarrow (i)^{\frac{n}{3}} = i^4$$

$$\Rightarrow \frac{n}{3} = 4 \Rightarrow n = 12$$

$m = 8 = 2^3$

$n = 12 = 2^2 \times 3$

HCF of $(8, 12) = 2^2 = 4$ ans. option (1)

4

Q 4 A pair of dice is thrown and sum of dice come up multiple of 4 then find probability that at least one dice shows 4.

(1) $\frac{2}{7}$ (2) $\frac{4}{9}$ (3) $\frac{1}{9}$ (4) $\frac{5}{8}$

Ans. Sum = 4, 8, 12

(1, 3), (2, 2), (3, 1)

(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)

(6, 6)

at least one dice shows 4 ⟶ (4, 4)

P (at least one 4 when Sum is multiple of 4) = $\frac{1}{9}$ ans. option (3).

5

Q 5 Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ be a 2×2 matrix such that $A^4 = [a_{ij}]_{2\times 2}$, $a_{11} = 109$, then find a_{22}

(1) 12 (2) 4 (3) -8 (4) 10

Ans.

$$A^2 = A \cdot A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\cdot\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} (x^2+1)^2 + x^2 & — \\ — & x^2+1 \end{bmatrix} = \begin{bmatrix} a_{11} = 109 & a_{12} \\ a_{21} & a_{22} = ? \end{bmatrix}$$

$$(x^2+1)^2 + x^2 = 109 \Rightarrow x^4 + 1 + 2x^2 + x^2 = 109$$

$$\Rightarrow (x^2)^2 + 3(x^2) - 108 = 0$$

$$\Rightarrow (x^2)^2 + 12(x^2) - 9(x^2) - 108 = 0$$

$$\Rightarrow (x^2+12)(x^2-9) = 0$$

$x^2 = 9$ $\qquad$ $x^2 = -12$

$a_{22} = x^2+1 = 10$ $\qquad$ $x^2+1 = -11$

↑ ans. Option (4)

6

Q.6 The equation of curve satisfying differential equation $(1+y^2)(e^x+1)\,dy = e^x y^2\,dx$ and also passes through the point $(0,1)$ is

(1) $y^2+1 = y\ln\left(\frac{1+e^x}{2}\right)$ (2) $y^2-1 = y\ln\left(\frac{1+e^x}{2}\right)$

(3) $y+1 = y\ln\left(\frac{1+e^x}{2}\right)$ (4) $2y^2+1 = y\ln\left(\frac{1+e^x}{2}\right)$

Ans $(1+y^2)(e^x+1)\,dy = e^x y^2\,dx$

$\Rightarrow \left(\frac{1+y^2}{y^2}\right)dy = \left(\frac{e^x}{e^x+1}\right)dx$ $\quad \begin{cases} e^x+1 = t \\ e^x dx = dt \end{cases}$

$\Rightarrow \int\left(\frac{1}{y^2}+1\right)dy = \int\frac{dt}{t}$

$\Rightarrow -\frac{1}{y}+y = \ln t + c$

$\Rightarrow -\frac{1}{y}+y = \ln(e^x+1)+c$

Curve passes through $(0,1)$

$\Rightarrow -\frac{1}{1}+1 = \ln(e^0+1)+c$

$\Rightarrow c = -\ln 2$ $\quad \begin{cases} \ln A-\ln B \\ = \ln\frac{A}{B} \end{cases}$

$\Rightarrow y-\frac{1}{y} = \ln(e^x+1)-\ln(2)$

$\Rightarrow y^2-1 = y\ln\left(\frac{e^x+1}{2}\right)$ ans. option (2)

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7

Q 7 Let $\frac{x^2}{4} + \frac{y^3}{3} = 1$ is an ellipse ~~which~~ and a hyperbola which is confocal with ellipse such that its transverse ~~excess~~ axis is $\sqrt{2}$ then which of following point does not lie on hyperbola

(1) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (2) $\left(-\sqrt{\frac{3}{2}}, 1\right)$ (3) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

(4) None of these

Ans. ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$a_1^2 = 4 \Rightarrow a_1 = 2$

$b_1^2 = 3 \Rightarrow b_1 = \sqrt{3}$

eccentricity of ellipse $= e_E = \sqrt{1 - \frac{b_1^2}{a_1^2}} = \sqrt{1 - \frac{3}{4}}$

