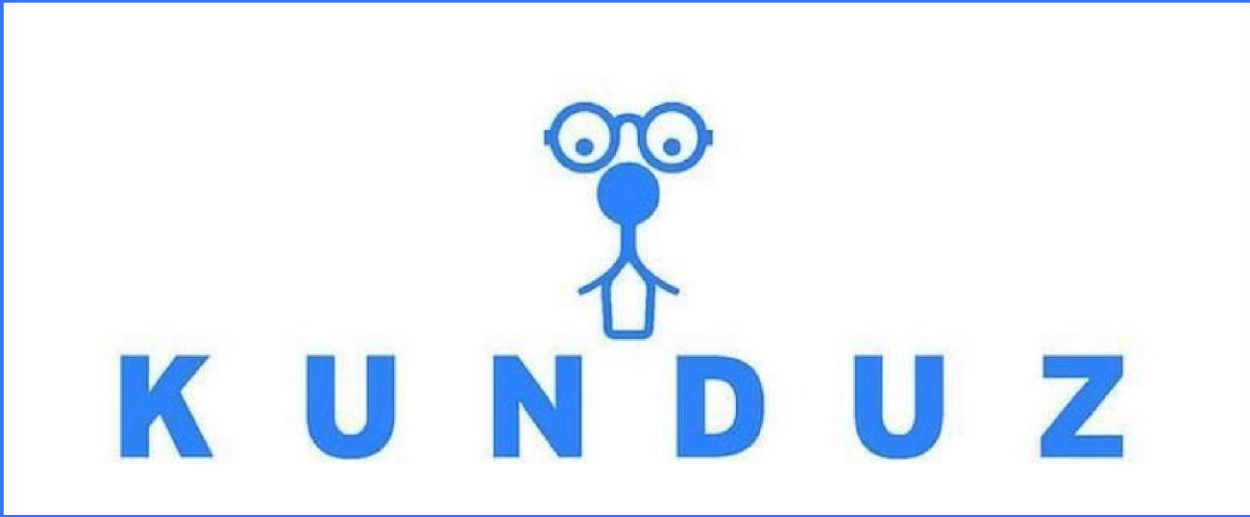


JEE Mains 4th September 2020 shift 1 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

Q 1 Let $Y = y(x)$ be the solution of the differential equation $xy' - y = x^2(x\cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to:

(1) $1 + \frac{\pi}{2}$ (2) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (3) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (4) $2 + \frac{\pi}{2}$

Ans. $xy' - y = x^2(x\cos x + \sin x)$

$$\frac{dy}{dx} - \frac{1}{x}y = x(x\cos x + \sin x)$$

$P(x)$ $Q(x)$

1st order linear differential equation

Integrating factor (I.F) $= e^{\int P(x)\,dx}$

$$I.F = e^{\int -\frac{1}{x}\,dx}$$

$$I.F = e^{-\ln x} = \frac{1}{x}$$

Solution of the given differential eq.

$$y\cdot\frac{1}{x} = \int \frac{1}{x}\cdot x(x\cos x + \sin x)\,dx + c$$

$$\Rightarrow \frac{y}{x} = \int (x\cos x + \sin x)\,dx + c$$

$$\Rightarrow \frac{y}{x} = x\sin x - \int \sin x\,dx - \cos x + c$$

$$\Rightarrow \frac{y}{x} = x\sin x + \cos x - \cos x + c$$

$$\Rightarrow \frac{y}{x} = x\sin x + c$$

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$$y = x^2 \sin x + cx$$

$\because\ y(\pi) = \pi$ $\quad \{ \sin \pi = 0$

$\therefore\ \pi = (\pi)^2 \sin \pi + c\pi$

$c\pi = \pi \Rightarrow c = 1$

$$y(x) = x^2 \sin x + x$$

$$y'(x) = 2x \sin x + x^2 \cos x + 1$$

$$y''(x) = 2\sin x + 2x\cos x + 2x\cos x - x^2 \sin x$$

$$y''(x) = 2\sin x + 4x\cos x - x^2 \sin x$$

$$y\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^2 \sin\frac{\pi}{2} + \frac{\pi}{2} \qquad \{ \sin\frac{\pi}{2} = 1$$

$$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2} \qquad \{ \cos\frac{\pi}{2} = 0$$

$$y''\left(\frac{\pi}{2}\right) = 2\sin\frac{\pi}{2} + 4\frac{\pi}{2}\cos\frac{\pi}{2} - \left(\frac{\pi}{2}\right)^2 \sin\frac{\pi}{2}$$

$$y''\left(\frac{\pi}{2}\right) = 2 + 0 - \frac{\pi^2}{4} = 2 - \frac{\pi^2}{4}$$

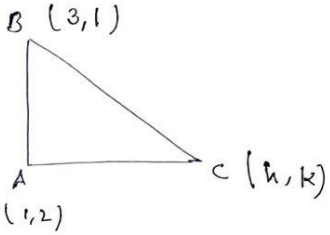
$$y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} + \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = 2 + \frac{\pi}{2}$$ ans. option (4)

2

Q 2. A triangle ABC lying in the first quadrant has two vertices as A(1,2) and B(3,1). If $\angle BAC = 90^\circ$ and area $(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is:

Ans. (1) $1+\sqrt{5}$ (2) $2+\sqrt{5}$ (3) $1+2\sqrt{5}$ (4) $2\sqrt{5}-1$



$BA \perp AC$

$m_{AB} \times m_{AC} = -1$

$\frac{2-1}{1-3} \cdot \frac{k-2}{h-1} = -1$

$\Rightarrow -\frac{1}{2}\left(\frac{k-2}{h-1}\right) = -1 \Rightarrow \frac{k-2}{h-1} = 2$

$\Rightarrow k-2 = 2h-2 \Rightarrow k = 2h$

area of $ABC = 5\sqrt{5}$

$\frac{1}{2} AB \times AC = 5\sqrt{5} \Rightarrow AB \times AC = 10\sqrt{5}$

$\Rightarrow (AB \times AC)^2 = (10\sqrt{5})^2$

$\Rightarrow AB^2 \times AC^2 = 500$

$\Rightarrow [(1-3)^2+(2-1)^2] \times [(k-2)^2+(h-1)^2] = 500$

$\Rightarrow (4+1)[(2h-2)^2+(h-1)^2] = 500$

$\Rightarrow 5[4(h-1)^2+(h-1)^2] = 500$

$\Rightarrow 5 \times 5 \times (h-1)^2 = 500$

$(h-1)^2 = 20$

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$(h-1)^2 = 20$

$|h-1| = 2\sqrt{5}$

$\therefore \quad h = 1 \pm 2\sqrt{5}$

$\because \triangle ABC$ lying in first quadrant

$\therefore \quad h = 1 + 2\sqrt{5}$ ans. option (3)

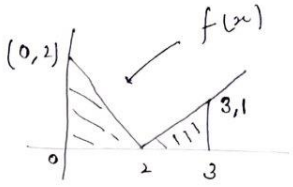
3

Q 3 let $f(x) = |x-2|$ and $g(x) = f(f(x))$, $x \in [0,4]$. Then $\int_0^3 (g(x) - f(x))\,dx$ is equal to:

(1) 0 (2) $\frac{1}{2}$ (3) $\frac{3}{2}$ (4) 1

Ans.

$f(x) = |x-2|$



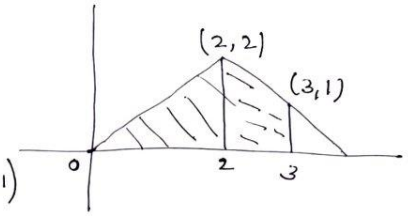
$\int_0^3 f(x)\,dx$ = Area of sheded region

$\int_0^3 f(x)\,dx = \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 1 = 2 + \frac{1}{2}$

$= \frac{5}{2}$

$g(x) = f(f(x)) = f(|x-2|) = ||x-2| - 2|$

$\int_0^3 g(x)\,dx$ = area of sheded region



$\int_0^3 g(x)\,dx = \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times (2+1)$

$= 2 + \frac{3}{2} = \frac{7}{2}$

$\int_0^3 g(x)\,dx - \int_0^3 f(x)\,dx = \frac{7}{2} - \frac{5}{2}$

$= 1$ ans. option (4)

4

Q 4 The integral $\int \left(\frac{x}{(x\sin x+\cos x)}\right)^2 dx$ is equal to

Ans. $\int \left(\frac{x}{x\sin x+\cos x}\right)^2 dx = \int \frac{x^2}{(x\sin x+\cos x)^2} dx$

$= \int (x\sec x)\cdot\frac{x\cos x}{(x\sin x+\cos x)^2} dx$

$\quad\quad\quad

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$$= \frac{-x \sec x}{x \sin x + \cos x} + \tan x + c$$

$$= \tan x - \frac{x \sec x}{x \sin x + \cos x} + c \quad \text{Ans.}$$

5

Q 5 If $1+(1-2^2 \cdot 1)+(1-4^2 \cdot 3)+(1-6^2 \cdot 5)+\cdots+(1-20^2 \cdot 19) = \alpha - 220\beta$ then ordered pair (α, β) is

(1) $(11, 97)$ (2) $(10, 103)$ (3) $(11, 103)$ (4) $(10, 97)$

Ans.

$$\text{Sum} = 1+\sum_{r=1}^{10}\left(1-(2r)^2(2r-1)\right)$$

$$= 1+\sum_{r=1}^{10}\left(1-4r^2(2r-1)\right)$$

$$= 1+\sum_{r=1}^{10}\left(1-8r^3+4r^2\right)$$

$$= 1+10-8\sum_{r=1}^{10}r^3+4\sum_{r=1}^{10}r^2$$

$$= 11-8\left(\frac{10\times 11}{2}\right)^3+4\left(\frac{10\times 11\times 21}{6}\right)$$

$$= 11-8(55)^2+55\times 28$$

$$= 11+55(28-440)$$

$$= 11+55(-412)$$

$$= 11-220\times 103 = \alpha - 220\beta$$

$\therefore \alpha = 11, \beta = 103$

$(\alpha, \beta) = (11, 103)$ ans. option (3)

6

Q 6 A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If X% of the people read both the newspapers then a possible value of X can be

