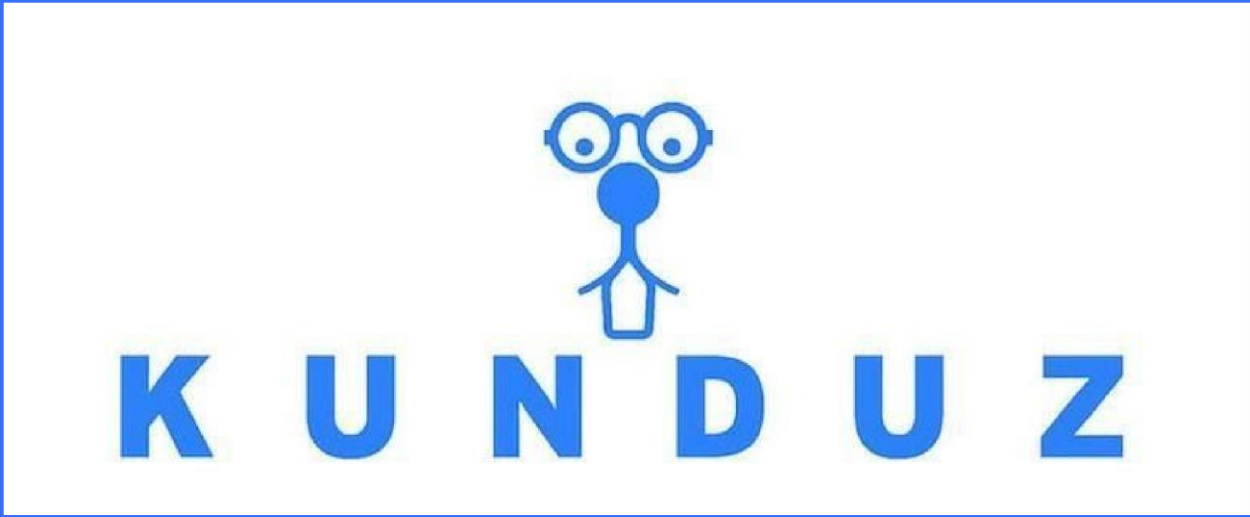


JEE Mains 3rd September 2020 shift 2 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

Q 1. Let A be a 3×3 matrix such that

$$\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix} \quad \text{and} \quad B = \text{adj}(\text{adj } A)$$

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$ then ordered pair $(|\lambda|, \mu)$ is equal to

Ans.

$$|\text{adj } A| = |A|^{n-1}$$

$$\therefore |A|^{3-1} = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix} = 2(0-(-4)) + (1)(1-2) + 1(2-0)$$

$$\Rightarrow |A|^2 = 2(4) + (-1) + (2)$$

$$\Rightarrow |A|^2 = 9 \Rightarrow |A| = \pm 3 = \lambda$$

$$|\lambda| = 3$$

$$B = \text{adj}(\text{adj } A)$$

$$|B| = |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$$

$$\Rightarrow |B| = |A|^{(3-1)^2} = |A|^4$$

$$\Rightarrow |B| = |\pm 3|^4$$

$$\Rightarrow |B| = 81$$

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$|(B^{-1})^T| = 4$

$|B^{-1}| = 4 \quad \Rightarrow \quad |B|^{-1} = 4$

$\Rightarrow \quad \frac{1}{|B|} = 4$

$\Rightarrow \quad 4 = \frac{1}{8}$

$(|\lambda|, 4) = \left(3, \frac{1}{8}\right)$ ans.

2

Q 2 If $x^3 dy + xy \cdot dx = x^2 dy + 2y\, dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to

(1) $\frac{\sqrt{e}}{2}$ (2) $\frac{1}{2} + \sqrt{e}$ (3) $\frac{3}{2}\sqrt{e}$ (4) $\frac{3}{2} + \sqrt{e}$

Ans. $x^3 dy + xy\, dx = x^2 dy + 2y\, dx$

By variable separation method

$\Rightarrow (x^3 - x^2)\, dy = (2 - x)\, y\, dx$

$\Rightarrow \frac{dy}{y} = \frac{2-x}{x^2(x-1)}\, dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{2-x}{x^2(x-1)}\, dx$

By Partial fraction

Let $\frac{2-x}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$\Rightarrow 2 - x = Ax(x-1) + B(x-1) + Cx^2$