$e_E = \frac{1}{2}$

Focus of ellipse $(\pm a_1 e_E, 0) = \left(\pm 2 \times \frac{1}{2}, 0\right)$

$= (\pm 1, 0)$

ellipse and hyperbola are confocal

then Focus of hyperbola is $(\pm 1, 0)$

transverse axis of hyperbola $= 2a_2 = \sqrt{2}$

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$$2a_2 = \sqrt{2} \quad \Rightarrow \quad a_2 = \frac{1}{\sqrt{2}}$$

$$a_2 e_H = 1 \quad \Rightarrow \quad \frac{1}{\sqrt{2}} e_H = 1 \quad \Rightarrow \quad e_H = \sqrt{2}$$

$$e_H = \sqrt{1 + \frac{b_2^2}{a_2^2}} \quad \Rightarrow \quad \sqrt{2} = \sqrt{1 + \frac{b_2^2}{\left(\frac{1}{\sqrt{2}}\right)^2}}$$

$$\Rightarrow \quad 2 = 1 + \frac{b_2^2}{1/2} \quad \Rightarrow \quad b_2^2 = 1/2$$

$$\Rightarrow \quad b_2 = \frac{1}{\sqrt{2}}$$

equation of hyperbola

$$\frac{x^2}{a_2^2} - \frac{y^2}{b_2^2} = 1$$

$$\frac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$x^2 - y^2 = \frac{1}{2}$$

---

now put the option in the equation of hyperbola and check which point is not on hyperbola

8

Q 8 $2\pi - \left[\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right] = ?$

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) $-\frac{\pi}{2}$

Ans.

$$\sin^{-1}(x) \pm \sin^{-1}(y) = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$2\pi - \left[\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right]$$

$$= 2\pi - \left[\sin^{-1}\left(\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right]$$

$$= 2\pi - \left[\sin^{-1}\left(\frac{4}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{3}{5}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right]$$

$$= 2\pi - \left[\sin^{-1}\left(\frac{48}{65} + \frac{15}{65}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right]$$

$$= 2\pi - \left[\sin^{-1}\left(\frac{63}{65}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right]$$

$$= 2\pi - \left[\sin^{-1}\left(\frac{63}{65}\sqrt{1-\left(\frac{16}{65}\right)^2} + \frac{16}{65}\sqrt{1-\left(\frac{63}{65}\right)^2}\right)\right]$$

$$= 2\pi - \left[\sin^{-1}\left(\frac{63}{65}\times\frac{63}{65} + \frac{16}{65}\times\frac{16}{65}\right)\right]$$

$$= 2\pi - \sin^{-1}\left(\frac{3969+256}{4225}\right)$$

$$= 2\pi - \sin^{-1}(1) \qquad \left\{\sin^{-1}(1) = \frac{\pi}{2}\right.$$

$$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$ ans. option (3)

9

Ques. 9 If $y^2 + \ln \cos^2 x = y$ then

(1) $|y''(0)| = 2$ (2) $|y'(0)| + |y''(0)| = 1$

(3) $|y'(0)| + |y''(0)| = 3$ (4) None of these

Ans. $y^2 + \ln(\cos^2 x) = y$ ---- (1)

$\Rightarrow 2yy' + \frac{2\cos x(-\sin x)}{\cos^2 x} = y'$

$\Rightarrow 2yy' - 2\tan x = y'$

$\Rightarrow y'(2y-1) = 2\tan x$

$\Rightarrow y' = \frac{2\tan x}{(2y-1)}$ ---- (2)

$\Rightarrow y'' = \frac{(2y-1)\cdot 2\sec^2 x - 2\tan x(2y')}{(2y-1)^2}$ --- (3)

equation (1)

$y^2 + \ln(\cos^2 x) = y$ Put $x = 0$

$\Rightarrow y^2(0) + \ln(\cos^2(0)) = y(0)$ $\{\ln(1) = 0$

$\Rightarrow y^2(0) - y(0) + \ln(1) = 0$

$\Rightarrow y(0)(y(0) - 1) = 0$

$y(0) = 0, 1$

equation (2)