(1) 65 (2) 55 (3) 37 (4) 29

Ans. A = Person who read newspaper of type A

B = Person who read newspaper of type B

$n(A) = 63\%$ $n(B) = 76\%$

$n(A \cap B) = X\%$

$n(A \cup B) = n(A) + n(B) - n(A \cap B) \leq 100$

$63 + 76 - X \leq 100$

$x \geq 39$

$n(A \cap B) \leq \min\{n(A), n(B)\}$

$n(A \cap B) \leq n(A)$

$X \leq 63$

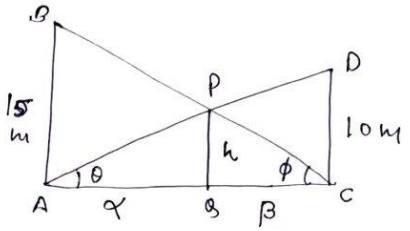
$\therefore 39 \leq X \leq 63$

$X = 55$ ans. option (2)

7

Q 7 two vertical poles AB = 15m and CD = 10m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :

Ans. (1) $10/3$ (2) 6 (3) 5 (4) $20/3$



let $AQ = \alpha$

$CQ = \beta$

$\triangle APQ$

$$\tan\theta = \frac{h}{\alpha}$$

$\triangle ADC$

$$\tan\theta = \frac{10}{\alpha+\beta}$$

$$\therefore \frac{h}{\alpha} = \frac{10}{\alpha+\beta} \quad \text{------ (1)}$$

$\triangle CPQ$

$$\tan\phi = \frac{h}{\beta}$$

$\triangle ABC$

$$\tan\phi = \frac{15}{\alpha+\beta} \quad \text{------ (2)}$$

$$\therefore \frac{h}{\beta} = \frac{15}{\alpha+\beta} \quad \text{------ (2)}$$

from eq. (1) and (2) { (1) ÷ (2) }

$$\frac{h/\alpha}{h/\beta} = \frac{10/(\alpha+\beta)}{15/(\alpha+\beta)}$$

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$$\frac{\beta}{\alpha} = \frac{2}{3} \quad \ldots (3)$$

using (1) and (3)

$$\frac{h}{\alpha} = \frac{10}{\alpha+\beta}$$

$$\frac{h}{\alpha} = \frac{10}{\alpha\left(1+\frac{\beta}{\alpha}\right)} \Rightarrow h = \frac{10}{1+\frac{2}{3}}$$

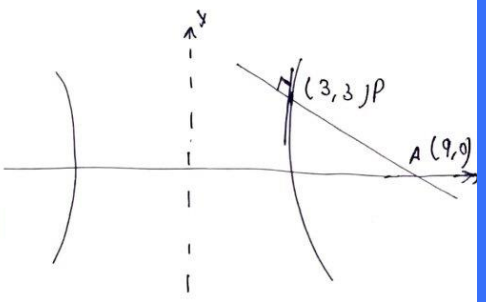
$$\Rightarrow h = \frac{10}{5/3} \Rightarrow h = 6 \quad \underline{\text{ans.}} \text{ option (2)}$$

8

Q 8 Let $P(3,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to:

(1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{3}{2}, \frac{9}{2}\right)$ (3) $(9, 3)$ (4) $\left(\frac{9}{2}, 2\right)$

Ans.



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\because$ P lies on it $\{P \equiv (3,3)\}$

$\therefore \quad \frac{9}{a^2} - \frac{9}{b^2} = 1$

$\Rightarrow \quad \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{9} \quad \text{-----} \quad (1)$

For slope of normal

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{x b^2}{y a^2} \Rightarrow \left.\frac{dy}{dx}\right)_{(3,3)} = \frac{b^2}{a^2}$

$\Rightarrow$ Slope of normal at P $= -\frac{1}{(dy/dx)} = -\frac{1}{b^2/a^2} = -\frac{a^2}{b^2}$

Slope of PA = slope of normal at P

$\frac{3-0}{3-9} = -\frac{a^2}{b^2} \Rightarrow \frac{1}{2} = \frac{a^2}{b^2}$

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$\Rightarrow \quad b^2 = 2a^2 \quad ---(2)$

from equation (1) and (2)

$$\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{9} \quad \Rightarrow \quad \frac{1}{a^2} - \frac{1}{2a^2} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{1}{2a^2} = \frac{1}{9}$$

$$a^2 = \frac{9}{2} \qquad b^2 = 9$$

$$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{2a^2}{a^2}$$

$$e^2 = 3$$

$$(a^2, e^2) \equiv \left(\frac{9}{2}, 3\right) \quad \underline{\text{ans.}} \quad \text{option (1)}$$

9

Q 9 Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x, $[x]^2 + 2[x+2] - 7 = 0$ has :