$\Rightarrow 2 - x = Ax^2 - Ax + Bx - B + Cx^2$

$\Rightarrow 2 - x = x^2(A + C) - x(A - B) - B$

by comparison

$-B = 2 \Rightarrow B = -2$

$A - B = 1 \Rightarrow A = 1 + B = 1 - 2 \Rightarrow A = -1$

$A + C = 0 \Rightarrow C = -A \Rightarrow C = 1$

$$\int \frac{1}{y}\,dy = \int \left(-\frac{1}{x} - \frac{2}{x^2} + \frac{1}{x-1} \right) dx$$

$$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln(x-1) + c$$

$$\because y(2) = e \qquad \{ \ln 1 = 0,\ \ln e = 1$$

$$\Rightarrow \ln e = -\ln 2 + \frac{2}{2} + \ln(2-1) + c$$

$$\Rightarrow 1 = -\ln 2 + 1 + 0 + c$$

$$\Rightarrow c = \ln 2$$

$$\ln y = -\ln x + \frac{2}{x} + \ln(x-1) + \ln 2$$

at $x = 4$

$$\ln(y(4)) = -\ln 4 + \frac{2}{4} + \ln(4-1) + \ln 2$$

$$\ln(y(4)) = \frac{1}{2} + \ln \frac{3 \cdot 2}{4} = \frac{1}{2} \ln \frac{3}{2}$$

$$y(4) = e^{\frac{1}{2} + \ln \frac{3}{2}} = e^{1/2}\, e^{\ln 3/2}$$

$$y(4) = \frac{3}{2} e^{1/2}$$

$$y(4) = \frac{3}{2}\sqrt{e}$$ ans. option (3)

3

Q.3 If the sum of the series is $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} +$ ----- upto n^{th} term is 488 and the n^{th} term is negative, then :

(1) n^{th} term is $-4\frac{2}{3}$ (2) $n = 41$

(3) n^{th} term is -4 (4) $n = 60$

Ans. $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} +$ ---- upto n^{th} term

this is an A.P. with first term $a = 20$ and

Common difference $d = 19\frac{3}{5} - 20 = -\frac{2}{5}$

Sum of n terms $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$\Rightarrow \frac{n}{2}\left(2a + (n-1)d\right) = 488$

$\Rightarrow \frac{n}{2}\left(2\times 20 + (n-1)\left(-\frac{2}{5}\right)\right) = 488$

$\Rightarrow 20n + \frac{(1-n)(n)}{5} = 488$

$\Rightarrow 100n + n - n^2 = 2440$

$\Rightarrow n^2 - 101 + 2440 = 0$

$\Rightarrow (n-40)(n-61) = 0$

$n = 40, 61$

$n = 40$ $\quad T_n = a + (n-1)d \Rightarrow T_n = 20 + 39\left(-\frac{2}{5}\right)$

$T_{40} > 0$ ($\because$ it is given that n^{th} term is negative)

$n = 61$ $\quad T_n = a + (n-1)d \Rightarrow T_n = 20 + 60\left(-\frac{2}{5}\right)$

$T_{61} < 0$

$T_{61} = 20 - \frac{120}{5} = 20 - 24$

$T_{61} = -4$

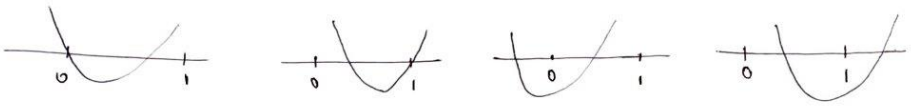
$n = 61$ and nth term is -4 ans. option (3)

4

Q 4 The set all of real values of λ for which the quadratic eq. $(\lambda^2+1)x^2 - 4\lambda x + 2 = 0$, always have exactly one root in the interval $(0,1)$ is

(1) $[-3,-1)$ (2) $(0,2)$ (3) $(1,3]$ (4) $(2,4]$

Ans. $f(x) = (\lambda^2+1)x^2 - 4\lambda x + 2$



$f(1)\,f(0) \leq 0$

$$\left[(\lambda^2+1)(1)^2 - 4\lambda(1) + 2\right]\left[(\lambda^2+1)(0)^2 - 4\lambda(0) + 2\right] \leq 0$$

$$\Rightarrow (\lambda^2 + 1 - 4\lambda + 2)(2) \leq 0$$

$$\Rightarrow (\lambda^2 - 4\lambda + 3) \leq 0$$

$$(\lambda - 3)(\lambda - 1) \leq 0$$

$$\lambda \in [1, 3]$$

at $\lambda = 1$ $\quad f(x) = 2x^2 - 4\lambda + 2 = 0$

$x = 1, 1$ both roots are same

$\therefore \lambda \neq 1$

at $\lambda = 3$ $\quad f(x) = 10x^2 - 12x + 2 = 0$

$x = 1, 1/5$

$\therefore \lambda \in (1, 3]$ ans. option (3)

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5

Q 5 Let R_1 and R_2 be two relations defined as follows:

$R_1 = \{(a,b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a,b) \in R^2 : a^2 + b^2 \notin Q\}$

where Q is the set of all rational numbers, then

(1) R_1 is transitive but R_2 is not transitive

(2) R_2 is transitive but R_1 is not transitive

(3) Neither R_1 nor R_2 is transitive

(4) R_1 and R_2 are both transitive

Ans. For R_1 let $a = (5+\sqrt{3})^{1/2}$, $b = (5-\sqrt{3})^{1/2}$, $c = (4+\sqrt{3})^{1/2}$