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$$y' = \frac{2\tan(x)}{(2y-1)}$$ Put $x = 0$

$$y'(0) = \frac{2\tan(0)}{(2y(0)-1)} \quad \{ \tan(0) = 0$$

$$y'(0) = 0$$

Equation (3)

$$y'' = \frac{(2y-1)\cdot 2\sec^2 x - 2\tan x\,(2y')}{(2y-1)^2}$$ Put $x = 0$

$$y''(0) = \frac{(2y(0)-1)\cdot 2\cdot \sec^2(0) - 2\tan(0)\,(2y'(0))}{(2y(0)-1)^2}$$

$\{ \sec(0) = 1, \quad \tan(0) = 0 \}$
$y(0) = 0, 1 \quad y'(0) = 0$

$$y''(0) = \frac{2(2y(0)-1)}{(2y(0)-1)^2} = 2(2y(0)-1)$$

$y''(0) = 2(2\cdot 0 - 1)$ $\quad$ $y''(0) = 2(2\cdot 1 - 1)$

$y''(0) = -2$ $\quad$ $y''(0) = 2$

$|y''(0)| = 2$ ans. option (1)

10

Q 10. $\lim_{x\to 0}\left(\frac{1-x+|x|}{1+[x]-\lambda}\right)=L$ (finite), where $[\cdot]$ denotes the greatest integer function then find L.

(1) 0 (2) $\frac{1}{2}$ (3) 1 (4) 2

Ans. $\lim_{x\to 0}\left(\frac{1-x+|x|}{1+[x]-\lambda}\right)=L$

L.H.L. $\lim_{x\to 0^-}\left(\frac{1-x+|x|}{1+[x]-\lambda}\right)=\lim_{h\to 0}\left(\frac{1-(0-h)+|0-h|}{1+[0-h]-\lambda}\right)$

$=\lim_{h\to 0}\left(\frac{1+h+h}{1-1-\lambda}\right)=\frac{1}{\lambda}$

R.H.L $\lim_{x\to 0^+}\left|\frac{1-x+|x|}{1+[x]-\lambda}\right|=\lim_{h\to 0}\left|\frac{1-(0+h)+|0+h|}{1+[0+h]-\lambda}\right|$

$\lim_{h\to 0}\left|\frac{1-h+h}{1+0-\lambda}\right|=\frac{1}{1-\lambda}$

L.H.L. = R.H.L.

$\frac{1}{\lambda}=\frac{1}{1-\lambda}\Rightarrow 1-\lambda=\lambda\Rightarrow\lambda=\frac{1}{2}$

$L=\lim_{x\to 0}\left|\frac{1-x+|x|}{1+[x]-1/2}\right|=\left|\frac{1-0+|0|}{1/2+[0]}\right|$

$L=\frac{1}{1/2}$

$L=2$ Ans. option (4)

11

Q 11 let α, β are roots of $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 - 2qx + 1 = 0$. Then find the value of

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)$$

Ans. α, β are roots of $x^2 + px + 2 = 0$

then $\alpha + \beta = -p$ and $\alpha\beta = 2$

$\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 - 2qx + 1$

then $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{(-2q)}{2} = q$

$$\frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{2} \Rightarrow \frac{1}{\alpha\beta} = \frac{1}{2} \Rightarrow \alpha\beta = 2$$

$(a+b)^2 = a^2 + b^2 + 2ab$

$a^2 + b^2 = (a+b)^2 - 2ab$

$$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)$$

$$= \left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)\left(\alpha\beta - \frac{\alpha}{\beta} - \frac{\beta}{\alpha} + \frac{1}{\alpha\beta}\right)$$

$$= \frac{(\alpha\beta + 1)^2}{\alpha\beta}\left(\alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha^2 + \beta^2}{\alpha\beta}\right)$$

$$= \frac{(2+1)^2}{2}\left(2 + \frac{1}{2} - \frac{(\alpha+\beta)^2 - 2\alpha\beta}{2}\right)$$

$$= \frac{9}{2}\left(\frac{5}{2} - \frac{(p)^2 - 2(2)}{2}\right) = \frac{9}{2}\left(\frac{5 - p^2 + 4}{2}\right)$$