(1) No integral solution (2) infinitely many solutions

(3) exactly two solutions (4) exactly four integral solutions

Ans. $\{ [x+I] = [x] + I \quad , I \in \text{Integer} \}$

$$[x]^2 + 2[x+2] - 7 = 0$$

$$\Rightarrow [x]^2 + 2([x]+2) - 7 = 0$$

$$\Rightarrow [x]^2 + 2[x] - 3 = 0 \qquad \{ [x] = t$$

$$\Rightarrow t^2 + 2t - 3 = 0$$

$$\Rightarrow t^2 + 3t - t - 3 = 0$$

$$(t+3)(t-1) = 0$$

$$t = -3, 1$$

$$[x] = -3, \underline{1}$$

$[x] = -3$ $\qquad$ $[x] = 1$

$-3 \leq x < -2$ $\qquad$ $1 \leq x < 2$

$\therefore \quad x \in [-3, -2) \cup [1, 2)$

Hence infinitely many solutions ans. option (2)

10

Q 10 If $(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos y) = a^2-b^2$, where $a>b>0$

then $\frac{dy}{dx}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is

(1) $\frac{2a+b}{2a-b}$ (2) $\frac{a+b}{a-b}$ (3) $\frac{a-b}{a+b}$ (4) $\frac{a-2b}{a+2b}$

Ans. $(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos y) = a^2-b^2$

differentiate w.r.t 'x'

$$(0-\sqrt{2}b\sin x)(a-\sqrt{2}b\cos y) + (a+\sqrt{2}b\cos x)(0+\sqrt{2}b\sin y)\frac{dy}{dx} = 0$$

at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

$$\Rightarrow (0-\sqrt{2}b\times\frac{1}{\sqrt{2}})(a-\sqrt{2}b\frac{1}{\sqrt{2}}) + (a+\sqrt{2}b\frac{1}{\sqrt{2}})(0+\sqrt{2}b\frac{1}{\sqrt{2}})\frac{dy}{dx} = 0$$

$$\Rightarrow (-b)(a-b) + (a+b)(b)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{b(a-b)}{b(a+b)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a-b}{a+b}$$ ans. option (3)

$$\frac{dx}{dy} = \frac{a+b}{a-b}$$ option (2)

11

Q 11 Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ at $x = x_0$ is

(1) -22 (2) 14 (3) -4 (4) -30

Ans. $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$

$$f(x) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$

$$f(x) = x\left(x^2 - (-2)(-1)\right) - (-2)\left((-2)(x) - 7(-1)\right) + 3\left((-2)(-2) - 7(x)\right)$$

$$f(x) = x(x^2 - 2) + 2(-2x + 7) + 3(4 - 7x)$$

$$f(x) = x^3 - 2x - 4x + 14 + 12 - 21x$$

$$f(x) = x^3 - 27x + 26$$

$$f'(x) = 3x^2 - 27 = 0 \Rightarrow x^2 = 9$$

$$x = \pm 3$$

$x = -3$ is the point of local maxima

$\therefore\ x_0 = -3$

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$$\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$$

$$= (x\hat{i} - 2\hat{j} + 3\hat{k})\cdot(-2\hat{i} + x\hat{j} - \hat{k}) + (-2\hat{i} + x\hat{j} - \hat{k})\cdot(7\hat{i} - 2\hat{j} + x\hat{k})$$

$$+ (7\hat{i} - 2\hat{j} + x\hat{k})\cdot(x\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= (-2x - 2x - 3) + (-14 - 2x - x) + (7x + 4 + 3x)$$

$$= 3x - 13$$

at $x = x_0$

$$= 3x_0 - 13$$

$$= 3(-3) - 13 = -22$$ ans. option (1)

12

Q 12 Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}\,dx \quad (x \geq 0)$. Then $f(3) - f(1)$ is equals to :

Ans. Let $\sqrt{x} = \tan\theta$

$x = \tan^2\theta \Rightarrow dx = 2\tan\theta \sec^2\theta\, d\theta$

$$I = f(x) = \int \frac{\sqrt{x}}{(1+x)^2}\,dx$$

$$I = \int \frac{\tan\theta \cdot 2\tan\theta \sec^2\theta\, d\theta}{(1+\tan^2\theta)^2}$$

$$I = \int \frac{2\tan^2\theta \cdot \sec^2\theta}{\sec^4\theta}\,d\theta$$

$$I = \int \frac{2(\sec^2\theta - 1)}{\sec^2\theta}\,d\theta$$

$$I = \int 2(1 - \cos^2\theta)\,d\theta$$

$$I = \int 2\sin^2\theta\,d\theta$$

$$I = \int (1 - \cos 2\theta)\,d\theta$$

$$I = \theta - \frac{\sin 2\theta}{2} + c$$

$$I = \theta - \frac{\tan\theta}{1+\tan^2\theta} + c$$

$\tan^2\theta = \sec^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\frac{\sin 2\theta}{2} = \frac{2\sin\theta\cos\theta}{2}$

$= \frac{\sin\theta\cos\theta}{\cos^2\theta\sec^2\theta}$

$= \frac{\tan\theta}{1+\tan^2\theta}$

$$I = f(x) = \tan^{-1}(\sqrt{x}) - \frac{\sqrt{x}}{1+x} + c$$

$$f(3) = \tan^{-1}(\sqrt{3}) - \frac{\sqrt{3}}{1+3} + c$$

$$f(3) = \frac{\pi}{3} - \frac{\sqrt{3}}{4} + c$$

$$f(1) = \tan^{-1} 1 - \frac{1}{1+1} + c$$

$$f(1) = \frac{\pi}{4} - \frac{1}{2} + c$$

$$f(3) - f(1) = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} + c\right) - \left(\frac{\pi}{4} - \frac{1}{2} + c\right)$$

$$f(3) - f(1) = \frac{\pi}{3} - \frac{\pi}{4} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

$$f(3) - f(1) = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4} \quad \text{ans.}$$