$a R_1 b \Rightarrow a^2 + b^2 = (5+\sqrt{3}) + (5-\sqrt{3}) = 10 \in Q$

$b R_1 c \Rightarrow b^2 + c^2 = (5-\sqrt{3}) + (4+\sqrt{3}) = 9 \in Q$

$a R_1 c \Rightarrow a^2 + c^2 = (5+\sqrt{3}) + (4+\sqrt{3}) = 9 + 2\sqrt{3} \notin Q$

$\therefore R_1$ is not transitive

For R_2 let $a = (5+\sqrt{3})^{1/2}$, $b = 2$, $c = (5-\sqrt{3})^{1/2}$

$a R_2 b \Rightarrow a^2 + b^2 = (5+\sqrt{3}) + 4 = 9 + \sqrt{3} \notin Q$

$b R_2 c \Rightarrow b^2 + c^2 = (4) + (5-\sqrt{3}) = 9 - \sqrt{3} \notin Q$

$a R_2 c \Rightarrow a^2 + c^2 = (5+\sqrt{3}) + (5-\sqrt{3}) = 10 \in Q$

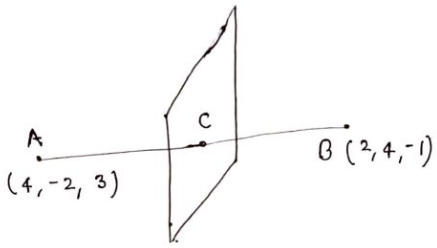
$\therefore R_2$ is not transitive

$\therefore$ Neither R_1 nor R_2 is transitive Ans. option (3)

6

Q 06 The plane which bisects the line joining the points $(4,-2,3)$ and $(2,4,-1)$ at right angles also passes through the point

(1) $(0,-1,1)$ (2) $(4,0,-1)$ (3) $(4,0,1)$ (4) $(0,1,-1)$



Ans. plane bisects line AB at C.

C is mid point of AB

$$C \equiv \left(\frac{4+2}{2}, -\frac{2+4}{2}, \frac{3-1}{2} \right)$$

$$C \left(3, 1, 1 \right)$$

normal of the plane is along AB

$$\vec{n} = 2\hat{i} - 6\hat{j} + 4\hat{k} \quad \text{(Normal of Plane)}$$

$$\vec{a} = \vec{C} = 3\hat{i} + \hat{j} + \hat{k} \quad \text{(point on Plane)}$$

equation of plane

$$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - 6\hat{j} + 4\hat{k}) = (3\hat{i} + \hat{j} + \hat{k}) \cdot (2\hat{i} - 6\hat{j} + 4\hat{k})$$

$$\Rightarrow 2x - 6y + 4z = 6 - 6 + 4$$

$$2x - 6y + 4z = 4 \Rightarrow x - 3y + 2z = 2$$

It passes through $(4, 0, -1)$ ans. option (2)

7

Q 7 Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. then the truth values of p, q, r are respectively:

(1) T, T, F (2) T, T, T (3) T, F, T

(4) F, T, F

Ans. $(p \wedge q) \rightarrow (\sim q \vee r)$

$A \rightarrow B$ = false

A should be true and B should be false

$p \wedge q$ true

$q \Rightarrow$ true

$p \Rightarrow$ true

$\sim q \vee r$ False

$\sim q \Rightarrow$ False

$r \Rightarrow$ False

$q \Rightarrow$ true

hence

$p \Rightarrow$ true T

$q \Rightarrow$ true T

$r \Rightarrow$ False F

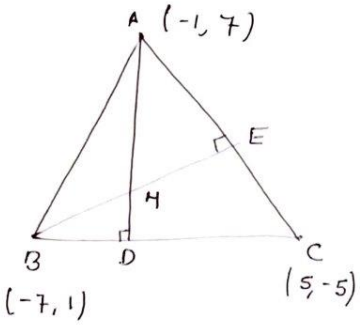
} ans. option (1)

8

Q 8 If a ΔABC has vertices $A(-1,7)$, $B(-7,1)$ and $C(5,-5)$, then its orthocenter has coordinates :

(1) $(-3,3)$ (2) $(3,-3)$ (3) $\left(-\frac{3}{5},\frac{3}{5}\right)$ (4) $\left(\frac{3}{5},-\frac{3}{5}\right)$

Ans.



