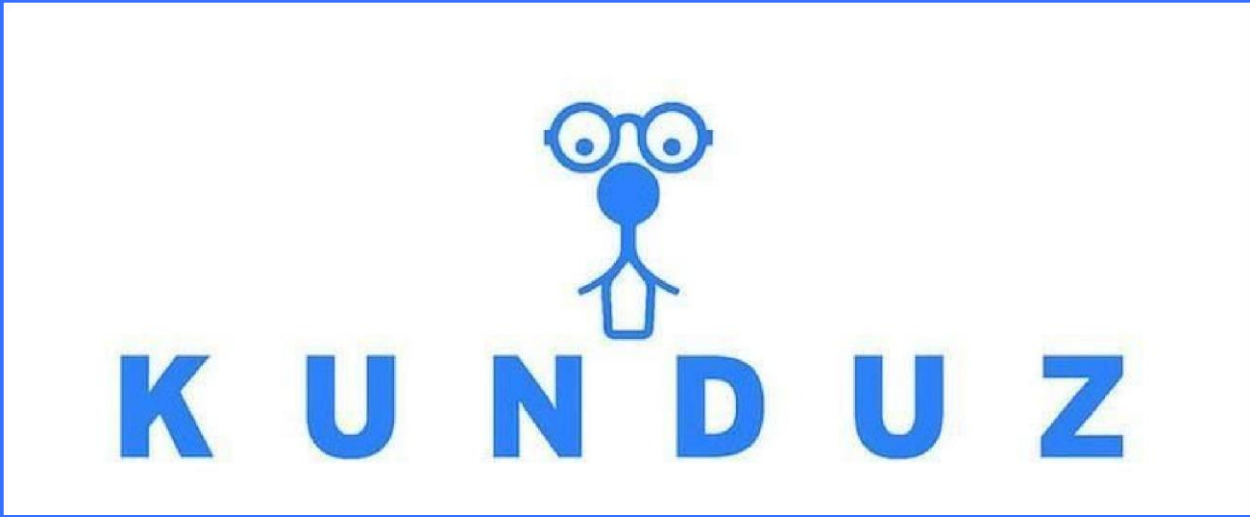

JEE Mains 4th September 2020 shift 2 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

Q1 Let $f:(0,\infty)\to(0,\infty)$ be differentiable function such that $f(1)=e$ and $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$. If $f(x)=1$ then x is equal to

Ans. $$\lim_{t\to x}\frac{-x^2f^2(t)+t^2f^2(x)}{t-x}=0$$

using L'Hopital's Rule

$$=\lim_{t\to x}\frac{2tf^2(x)-x^2\,2f(t)f'(t)}{1}=0$$

$$=2xf^2(x)-2x^2f(x)f'(x)=0$$

$$=2xf(x)\left(f(x)-xf'(x)\right)=0$$

$f(x)\neq 0 \quad \therefore f(x)-xf'(x)=0$

$$xf'(x)=f(x)$$

$$x\frac{dy}{dx}=y \Rightarrow \frac{1}{y}dy=\frac{1}{x}dx$$

$$\int\frac{1}{y}dy=\int\frac{1}{x}dx$$

$$\ln y=\ln x+\ln C$$

$$\ln y=\ln Cx \Rightarrow y=Cx=f(x)$$

$$f(1)=C(1)=e \Rightarrow C=e$$

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$\therefore \quad f(x) = ex$

$f(x) = 1 \quad \ldots$ given

$ex = 1$

$x = \frac{1}{e}$ ans.

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2

Q2 Contrapositive of the statement :

'If a function f is differentiable at a, then it is also continuous at a' is :

(1) If a function is not continuous at a, then it is not differentiable at a.

(2) If a function is continuous at a, then it is differentiable at a

(3) If a function is not continuous at a, then it is differentiable at a

(4) If a function f is continuous at a, then it is not differentiable at a.

Ans. Contrapositive of $p \to q$ is $\sim q \to \sim p$

option (1)

If a function is not continuous at a, then it is not differentiable at a.

3

Q 3 The solution of the differential equation

$$\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0 \text{ is}$$

(where c is a constant of integration)

(1) $x - 2\log_e(y+3x) \pm c$ (2) $x - \log_e(y+3x) = c$

(3) $y+3x - \frac{1}{2}(\log_e x)^2 = c$ (4) $x - \frac{1}{2}(\log_e(y+3x))^2 = c$