$$= \frac{9}{4}(9 - p^2) \quad \text{ans.}$$

12

Q.12 Evaluate $\int_{-\pi}^{\pi} | \pi - |x| | \, dx$

(1) π^2 (2) $\frac{\pi^2}{2}$ (3) $\frac{\pi^2}{3}$ (4) $\frac{\pi^2}{4}$

Ans. for an even function $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

$f(x) = Y = | \pi - |x| |$ is an even function as $f(x) = f(-x)$

$$I = \int_{-\pi}^{\pi} | \pi - |x| | \, dx$$

$$I = 2\int_{0}^{\pi} (\pi - x)\, dx = 2 \left(\pi x - \frac{x^2}{2} \right)_0^{\pi}$$

$$I = 2\left[\left(\pi(\pi) - \frac{(\pi)^2}{2} \right) - \left(\pi(0) - \frac{(0)^2}{2} \right) \right]$$

$$I = 2 \left(\pi^2 - \frac{\pi^2}{2} \right)$$

$$I = 2 \times \frac{\pi^2}{2}$$

$I = \pi^2$ ans. option 1

13

Q 13 $\lim_{x\to 0} \dfrac{\left(1-\cos^2\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)}{x^8} = 2^{-k}$, find k

Ans. $\Rightarrow \lim_{x\to 0} \dfrac{\left(1-\cos^2\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)}{x^8} = 2^{-k}$

$\Rightarrow \lim_{x\to 0} \dfrac{\left(1-\cos\frac{x^2}{2}\right)}{x^4} \cdot \dfrac{\left(1-\cos\frac{x^2}{4}\right)}{x^4}$

$\left\{ \lim_{x\to 0} \dfrac{(1-\cos x)}{x^2} = \dfrac{1}{2} \right.$

$\Rightarrow \lim_{x\to 0} \dfrac{\left(1-\cos\frac{x^2}{2}\right)}{\frac{x^4}{4}\times 4} \cdot \dfrac{\left(1-\cos\frac{x^2}{4}\right)}{\frac{x^4}{16}\cdot 16}$

$\Rightarrow \lim_{x\to 0} \dfrac{\left(1-\cos\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)^2}\cdot\dfrac{1}{4} \cdot \dfrac{\left(1-\cos\frac{x^2}{4}\right)}{\left(\frac{x^2}{4}\right)^2}\cdot\dfrac{1}{16} = 2^{-k}$

$\Rightarrow \dfrac{1}{2}\cdot\dfrac{1}{4}\cdot\dfrac{1}{2}\cdot\dfrac{1}{16} = 2^{-k}$

$\Rightarrow \dfrac{1}{2^8} = 2^{-k}$

$\Rightarrow 2^{-8} = 2^{-k}$

$\Rightarrow k = 8$ ans.

14

Q 14. The value of $0.16^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\infty\right)}$ is

Ans. Sum of an infinite G.P. is given by $= \frac{a}{1-r}$

a = first term, r = common ratio

$$(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\infty\right)}$$

$$= \left(\frac{16}{100}\right)^{\log_{\frac{5}{2}}\left(\frac{1/3}{1-1/3}\right)}$$

$$= \left(\frac{4}{25}\right)^{\log_{\frac{5}{2}}\left(\frac{1/3}{2/3}\right)}$$

$$= \left(\frac{2}{5}\right)^{2\log_{\frac{5}{2}}\left(\frac{1}{2}\right)}$$

$$= \left(\frac{2}{5}\right)^{\log_{\frac{5}{2}}\left(\frac{1}{2}\right)^2}$$

$$= \left(\frac{2}{5}\right)^{\log_{\frac{5}{2}}\left(\frac{1}{4}\right)}$$

$$= \left(\frac{2}{5}\right)^{\frac{\log\frac{1}{4}}{\log 5/2}} = \left(\frac{2}{5}\right)^{\frac{\log 1-\log 4}{\log 5-\log 2}}$$

$$= \left(\frac{2}{5}\right)^{\frac{\log 4}{\log 2-\log 5}} = \left(\frac{2}{5}\right)^{\frac{\log 4}{\log 2/5}} = \left(\frac{2}{5}\right)^{\log_{\frac{2}{5}}(4)}$$

$$= 4 \quad \text{ans.}$$

$\log 1 = 0$

$\log\frac{A}{B} = \log A - \log B$

$\log_B A = \frac{\log A}{\log B}$

$a^{\log_a B} = B$

15

Q 15 $S = (2\ {}^{1}P_0 - 3\ {}^{2}P_1 + 4\ {}^{3}P_2 - \ldots 51 \text{ terms}) + (1!_0 - 2!_0 + 3!_0 - 4!_0 + \ldots 51 \text{ terms})$. then find S ?