Slope of $BC = m_{BC} = \frac{-5-1}{5-(-7)}$

$m_{BC} = -\frac{6}{12} = -\frac{1}{2}$

Slope of $AD = m_{AD} = -\frac{1}{m_{BC}}$

$m_{AD} = -\frac{1}{(-1/2)} \Rightarrow m_{AD} = 2$

equation of $AD \Rightarrow y-7 = 2(x-(-1))$

$\Rightarrow y = 2x+9$ ... ①

Slope of $AC = m_{AC} = \frac{-5-7}{5-(-1)} = -\frac{12}{6} = -2$

Slope of $BE = m_{BE} = -\frac{1}{m_{AC}} = -\frac{1}{(-2)} = \frac{1}{2}$

equation of $BE \Rightarrow y-1 = \frac{1}{2}(x-(-7))$

$\Rightarrow y = \frac{x}{2} + \frac{9}{2}$ ②

Orthocenter is intersection point of AD and BE

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from equation (1) and (2)

$$2x + 9 = \frac{x}{2} + \frac{9}{2}$$

$$\Rightarrow 4x + 18 = x + 9$$

$$\Rightarrow 3x = -9$$

$$x = -3$$

$$Y = 2x + 9 \Rightarrow Y = 2(-3) + 9$$

$$Y = 3$$

orthocenter $(-3, 3)$ ans. option (1)

9

(9) Suppose $f(x)$ is a polynomial of degree four, having critical points at $-1, 0, 1$. If $T = \{x \in R, | f(x) = f(0)\}$ then sum of squares of all the elements of T is :

(1) 4 (2) 6 (3) 2 (4) 8

Ans $f(x)$ have critical points at $-1, 0, 1$

$$\therefore \quad f'(x) = A(x+1)(x-1)x$$

$$f'(x) = A(x^2-1)x = A(x^3-x)$$

$$f(x) = \int A(x^3-x)\,dx$$

$$f(x) = \frac{Ax^4}{4} - \frac{Ax^2}{2} + K$$

Given that $f(x) = f(0)$

$$\frac{Ax^4}{4} - \frac{Ax^2}{2} + K = \frac{A(0)^4}{4} - \frac{A(0)^2}{2} + K$$

$$\Rightarrow \quad \frac{Ax^4}{4} - \frac{Ax^2}{2} = 0$$

$$\Rightarrow \quad \frac{A}{4}\left(x^4 - 2x^2\right) = 0$$

$$\Rightarrow \quad \frac{A}{4}x^2\left(x^2 - 2\right) = 0$$

$$x = 0, \sqrt{2}, -\sqrt{2}$$

Sum of squares of all elements of T

$$(0)^2 + (\sqrt{2})^2 + (-\sqrt{2})^2 = 4 \quad \text{ans. option 1}$$

10

Q 10 If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then $18k$ is equal to

Ans. Let T_{r+1} be a term independent of x

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$$

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-2r-r}$$

$$18 - 2r - r = 0$$

$$r = 6$$

$$T_{6+1} = {}^9C_6 \left(\frac{3}{2}\right)^{9-6} \left(-\frac{1}{3}\right)^6$$

$$T_{6+1} = \frac{9 \times 8 \cdot 7}{3 \times 2 \times 1} \times \frac{3^3}{2^3} \times \frac{1}{3^6}$$

$$T_7 = \frac{7}{18} = k$$

$$18k = 18 \times \frac{7}{18}$$

$$18k = 7 \quad \text{ans.}$$

11

Q 11 If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 - 1|$, $Re(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $Im(z_1 + z_2)$ is equal to

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{3}}$ (3) $2\sqrt{3}$ (4) $\frac{2}{\sqrt{3}}$

Ans. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$Re(z) = |z_1 - 1|$

$\Rightarrow x_1 = |x_1 + iy_1 - 1| \Rightarrow x_1 = \sqrt{(x_1 - 1)^2 + y_1^2}$

$\Rightarrow x_1^2 = x_1^2 + 1 - 2x_1 + y_1^2$

$\Rightarrow y_1^2 + 1 = 2x_1 \quad \cdots (1)$

$Re(z_2) = |z_2 - 1|$

$\Rightarrow x_2 = |x_2 + iy_2 - 1| \Rightarrow x_2 = \sqrt{(x_2 - 1)^2 + y_2^2}$

$\Rightarrow x_2^2 = x_2^2 + 1 - 2x_2 + y_2^2$

$\Rightarrow y_2^2 + 1 = 2x_2 \quad \cdots (2)$

By (1) - (2)

$y_1^2 - y_2^2 = 2(x_1 - x_2)$

$(y_1 + y_2)(y_1 - y_2) = 2(x_1 - x_2)$

$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{2}{y_1 + y_2} \quad \cdots (3)$

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$$\arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$$

$$\frac{y_1 - y_2}{x_1 - x_2} = \tan\frac{\pi}{6}$$

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}} \quad \text{--- (4)}$$

from (3) and (4)

$$\frac{2}{y_1 + y_2} = \frac{1}{\sqrt{3}}$$

$$y + y_2 = 2\sqrt{3}$$

$$\text{Img}(z_1 + z_2) = 2\sqrt{3} \quad \underline{\text{ans.}}$$

$$\begin{cases} z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2) \\ z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2) \\ \arg(z_1 - z_2) = \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) \end{cases}$$

12

Q 12 Let x_i $(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10} (x_i - p) = 3$ and $\sum_{i=1}^{10} (x_i - p)^2 = 9$ where $p \neq 0$, $p \in R$, then the standard deviation of these observation is :