13

Q.13 The mean and variance of 8 observations are 10 and 13.5, resp. If 6 of these observations are 5, 7, 10, 12, 14, 15 then the absolute difference of remaining two observations is :

(1) 9 (2) 5 (3) 3 (4) 7

Ans. let remaining two observations be a and b

$$\text{Mean} = 10 = \frac{5+7+10+12+14+15+a+b}{8} = \bar{x}$$

$$\Rightarrow \quad 80 = 63 + a + b$$

$$a + b = 17$$

$$\text{Variance} = \sigma^2 = \frac{1}{n}\sum x_i^2 - (\bar{x})^2$$

$$13.5 = \frac{1}{8}\left(25+49+100+144+196+225 + a^2+b^2\right) - 100$$

$$\Rightarrow \quad 108 = 739 + a^2 + b^2 - 800$$

$$\Rightarrow \quad a^2 + b^2 = 169$$

$$(a+b)^2 = a^2 + b^2 + 2ab \qquad \left\{ \begin{array}{l} a+b = 17 \\ a^2+b^2 = 169 \end{array} \right\}$$

$$\Rightarrow \quad (17)^2 = 169 + 2ab$$

$$\Rightarrow \quad 289 - 169 = 2ab$$

$$\Rightarrow \quad 2ab = 120 \quad \Rightarrow \quad ab = 60$$

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$$(a-b)^2 = (a+b)^2 - 4ab$$

$$\begin{cases} a+b = 17 \\ ab = 60 \end{cases}$$

$$(a-b)^2 = (17)^2 - 4(60)$$

$$(a-b)^2 = 289 - 240$$

$$(a-b)^2 = 49 \Rightarrow a-b = \pm 7$$

$\therefore |a-b| = 7$ ans. option (4)

14

Q 14 Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function $\phi(t) = \frac{5}{12} + t - t^2$ then $a^2 + b^2$ is equal to:

Ans. (1) 145 (2) 126 (3) 116 (4) 135

$$\phi(t) = \frac{5}{12} + t - t^2$$

$$\phi'(t) = 0 + 1 - 2t = 0 \Rightarrow t = 1/2$$

$$\phi''(t) = -2 < 0 \quad \therefore$$

$\therefore \phi(t)$ will be maximum at $t = 1/2$

$$\text{Max } \phi(t) = \frac{5}{12} + \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{5}{12} + \frac{1}{2} - \frac{1}{4}$$

$$\text{Max } \phi(t) = \frac{2}{3} = e$$

length of latus rectum of an ellipse is given by $LR = \frac{2b^2}{a}$

$$\frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$$

$$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{4}{9}\left(\frac{2}{3}\right)^2 = 1 - \frac{5a}{a^2}$$

$$\Rightarrow \frac{4}{9} = 1 - \frac{5}{a} \Rightarrow \frac{5}{a} = \frac{5}{9}$$

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$a = 9 \Rightarrow a^2 = 81$

$b^2 = 5a \Rightarrow b^2 = 5 \times 9$

$b^2 = 45$

$a^2 + b^2 = 81 + 45$

$a^2 + b^2 = 126$ ans. option (2)

15

Q 15 Let f be a twice differentiable function on $(1,6)$. If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$ for all $x \in (1,6)$ then

(1) $f(5) + f'(5) \geq 28$

(2) $f'(5) + f''(5) \leq 20$

(3) $f(5) \leq 10$

(4) $f(5) + f'(5) \leq 26$

Ans. $f'(x) \geq 1$ -- given

$$\int_2^5 f'(x)\,dx \geq \int_2^5 1 \cdot dx$$

$$\Rightarrow [f(x)]_2^5 \geq [x]_2^5$$

$$\Rightarrow f(5) - f(2) \geq 5 - 2$$

$\{ f(2) = 8$ -- given

$$\Rightarrow f(5) \geq 3 + f(2)$$

$$\Rightarrow f(5) \geq 3 + 8$$

$$f(5) \geq 11 \quad -- (1)$$

$f''(x) \geq 4$ ---- given

$$\Rightarrow \int_2^5 f''(x)\,dx \geq \int_2^5 4 \cdot dx$$

$$\Rightarrow [f'(x)]_2^5 \geq [4x]_2^5$$

$$\Rightarrow f'(5) - f'(2) \geq 4(5-2)$$

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$\Rightarrow f'(5) \geq 12 + f'(2)$ $\quad \left\{ f'(2) = 5 \text{ ... given} \right.$

$\Rightarrow f'(5) \geq 12 + 5$

$\Rightarrow f'(5) \geq 17 \quad \text{---} \quad (2)$

from equation (1) and (2)

$f(5) + f'(5) \geq 11 + 17$

$f(5) + f'(5) \geq 28$ ans. option (1)