(1) $1 + 51!_0$ (2) $1 + 52!_0$ (3) $1 + 50(51!_0)$

(4) $1 + 51(51!_0)$

Ans. $${}^{n}P_r = \frac{n!_0}{(n-r)!_0} \Rightarrow {}^{n}P_{n-1} = \frac{n!_0}{(n-(n-1))!_0}$$

$${}^{n}P_{n-1} = \frac{n!_0}{(n-n+1)!_0} \Rightarrow {}^{n}P_{n-1} = n!_0$$

$$S = (2\ {}^{1}P_0 - 3\ {}^{2}P_1 + 4\ {}^{3}P_2 - \ldots 51 \text{ terms}) + (1!_0 - 2!_0 + 3!_0 - 4!_0 + \ldots + 51!_0) + 52\ {}^{51}P_{50}$$

$$S = (2 \cdot 1!_0 - 3 \cdot 2!_0 + 4 \cdot 3!_0 - \ldots + 52 \cdot 51!_0) + (1!_0 - 2!_0 + 3!_0 - 4!_0 + \ldots + 51!_0)$$

$$S = (2!_0 - 3!_0 + 4!_0 - \ldots + 52!_0) + (1!_0 - 2!_0 + 3!_0 - 4!_0 \ldots + 51!_0)$$

$S = 1!_0 + 52!_0$

$S = 1 + 52!_0$

↕ ans. option (2)

$1!_0 = 1$

$n \times (n-1)!_0 = n!_0$

16

Ques 16 The function $f(x) = (3x-7)\,x^{2/3}$, $x \in R$ is increasing for all x lying in :

(1) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$ (2) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(3) $\left(-\infty, \frac{14}{15}\right)$ (4) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

Ans. $f(x) = (3x-7)\,x^{2/3}$

$$f'(x) = (3x-7)\,\frac{2}{3}\cdot\frac{1}{x^{1/3}} + x^{2/3}\cdot 3$$

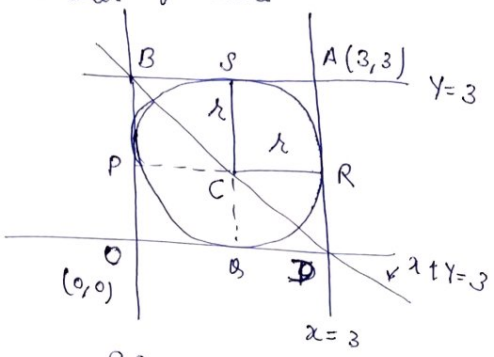
$$f'(x) = \frac{15x-14}{3x^{1/3}} \geq 0 \text{ for increasing}$$

$x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$ ans. option (2)

$f'(x) \geq 0$ for $f(x)$ to be increasing

17

Q 17 The center of a circle lies on $x+y=3$ and touching the lines $x=3$, $y=3$, then find diameter of circle



Ans. Center lies on $x+y=3$

coordinates of center

$C(x,y) \equiv (\alpha, 3-\alpha)$

$PC = \alpha$, $CR = r$, $PR = 3$

$QC = 3-\alpha$, $CS = r$, $QS = 3$

$CR = PR - PC \Rightarrow r = 3-\alpha$ ---①

$CS = QS - QC \Rightarrow r = 3-(3-\alpha) \Rightarrow r = \alpha$ --②

from equation ① and ②

$3-\alpha = \alpha \Rightarrow \alpha = 3/2$

$\Rightarrow \alpha = r = \frac{3}{2}$

$\Rightarrow 2r = 3$

diameter = 3 units ans.