(1) $\sqrt{\frac{3}{5}}$ (2) $\frac{4}{5}$ (3) $\frac{9}{10}$ (4) $\frac{7}{10}$

Ans.

$$\sum_{i=1}^{10} (x_i - p) = 3 \quad , \quad \sum_{i=1}^{10} (x_i - p)^2 = 9$$

If we add or subtract a particular number from all observations then standard deviation remains unchanged

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} X_i^2 - (\bar{X})^2$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{10} (x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10} (x_i - p)}{10} \right)^2}$$

$$\sigma = \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2} = \sqrt{\frac{9}{10} - \frac{9}{100}}$$

$$\sigma = \sqrt{\frac{90 - 9}{100}} = \sqrt{\frac{81}{100}}$$

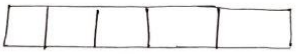
$$\sigma = \frac{9}{10}$$ Ans. option (3)

Standard deviation

13

Q 13 The probability that a randomly chosen 5 digit number is made from exactly two digits is :

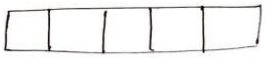
(1) $\frac{135}{10^4}$ (2) $\frac{150}{10^4}$ (3) $\frac{134}{10^4}$ (4) $\frac{121}{10^4}$

Ans. 5 digit number 

$9 \times 10 \times 10 \times 10 \times 10$

$= 9 \times 10^4$

Required way

without '0', (1, 2, ... 9) 

${}^9C_2 \left(2 \times 2 \times 2 \times 2 \times 2 - 2\right)$

$= {}^9C_2 \left(2^5 - 2\right)$

with '0' (0, 1, 2, ... 9) 

$= {}^9C_1 \left(1 \times 2 \times 2 \times 2 \times 2 - 1\right)$

$= {}^9C_1 \left(2^4 - 1\right)$

$$\text{Probability} = \frac{{}^9C_2 \left(2^5 - 2\right) + {}^9C_1 \left(2^4 - 1\right)}{9 \times 10^4}$$

$$= \frac{36 \times 2 \left(2^4 - 1\right) + 9 \left(2^4 - 1\right)}{9 \times 10^4}$$

$$= \frac{135}{10^4}$$ ans. option (1)

14

Q 14 Let $a, b, c \in R$ such that $a^2 + b^2 + c^2 = 1$. If $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right)$ $= c\cos\left(\theta + \frac{4\pi}{3}\right)$ where $\theta = \frac{\pi}{9}$, then the angle between the the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is :

(1) 0 (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{9}$

Ans.

$$a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) + c\cos\left(\theta + \frac{4\pi}{3}\right) = k$$

$$\frac{k}{a} = \cos\theta \quad , \quad \frac{k}{b} = \cos\left(\theta + \frac{2\pi}{3}\right) \quad , \quad \frac{k}{c} = \cos\left(\theta + \frac{4\pi}{3}\right)$$

$$\Rightarrow \frac{k}{a} + \frac{k}{b} + \frac{k}{c} = \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{k}\left(\cos\theta + \cos\left(\frac{\pi}{2} + \left(\frac{\pi}{6} + \theta\right)\right) + \cos\left(\frac{3\pi}{2} - \left(\frac{\pi}{6} - \theta\right)\right)\right)$$

$$= \frac{1}{k}\left(\cos\theta - \sin\left(\frac{\pi}{6} + \theta\right) - \sin\left(\frac{\pi}{6} - \theta\right)\right)$$

$$\left\{ \sin(A+B) + \sin(A-B) = 2\sin A\cos B \right\}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{k}\left(\cos\theta - \left(2\sin\frac{\pi}{6}\cos\theta\right)\right)$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{k}\left(\cos\theta - 2\times\frac{1}{2}\cos\theta\right) = 0$$

$$\Rightarrow \frac{ab + bc + ca}{abc} = 0 \Rightarrow ab + bc + ca = 0$$

$$\cos\alpha = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (b\hat{i} + c\hat{j} + a\hat{k})}{\sqrt{a^2 + b^2 + c^2}\ \sqrt{b^2 + c^2 + a^2}}$$

$$\cos(\alpha) = \frac{ab + bc + ca}{\sqrt{a^2 + b^2 + c^2}\ \sqrt{b^2 + c^2 + a^2}} \qquad \left\{ ab + bc + ca = 0 \right.$$

$$\cos\alpha = 0$$

$$\alpha = \cos^{-1}(0)$$

$$\alpha = \frac{\pi}{2} \quad \text{ans. Option (3)}$$

15

Q 15 If the surface area of a cube is increasing at a rate of $3.6\ cm^2/sec$, retaining its shape : then the rate of change of its volume (in cm^3/sec) when the length of a side of the cube is 10 cm is.