16

Q 16 Let $u = \dfrac{2z + i}{z - ki}$, $z = x + iy$ and $k > 0$. If the curve represented by $Re(u) + Im(u) = 1$, intersects the Y-axis at points P and Q, where $PQ = 5$ then the value of k is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 4 (4) 2

Ans.

$$u = \frac{2z + i}{z - ki}, \quad z = x + iy, \quad k > 0$$

Put $z = x + iy$

$$u = \frac{2x + 2yi + i}{x + yi - ki}$$

$$u = \frac{2x + (2y+1)i}{x + (y-k)i} \times \frac{x - (y-k)i}{x - (y-k)i}$$

$$u = \frac{2x^2 - 2x(y-k)i + x(2y+1)i + (2y+1)(y-k)}{x^2 + (y-k)^2}$$

$$Re(u) = \frac{2x^2 + (2y+1)(y-k)}{x^2 + (y-k)^2}$$

$$Im(u) = \frac{(2y+1)x - 2x(y-k)}{x^2 + (y-k)^2} = \frac{x(1+2k)}{x^2 + (y-k)^2}$$

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Let Curve C : $Re(u) + Im(u) = 1$

$$\Rightarrow \frac{2x^2 + (2y+1)(y-k)}{x^2 + (y-k)^2} + \frac{x(1+2k)}{x^2+(y-k)^2} = 1$$

$$\Rightarrow 2x^2 + (2y+1)(y-k) + x(1+2k) = x^2 + (y-k)^2 \quad \ldots\ldots (1)$$

Curve C cuts Y-axis at two points P and Q,

For point of intersection at Y-axis, Put $x = 0$ in the equation (1)

$$\Rightarrow 0 + (2y+1)(y-k) + 0 = 0 + (y-k)^2$$

$$\Rightarrow (y-k)^2 - (2y+1)(y-k) = 0$$

$$\Rightarrow (y-k)(y-k-2y-1) = 0$$

$$\Rightarrow (y-k)(-k-1-y) = 0$$

$$y = k \quad , \quad y = -(k+1)$$

$P(0, k)$ and $Q(0, -(k+1))$

$$PQ = |k - (-(k+1))| = 5$$

$$|k + k + 1| = 5 \Rightarrow 2k+1 = \pm 5$$

$$k = -3, 2$$

$\because k > 0$ -- given, $\therefore k = 2$ ans option (4)

17

Q 17 If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true ?

(1) $a \leq a^2 + b^2 \leq 1$ (2) $a^2 - d^2 = 0$

(3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

Ans. $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$ $\left\{ i^2 = -1 \right.$

$$A^2 = A \cdot A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2i\sin\theta\cos\theta \\ 2i\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix} \quad \left\{ \begin{array}{l} \cos 2\theta = \cos^2\theta - \sin^2\theta \\ \sin 2\theta = 2\sin\theta\cos\theta \end{array} \right.$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\theta\cos\theta - \sin 2\theta\sin\theta & i(\sin\theta\cos 2\theta + \sin 2\theta\cos\theta) \\ i(\sin 2\theta\cos\theta + \cos 2\theta\sin\theta) & -\sin 2\theta\sin\theta + \cos 2\theta\cos\theta \end{bmatrix}$$

$$\left\{\begin{array}{l} \sin(A+B) = \sin A\cos B + \cos A\sin B \\ \cos(A+B) = \cos A\cos B - \sin A\sin B \end{array}\right\}$$

$$A^3 = \begin{bmatrix} \cos 3\theta & i\sin 3\theta \\ i\sin 3\theta & \cos 3\theta \end{bmatrix}$$

Similarly

$$A^5 = \begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a = \cos 5\theta \qquad b = i\sin 5\theta$

$c = i\sin 5\theta \qquad d = \cos 5\theta$

$\underline{a^2 - b^2} = \cos^2 5\theta - i^2\sin^2 5\theta = \cos^2 5\theta + \sin^2 5\theta = 1$

$\underline{a^2 - d^2} = \cos^2 5\theta - \cos^2 5\theta = 0$

$\underline{a^2 + b^2} = \cos^2 5\theta + i^2\sin^2 5\theta$

$a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta$

$a^2 + b^2 = \cos 10\theta = \cos\left(10 \times \frac{\pi}{24}\right) = \cos\left(\frac{5\pi}{12}\right) \in (0, 1)$

$a^2 - c^2 = \cos^2 5\theta - i^2\sin^2 5\theta$

$\underline{a^2 - c^2} = \cos^2 5\theta + \sin^2 5\theta = 1$

18

Q 18 Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a Geometric Progression. The ratio $(2q+p) : (2q-p)$ is

(1) 3 : 1 (2) 5 : 3 (3) 9 : 7 (4) 33 : 31

Ans. α, β are roots of $x^2 - 3x + p = 0$

$\therefore \quad \alpha + \beta = 3 \quad , \quad \alpha\beta = p$

γ, δ are roots of $x^2 - 6x + q = 0$

$\gamma + \delta = 6 \quad , \quad \gamma\delta = q$

$\because \alpha, \beta, \gamma, \delta$ are in G.P.

Let $a, \beta = ar, \gamma = ar^2, \delta = ar^3$, where r is common ratio of the G.P.