18

Q18 $(2^{1/2} + 5^{1/8})^n$ has 33 integral terms find least value of n

(i) 256 (2) 257 (3) 258 (4) 259

Ans. $T_{r+1} = {}^nC_r \, 2^{\frac{n-r}{2}} \, 5^{\frac{r}{8}}$

for integral terms $\left\{ \frac{r}{8} \text{ and } \frac{n-r}{2} \right.$ should be integers

then $r = 0, 8, 16, 24, \ldots$

$(2^{1/2} + 5^{1/8})$ has 33 integral terms

$a = 0 \quad , \quad d = 8$

$n = t_{33} = a + (33-1)d$

$n = 0 + (33-1)\,8$

$n = 32 \times 8$

$n = 256$ ans. option (i)

$\left\{ \frac{r}{8} \text{ and } \frac{n-r}{2} \right.$ should be integers { If a number is divisible by 8, then it is divisible by 2 also }

$\frac{n}{8} + 1 = 33 \Rightarrow \frac{n}{8} = 32$

$n = 32 \times 8 = 256$

19

Ques 19 Consider the two sets :

$A = \{ m \in R : \text{both the roots of } x^2 - (m+1)x + m + 4 = 0 \text{ are real} \}$

and $B = [-3, 5)$

which of the following is not true ?

(1) $A - B = (-\infty, -3) \cup (5, \infty)$

(2) $A \cap B = \{-3\}$

(3) $B - A = (-3, 5)$

(4) $A \cup B = R$

Ans. $x^2 - (m+1)x + (m+4) = 0$

both the roots are real

$\therefore \quad D = b^2 - 4ac \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$

$\Rightarrow m^2 + 1 + 2m - 4m - 16 \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$

$(m-5)(m+3) \geq 0$

$m \in (-\infty, -3] \cup [5, \infty)$

$A = (-\infty, -3] \cup [5, \infty)$, $B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$ ans option (1)

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20

Ques 20 For the frequency distribution

Variate (x) : $x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_{15}$

Frequency (f) : $f_1 \quad f_2 \quad f_3 \quad \cdots \quad f_{15}$

where $0 < x_1 < x_2 < x_3 < \cdots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be

(1) 2 (2) $\frac{1}{4}$ (3) 4 (4) 6

Ans. Standard deviation will be maximum when half x_i approch to 0 and half to 10.

$\bar{x}$ will be middle value = 5

$$(x_i - \bar{x})^2 \leq 25$$

$$(x_1 - \bar{x}) \leq 5 \quad \Rightarrow \quad \sigma \leq 5$$

Standard deviation cannot be more than 5

$\therefore$ option (4) 6 ans.

21

Ques. 2] The lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) Intersect when $\lambda = 1$ and $m = 2$

(2) Intersect when $\lambda = 2$ and $m = 1/2$

(3) Do not intersect for any values of λ and m

(4) Intersect for all values of λ and m

Ans. $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$

$L_1 : \vec{r} = (1 + 2\lambda, -1, \lambda)$

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

$L_2 : \vec{r} = (2 + m, -1 + m, -m)$

$1 + 2\lambda = 2 + m \Rightarrow 2\lambda - m = 1$

$-1 = -1 + m \Rightarrow m = 0$

$\lambda = -m \Rightarrow \lambda = 0$

does not satisfy the above equations

Skew lines

do not intersect for any values of λ and m

ans. option (3)

22

Ques. 22 If $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$

then find the absolute value of $B + C$?

Ans.

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\Rightarrow \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$C_2 \to C_2 - 2C_1$, $C_3 \to C_3 - 3C_1$

$$\Rightarrow \begin{vmatrix} x-2 & 1 & 2 \\ 1 & -x & -2x \\ 1 & -x-1 & x-8 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$$\Rightarrow (x-2)\left(-x(x-8) - (-x-1)(-2x)\right) - 1\left((x-8) - (-2x)\right) + 2\left((-x-1) - (-x)\right) = Ax^3 + Bx^2 + Cx + D$$

$\Rightarrow (x-2)\left(-x^2+8x-2x^2-2x\right)-(3x-8)+2(-1)$

$= Ax^3+Bx^2+Cx+D$

$\Rightarrow (x-2)(-3x^2+6x)-3x+8-2 = Ax^3+Bx^2+Cx+D$

$\Rightarrow -3x^3+6x^2+6x^2-12x-3x+6 = Ax^3+Bx^2+Cx+D$

$\Rightarrow -3x^3+12x^2-15x+6 = Ax^3+Bx^2+Cx+D$

$A = -3$

$B = 12$

$C = -15$

$D = 6$

$|B+C| = |12+(-15)| = |12-15| = |-3|$

$|B+C| = 3$ ans.