(1) 20 (2) 10 (3) 18 (4) 9

Ans. Side of the cube $= l$

Surface area of the cube $= 6l^2 = A$

$$\frac{dA}{dt} = 12l \cdot \frac{dl}{dt} = 3.6$$

$\frac{dA}{dt} = 3.6\ cm^2/sec$ given

at $l = 10$ cm $\frac{dl}{dt} = ?$

$$12(10)\frac{dl}{dt} = 3.6 \Rightarrow \frac{dl}{dt} = 0.03$$

Volume of the cube $V = l^3$

$$\frac{dV}{dt} = 3l^2\frac{dl}{dt}$$

at $l = 10$ cm $\frac{dV}{dt} = ?$

$$\frac{dV}{dt} = 3(10^2)(0.03) = 3 \times 100 \times 0.03$$

$$\frac{dV}{dt} = 9\ cm^3/sec$$ ans. option (4)

16

Q 16 $\lim_{x \to a} \frac{(a+2x)^{1/3} - (3x)^{1/3}}{(3a+x)^{1/3} - (4x)^{1/3}}$ $(a \neq 0)$ is equal to :

(1) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{1/3}$ (2) $\left(\frac{2}{3}\right)^{4/3}$ $\left(\frac{2}{9}\right)^{4/3}$ (4) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{1/3}$

Ans. $\lim_{x \to a} \frac{(a+2x)^{1/3} - (3x)^{1/3}}{(3a+x)^{1/3} - (4x)^{1/3}}$

when we put $x = a$, it is $\frac{0}{0}$ form

$\therefore$ differentiate numerator and denominator

$$= \lim_{x \to a} \frac{\frac{1}{3}(a+2x)^{-2/3} \cdot 2 - \frac{1}{3}(3x)^{-2/3} \cdot 3}{\frac{1}{3}(3a+x)^{-2/3} \cdot 1 - \frac{1}{3}(4x)^{-2/3} \cdot 4}$$

$$= \lim_{x \to a} \frac{2(a+2x)^{-2/3} - 3(3x)^{-2/3}}{(3a+x)^{-2/3} - 4(4x)^{-2/3}}$$

now, at $x = a$ it is not $\frac{0}{0}$ form

$$= \frac{2(3a)^{-2/3} - 3(3a)^{-2/3}}{(4a)^{-2/3} - 4(4a)^{-2/3}}$$

$$= \frac{-(3a)^{-2/3}}{-3(4a)^{-2/3}} = \frac{1}{3}\left(\frac{3a}{4a}\right)^{-2/3} = \frac{1}{3}\frac{3^{-2/3}}{4^{-2/3}}$$

$$= \frac{1}{3}\frac{4^{2/3}}{3^{2/3}} = \frac{2}{3} \cdot \frac{2^{1/3}}{3^{1/3}} = \frac{2}{3}\left(\frac{2}{3}\right)^{1/3}$$ ans. option (4)

17

Q.17 Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$, $(b<5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1 e_2 = 1$. If α and β are the distance between the foci of the ellipse and the foci of the hyperbola resp. then the ordered pair (α, β) is equal to:

Ans. $e_1 = \sqrt{1 - \frac{b^2}{25}}$; $e_2 = \sqrt{1 + \frac{b^2}{16}}$

$e_1 e_2 = 1$

$\Rightarrow (e_1 e_2)^2 = 1 \Rightarrow \left(1 - \frac{b^2}{25}\right)\left(1 + \frac{b^2}{16}\right) = 1$

$\Rightarrow 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{16 \cdot 25} = 1$

$\Rightarrow \frac{9 b^2}{16 \cdot 25} - \frac{b^4}{16 \cdot 25} = 0$

$\Rightarrow b^2 (9 - b^2) = 0 \Rightarrow b^2 = 9$

$e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

distance between the foci of the ellipse

$$\alpha = 2ae_1 = 2 \times 5 \times \frac{4}{5} = 8$$

distance between the foci of the hyperbola

$$\beta = 2ae_2 = 2 \times 4 \times \frac{5}{4} = 10$$

$$(\alpha, \beta) \equiv (8, 10) \text{ ans.}$$

18

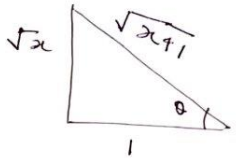
Q 18. If $\int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be :

(1) $(x-1, \sqrt{x})$ (2) $(x-1, -\sqrt{x})$

(3) $(x+1, \sqrt{x})$ (4) $(x+1, -\sqrt{x})$

Ans.

$$\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) = \theta$$



$$\sin\theta = \frac{\sqrt{x}}{\sqrt{1+x}} \quad ; \quad \tan\theta = \sqrt{x}$$

$$\theta = \tan^{-1}\sqrt{x} = \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)$$

$$I = \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+1}}\right) dx = \int \underset{I}{\tan^{-1}(\sqrt{x})} \cdot \underset{II}{1}\, dx$$

$$I = x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \cdot x\, dx + C$$

$$I = x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{x}{\sqrt{x}(x+1)}\, dx + C$$

$$\{ x = t^2 \Rightarrow dx = 2t\, dt \}$$

$$I = x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{t^2 \cdot 2t\, dt}{t(t^2+1)}\, dt$$

$$I = x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}\,dt + c$$

$$I = x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + c$$

$$I = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$$

$$I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c = A(x)\tan^{-1}\sqrt{x} + B(x) + c$$

$$A(x) = x+1$$

$$B(x) = -\sqrt{x}$$

$$(A(x), B(x)) = (x+1, -\sqrt{x})$$ ans.