$\alpha + \beta = 3 \Rightarrow a + ar = 3 \quad \ldots (1)$

$\gamma + \delta = 6 \Rightarrow ar^2 + ar^3 = 6 \quad \ldots (2)$

from equation (1) and (2)

$$\frac{a + ar}{ar^2 + ar^3} = \frac{3}{6} \Rightarrow \frac{(a + ar)}{r^2(a + ar)} = \frac{1}{2}$$

$\Rightarrow r^2 = 2$

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$$\frac{2q+p}{2q-p} = \frac{2\gamma\delta+\alpha\beta}{2\gamma\delta-\alpha\beta} \qquad \begin{cases} q = \gamma\delta \\ p = \alpha\beta \end{cases}$$

$$\Rightarrow \frac{2q+p}{2q-p} = \frac{2(\alpha r^2)(\alpha r^3)+\alpha(\alpha r)}{2(\alpha r^2)(\alpha r^3)-\alpha(\alpha r)}$$

$$= \frac{2\alpha^2 r^5+\alpha^2 r}{2\alpha^2 r^5-\alpha^2 r}$$

$$= \frac{\alpha^2 r(2r^4+1)}{\alpha^2 r(2r^4-1)} = \frac{(2(2)^2+1)}{(2(2)^2-1)}$$

$$= \frac{9}{7} \quad \text{ans. option (3)}$$

19

Q 19 The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to

(1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{51}C_7 + {}^{30}C_7$

(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Ans. ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$$\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \cdots + {}^{31}C_6 + {}^{30}C_6$$

$$= {}^{50}C_6 + {}^{49}C_6 + \cdots + {}^{31}C_6 + {}^{30}C_6 + {}^{30}C_7 - {}^{30}C_7$$

$$= {}^{50}C_6 + {}^{49}C_6 + \cdots + {}^{31}C_6 + {}^{31}C_7 - {}^{30}C_7$$

$$= {}^{50}C_6 + {}^{49}C_6 + \cdots + {}^{31}C_7 - {}^{30}C_7$$

$$= {}^{51}C_7 - {}^{30}C_7$$ Ans. option (1)

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20

Q 20 Given the following two statements

(S_1) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology

(S_2) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy then :

(1) Only (S_2) is correct (2) both (S_1) and (S_2) are not correct

(3) both (S_1) and (S_2) are correct (4) only (S_1) is correct

Ans.

(S_1)

p	q	$\sim p$	$\sim q$	$q \vee p$	$p \leftrightarrow \sim q$	$(q \vee p) \rightarrow (p \leftrightarrow \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	F	T

↓

Not a tautology

(S_2)

$\sim p \leftrightarrow q$	$\sim q \wedge (\sim p \leftrightarrow q)$
F	F
T	T
T	F
F	F

Not a Fallacy

So both (S_1) and (S_2) are not correct ans. option (2)

21

Q 21 If the equation of a plane P, passing through the intersection of the planes, $x+4y-z+7=0$ and $3x+y+5z=8$ is $ax+by+6z=15$ for some $a, b \in R$, then the distance of the point $(3,2,-1)$ from the plane P is ...

Ans. $P: \quad ax+by+6z=15$

$P_1: \quad x+4y-z+7=0$

$P_2: \quad 3x+y+5z-8=0$

$P = P_1 + \lambda P_2$

$x+4y-z+7+\lambda(3x+y+5z-8)=0$

$\Rightarrow x(1+3\lambda)+y(4+\lambda)+z(5\lambda-1)+7-8\lambda=0$

$$\Rightarrow \frac{x}{\frac{8\lambda-7}{1+3\lambda}} + \frac{y}{\frac{8\lambda-7}{4+\lambda}} + \frac{z}{\frac{8\lambda-7}{5\lambda-1}} = 1 \quad \cdots (1)$$

$P: \quad ax+by+6z=15$

$$\Rightarrow \frac{x}{\frac{15}{a}} + \frac{y}{\frac{15}{b}} + \frac{z}{\frac{15}{6}} = 1 \quad \cdots (2)$$

Since (1) and (2) represent same plane P

$$\therefore \frac{8\lambda-7}{5\lambda-1} = \frac{15}{6} \Rightarrow 48\lambda-42 = 75\lambda-15$$

$$\Rightarrow 27\lambda = -27$$

$$\lambda = -1$$

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Equation of Plane P is

$\lambda = -1$

$\Rightarrow x(1+3\lambda) + y(4+\lambda) + z(5\lambda-1) + 7 - 8\lambda = 0$

$\Rightarrow x(1-3) + y(4-1) + z(-5-1) + 7 + 8 = 0$

$\Rightarrow -2x + 3y - 6z + 15 = 0$

$\Rightarrow 2x - 3y + 6z - 15 = 0$

distance of the point $(3, 2, -1)$ from the plane P

is $= \dfrac{|2\cdot 3 - 3\cdot 2 + 6\cdot(-1) - 15|}{\sqrt{(2)^2 + (-3)^2 + (6)^2}}$

$= \dfrac{|6 - 6 - 6 - 15|}{\sqrt{4+9+36}}$

$= \dfrac{21}{7} = 3$ ans.