23

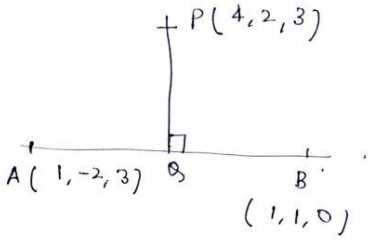
Ques 23 The foot of the perpendicular drawn from the point $(4, 2, 3)$ to the line joining the points $(1, -2, 3)$ and $(1, 1, 0)$ lies on the plane :

(1) $x + 2y - z = 1$ (2) $x - 2y + z = 1$

(3) $x - y - 2z = 1$ (4) $2x + y - z = 1$

Ans.

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$



line AB : $\frac{x-1}{0} = \frac{y+2}{1} = \frac{z-3}{-1} = \lambda$

$Q \equiv (1, -2+\lambda, 3-\lambda)$

$\overrightarrow{PQ} = -3\hat{i} + (-4+\lambda)\hat{j} - \lambda\hat{k}$

$\overrightarrow{AB} \cdot \overrightarrow{PQ} = 0$ --- ($PQ \perp AB$)

$\Rightarrow (3\hat{j} - 3\hat{k}) \cdot (-3\hat{i} + (-4+\lambda)\hat{j} - \lambda\hat{k}) = 0$

$\Rightarrow 3(-4+\lambda) + 3\lambda = 0$

$\Rightarrow -12 + 3\lambda + 3\lambda = 0 \Rightarrow \lambda = 2$

$Q \equiv (1, -2+2, 3-2)$

$Q \equiv (1, 0, 1)$

Now check the options on which plane Q lies

Q lies on $2x + y - z = 1$ ans. option (4)

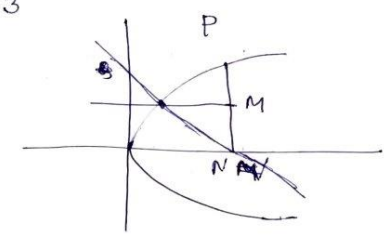
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Ques 24 Let P be a point on the parabola $Y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid point M of PN, parallel to its axis which meets the parabola at Q. If the Y-intercept of the line NQ is $\frac{4}{3}$, then:

(1) $MQ = \frac{1}{3}$ (2) $PN = 3$ (3) $MQ = \frac{1}{4}$ (4) $PN = 4$

Ans. Parabola : $Y^2 = 12x$ $\quad 4a = 12$ $\Rightarrow a = 3$



Let $P(3t^2, 6t)$

$\therefore N(3t^2, 0)$

$M(3t^2, 3t)$

for Q $\quad Y = 3t$

$9t^2 = 12x \quad \Rightarrow x = \frac{3}{4}t^2$

$Q\left(\frac{3}{4}t^2, 3t\right)$

Slope of QN $= \dfrac{3t - 0}{\frac{3}{4}t^2 - 3t^2} = -\dfrac{4}{3t^2}$

$(Y - 0) = \frac{-4}{3t}\left(x - 3t^2\right)$

Passing through $\left(0, \frac{4}{3}\right)$

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$$\frac{4}{3} - 0 = -\frac{4}{3t}(0 - 3t^2)$$

$$\Rightarrow \frac{4}{3} = 4t$$

$$\Rightarrow t = \frac{1}{3}$$

$$P(3t^2, 6t) \equiv P\left(\frac{1}{3}, 2\right)$$

$$Q\left(\frac{3}{4}t^2, 3t\right) \equiv Q\left(\frac{1}{12}, 1\right)$$

$$M(3t^2, 3t) \equiv M\left(\frac{1}{3}, 1\right)$$

$$N(3t^2, 0) \equiv N\left(\frac{1}{3}, 0\right)$$

$PN = 2$ $\quad MQ = \frac{1}{4}$ $\quad$ ans. option (3)

25

Ques 25 The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to:

(1) $(\sim p) \vee q$

(2) q

(3) $(\sim p) \wedge q$

(4) $(\sim p) \vee (\sim q)$

Ans.

$$p \rightarrow \sim(p \wedge \sim q)$$

$$\equiv \sim p \vee (\sim(p \wedge \sim q))$$

$$\equiv \sim p \vee (\sim p \vee q)$$

$$\equiv \sim p \vee q \quad \text{ans.}$$

$$= (\sim p) \vee q \quad \text{ans. option (1)}$$

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