19

Q 19 If the value of the integration $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}}\,dx$ is $\frac{k}{6}$, then k is equal to:

(1) $2\sqrt{3} + \pi$ (2) $2\sqrt{3} - \pi$ (3) $3\sqrt{2} + \pi$ (4) $3\sqrt{2} - \pi$

Ans.

$$\frac{k}{6} = \int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}}\,dx$$

$$\{ x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta \}$$

$$\Rightarrow \frac{k}{6} = \int_0^{\pi/6} \frac{\sin^2\theta}{(1-\sin^2\theta)^{3/2}} \cdot \cos\theta\, d\theta$$

$$\Rightarrow \frac{k}{6} = \int_0^{\pi/6} \frac{\sin^2\theta \cdot \cos\theta}{(\cos^2\theta)^{3/2}}\,d\theta = \int_0^{\pi/6} \frac{\sin^2\theta \cdot \cos\theta}{\cos^3\theta}\,d\theta$$

$$\Rightarrow \frac{k}{6} = \int_0^{\pi/6} \tan^2\theta\, d\theta = \int_0^{\pi/6} (\sec^2\theta - 1)\,d\theta$$

$$\Rightarrow \frac{k}{6} = \left[\tan\theta - \theta\right]_0^{\pi/6}$$

$$\Rightarrow \frac{k}{6} = \left[\tan\frac{\pi}{6} - \frac{\pi}{6} - \tan 0 + 0\right] = \frac{1}{\sqrt{3}} - \frac{\pi}{6}$$

$$\Rightarrow \frac{k}{6} = \frac{2\sqrt{3} - \pi}{6}$$

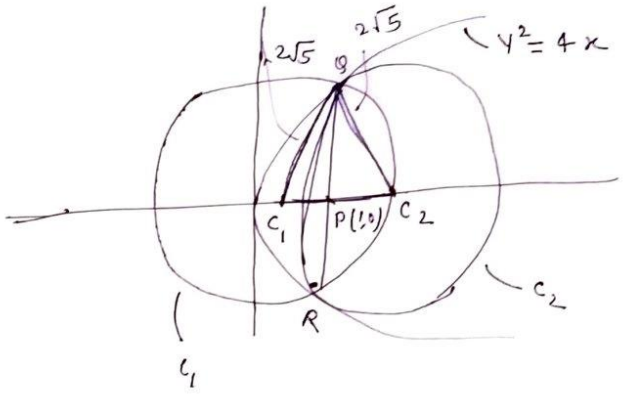
$$\Rightarrow k = 2\sqrt{3} - \pi$$ ans. option (2)

20

Ques 20 let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centers of the circles C_1 and C_2 is

(1) 12 (2) 8 (3) $8\sqrt{5}$ (4) $4\sqrt{5}$

ans.



$C_1Q = C_2Q = R = 2\sqrt{5}$

latus rectum of $y^2 = 4x$ = QR = Common cord of C_1 and C_2

$y^2 = 4x$

$4a = 4 \Rightarrow a = 1$

length of latus rectum $= 4a = 4 = QR$

$QP = \frac{QR}{2} = \frac{4}{2} = 2$

$C_1P = \sqrt{(2\sqrt{5})^2 - (2)^2} = \sqrt{20-4} = 4$

$C_2P = \sqrt{(2\sqrt{5})^2 - (2)^2} = \sqrt{20-4} = 4$

21

Q. 21 If the tangent to the curve $y = e^x$ at a point (c, e^c) and the normal to the parabola $y^2 = 4x$ at the point $(1, 2)$ intersect at the same point on the x-axis, then value of c is

Ans. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$\left.\frac{dy}{dx}\right)_{(c,e^c)} = e^c$ it is slope of tangent at (c, e^c)

eq. of tangent on $y = e^x$ at (c, e^c)

$y - e^c = e^c(x - c)$

at $y = 0$, (x-axis)

$0 - e^c = e^c(x - c) \Rightarrow x = c - 1 \quad \cdots (1)$

$y^2 = 4x \Rightarrow 2y\frac{dy}{dx} = 4$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left.\frac{dy}{dx}\right)_{(1,2)} = \frac{2}{2} = 1$

slope of normal $= -\frac{1}{(dy/dx)} = -\frac{1}{1} = -1$

equation of normal to $y^2 = 4x$ at $(1, 2)$

$y - 2 = -1(x - 1)$

at $y = 0$, (x-axis)

$0 - 2 = -x + 1 \Rightarrow x = 3 \quad \cdots (2)$

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from equation (1) and (2)