22

Q 22. Suppose a differentiable function $f(x)$ satisfies the identity $f(x+y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim_{x\to 0} \frac{f(x)}{x} = 1$, then $f'(3)$ is equal to:

Ans

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x) + f(h) + x^2h + h^2x - f(x)}{h}$$

$$= \lim_{h\to 0} \left[\frac{f(h)}{h} + x^2 + hx \right]$$

$$= \lim_{h\to 0} \frac{f(h)}{h} + \lim_{h\to 0} x^2 + \lim_{h\to 0} x$$

$$f'(x) = 1 + x^2 + 0$$

$$f'(x) = 1 + x^2$$

$$f'(3) = 1 + (3)^2 = 1 + 9 = 10 \quad \text{ans.}$$

23

Ques 23 The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ....

Ans. let the number of shots required $= n$

Probability (To hit the target at least once)

$= 1 - P$(not hitting in n attempts)

H = an event of hitting the target

$P(H) = \frac{1}{10}$, $P(\bar{H}) = \frac{9}{10}$

$\therefore$ Prob (to hit the target at least once)

$= 1 - \left(\frac{9}{10}\right)^n$

$1 - \left(\frac{9}{10}\right)^n > \frac{1}{4} \Rightarrow \left(\frac{9}{10}\right)^n < \frac{3}{4}$

$n = 1$ $\quad \frac{3}{4} > \frac{9}{10}$ $\quad \times$

$n = 2$ $\quad \frac{3}{4} > \frac{81}{100}$ $\quad \times$

$n = 3$ $\quad \frac{3}{4} > \frac{729}{100}$ $\quad \checkmark$

minimum number of shots $= 3$ ans.

24

Q 24 let $(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to. . .

ans. $(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r x^r$

General term of $(2x^2+3x+4)^{10}$

$$= \frac{10!}{P_1!\, P_2!\, P_3!} (2x^2)^{P_1} (3x)^{P_2} (4)^{P_3}$$

where $P_1 + P_2 + P_3 = 10$

and $0 \le P_i \le 10$; $P_i \in W$, $i = 1, 2, 3$

a_7 = Coefficient of x^7

a_{13} = Coefficient of x^{13}

$$= \frac{10!}{P_1!\, P_2!\, P_3!} 2^{P_1} 3^{P_2} 4^{P_3} x^{2P_1 + P_2}$$

For a_7 : $2P_1 + P_2 = 7$

P_1	P_2	P_3		Coefficient of x^7
3	1	6	$\longrightarrow$	$\frac{10!}{3!\,1!\,6!} 2^3 \cdot 3^1 \cdot 4^6 = \frac{10!}{3!\,1!\,6!} 2^{15} \cdot 3^1$
2	3	5	$\longrightarrow$	$\frac{10!}{2!\,3!\,5!} 2^2 \cdot 3^3 \cdot 4^5 = \frac{10!}{2!\,3!\,5!} 2^{12} \cdot 3^3$

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1 5 4 $\quad \frac{10!}{11!\,5!\,4!}\,2^1\cdot 3^5\cdot 4^4 = \frac{10!}{11!\,5!\,4!}\,2^9\cdot 3^5$

0 7 3 $\quad \frac{10!}{7!\,3!}\,1\cdot 3^7\cdot 4^3 = \frac{10!}{7!\,3!}\,2^6\cdot 3^7$

$$a_7 = 2^3\left(\frac{10!}{3!\,11!\,6!}\,2^{12}\cdot 3^1 + \frac{10!}{2!\,3!\,5!}\,2^9\cdot 3^3 + \frac{10!}{11!\,5!\,4!}\,2^6\cdot 3^5 + \frac{10!}{7!\,3!}\,2^3\cdot 3^7\right)$$

For a_{13} : $2p_1 + p_2 = 13$

$$a_{13} = \left(\frac{10!}{6!\,11!\,3!}\,2^{12}\cdot 3^1 + \frac{10!}{5!\,3!\,2!}\,2^9\cdot 3^3 + \frac{10!}{4!\,5!\,1!}\,2^6\cdot 3^5 + \frac{10!}{3!\,7!}\,2^3\cdot 3^7\right)$$

$$\frac{a_7}{a_{13}} = 2^3 = 8 \quad \text{ans.}$$

25

Q 25 If the System of equations

$x - 2y + 3z = 9$

$2x + y + z = b$

$x - 7y + az = 24$, has infinitely many solutions, then $a - b$ is equal to :

Ans. For infinitely many solutions $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$\Rightarrow 1(a - (-7)) - (-2)(2a - 1) + 3(-14 - 1) = 0$

$\Rightarrow a + 7 + 4a - 2 - 45 = 0$

$\Rightarrow 5a - 40 = 0$

$a = 8$

$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & a \end{vmatrix} = 0 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix}$$

$\Rightarrow 9(8 - (-7)) - (-2)(8b - 24) + 3(-7b - 24) = 0$

$\Rightarrow 135 + 16b - 48 - 21b - 72 = 0$

$\Rightarrow 5b = 15$

$b = 3$

$a - b = 8 - 3$

$a - b = 5$ ans.

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