$c - 1 = 3$

$c = 4$ ans

22

Ques 22 Let a plane P contains two lines

$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j})$ $\lambda \in R$ and

$\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k})$, $\mu \in R$

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point $M(1,0,1)$ to P, then $3(\alpha+\beta+\gamma)$ is equals to:

Ans. Lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j})$

$\vec{r} = \hat{j} + \mu(\hat{j} - \hat{k})$

$\vec{p} = \hat{i} + \hat{j} + 0\hat{k}$ $\vec{q} = 0\hat{i} + \hat{j} - \hat{k}$

normal to the plane $\vec{\eta}$

$$\vec{\eta} = \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

$\vec{\eta} = \hat{i}(-1-0) - \hat{j}(-1-0) + \hat{k}(1-0)$

$\vec{\eta} = -\hat{i} + \hat{j} + \hat{k}$

Point on Plane $\vec{a} = \hat{i} + 0\hat{j} + 0\hat{k}$

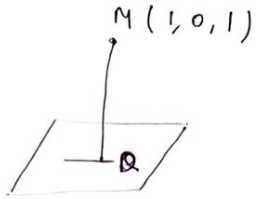
equation of Plane

$\vec{r} \cdot \vec{\eta} = \vec{a} \cdot \vec{\eta}$

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (-\hat{i} + \hat{j} + \hat{k}) = (\hat{i}) \cdot (-\hat{i} + \hat{j} + \hat{k})$

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$-x + y + z = -1$



$Q\ (1-\lambda,\ 0+\lambda,\ 1+\lambda) \equiv (\alpha, \beta, \gamma)$

Q on the plane

$-x + y + z = -1$

$\Rightarrow -(1-\lambda) + \lambda + 1 + \lambda = -1$

$\Rightarrow \lambda - 1 + \lambda + 1 + \lambda = -1$

$3\lambda = -1 \Rightarrow \lambda = -1/3$

$\alpha = 1 - \lambda = 1 - (-1/3) = 1 + 1/3$

$\alpha = 4/3$

$\beta = \lambda = -1/3$

$\gamma = 1 + \lambda = 1 + \left(-\frac{1}{3}\right) = 1 - 1/3$

$\gamma = 2/3$

$3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right)$

$= 3\left(\frac{5}{3}\right)$

$= 5$ ans.

23

Q 23 If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M then m is equal to

Ans. $3, A_1, A_2, A_3, \ldots, A_m, 243$ A.M.

first term $a = 3$, common difference $= d = ?$

$$243 = 3 + (m+2-1)d$$

$$\Rightarrow d = \frac{240}{m+1}$$

$3, G_1, G_2, G_3, 243$ G.P.

first term $= 3$, Common ratio $= r = ?$

$$243 = 3r^{5-1}$$

$$\Rightarrow r = \left(\frac{243}{3}\right)^{\frac{1}{4}} = (81)^{1/4}$$

$$r = 3$$

$$G_2 = A_4$$

$$3(3)^{3-1} = 3 + (5-1)\left(\frac{240}{m+1}\right)$$

$$\Rightarrow 27 = 3 + 4\left(\frac{240}{m+1}\right)$$

$$\Rightarrow 24(m+1) = 4(240)$$

$$\Rightarrow m+1 = 40$$

$$m = 39 \quad \text{ans.}$$

24

Q 24 The total number of 3-digit numbers, whose sum of digits is 10, is

Ans. Let three digit number be ABC

$A+B+C = 10$ $\quad A \geq 1,\ B \geq 0,\ C \geq 0$

Let $A = Y+1$

$Y+1+B+C = 10$

$Y+B+C = 9$ $\quad Y \geq 0$

$0 \leq Y, B, C \leq 9$

Number of solutions which are non-negative

$= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11 \times 10}{1 \times 2}$

$= 55$

but If $Y = 9 \Rightarrow A = Y+1 = 10$

then total number of solutions $= 55-1 = 54$

25

Q 25 Let S be the set of all integers solutions (x, y, z) of the system of equations $x - 2y + 5z = 0$, $-2x + 4y + z = 0$, $-7x + 14y + 9z = 0$, such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to...

Ans.

$$x - 2y + 5z = 0 \quad \text{---- (1)}$$

$$-2x + 4y + z = 0 \quad \text{--- (2)}$$

$$-7x + 14y + 9z = 0 \quad \text{---- (3)}$$

$2 \times (1) + (2)$

$$2x - 4y + 10z = 0$$
$$-2x + 4y + z = 0$$
$$11z = 0 \Rightarrow z = 0$$

$$x - 2y + 5z = 0, \quad z = 0$$

$$x = 2y$$

$$15 \le x^2 + y^2 + z^2 \le 150$$
$$15 \le (2y)^2 + y^2 + (0)^2 \le 150$$
$$15 \le 5y^2 \le 150$$
$$3 \le y^2 \le 30$$
$$y = \pm 2, \pm 3, \pm 4, \pm 5$$

Total number of solutions are 8

Number of elements in the set S is 8 ans.

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