Ans. $$\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$$

$$\frac{dy}{dx} + 3 = \frac{y+3x}{\log_e(y+3x)}$$

$$\left\{ \begin{array}{l} y+3x = t \\ \frac{dy}{dx} + 3 = \frac{dt}{dx} \end{array} \right\}$$

$$\Rightarrow \frac{dt}{dx} = \frac{t}{\log_e(t)}$$

$$\Rightarrow \int \frac{\log_e(t)}{t}\, dt = \int dx$$

$$\left\{ \log_e(t) = p \Rightarrow \frac{1}{t}\, dt = dp \right\}$$

$$\Rightarrow \int p\, dp = \int dx$$

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$$\Rightarrow \frac{p^2}{2} = x + c_1$$

$$\Rightarrow \frac{(\log_e(t))^2}{2} = x + c_1$$

$$\Rightarrow \frac{(\log_e(y+3x))^2}{2} = x + c_1$$

$$\Rightarrow x - \frac{1}{2}(\log_e(y+3x))^2 = -c_1$$

$$x - \frac{1}{2}(\log_e(y+3x))^2 = c \quad \underline{\text{ans.}} \text{ option (4)}$$

4

Q 4 If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in the expansion is :

(1) 330 (2) 252 (3) 792 (4) 462

Ans. Let three consecutive term are T_r, T_{r+1}, T_{r+2}

$$T_r : T_{r+1} : T_{r+2} = 5 : 10 : 14$$

$$\frac{T_r}{T_{r+1}} = \frac{5}{10} \Rightarrow \frac{{}^{n+5}C_{r-1}}{{}^{n+5}C_r} = \frac{1}{2}$$

$$\Rightarrow \frac{(n+5)!/(n+5-(r-1))!\,(r-1)!}{(n+5)!/(n+5-r)!\,r!} = \frac{1}{2}$$

$$\Rightarrow \frac{(n+5-r)!\; r \times (r-1)!}{(n+5-r+1)!\,(r-1)!} = \frac{1}{2}$$

$$\Rightarrow \frac{r}{(n+5-r+1)} = \frac{1}{2}$$

$$\Rightarrow n+5-r+1 = 2r$$

$$\Rightarrow n-3r+6 = 0 \quad \text{---} (1)$$

$$\frac{T_{r+1}}{T_{r+2}} = \frac{10}{14} \Rightarrow \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{5}{7}$$

$$\Rightarrow \frac{(n+5)!/(n+5-r)!\,r!}{(n+5)!/(n+5-(r+1))!\,(r+1)!} = \frac{5}{7}$$

$$\Rightarrow \frac{(n+5-r-1)!\,(r+1)\times r!}{(n+5-r)(n+5-r-1)!\,r!} = \frac{5}{7}$$

$$\Rightarrow \frac{r+1}{n+5-r} = \frac{5}{7}$$

$$\Rightarrow 5n + 25 - 5r = 7r + 7$$

$$\Rightarrow 5n - 12r + 18 = 0 \quad \text{--- (2)}$$

from (1) and (2)

$$\begin{array}{r} 5n - 15r + 30 = 0 \\ \underset{-}{5n} \underset{+}{-12r} \underset{-}{+18} = 0 \\ \hline -3r + 12 = 0 \end{array} \quad \Rightarrow r = 4$$

$$5n - 12r + 18 = 0$$

$$\Rightarrow 5n - 12(4) + 18 = 0 \quad \Rightarrow n = 6$$

hence greatest coefficient will be the middle-term T_{r+1}

$$= {}^{n+5}C_{r+1} = {}^{6+5}C_{4+1}$$

$$= {}^{11}C_5 = \frac{11\times10\times9\times8\times7}{1\times2\times3\times4\times5}$$

$= 462$ ans. option (4)

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5

Q 5 The circle passing through the intersection of the circles $x^2+y^2-6x=0$ and $x^2+y^2-4y=0$, having its center on the line $2x-3y+12=0$, also passes through the point :

(1) $(1,-3)$ (2) $(-1,3)$ (3) $(-3,6)$ (4) $(-3,1)$

Ans. $S: \quad S_1+\lambda S_2=0$

$$S: \quad (x^2+y^2-6x)+\lambda(x^2+y^2-4y)=0$$

$$x^2(1+\lambda)+y^2(1+\lambda)-6x-4\lambda y=0$$

$$\Rightarrow \quad x^2+y^2-\left(\frac{6}{\lambda+1}\right)x-\left(\frac{4\lambda}{\lambda+1}\right)y=0$$

Center $\left(\frac{3}{\lambda+1}, \frac{2\lambda}{\lambda+1}\right)$

Center is on the line $2x-3y+12=0$

$$2\left(\frac{3}{\lambda+1}\right)-3\left(\frac{2\lambda}{\lambda+1}\right)+12=0$$

$$\Rightarrow \quad 6-6\lambda+12\lambda+12=0$$

$$\Rightarrow \quad 6\lambda=-18 \quad \Rightarrow \quad \lambda=-3$$

Equation of circle S: $x^2+y^2-\frac{6}{(-3+1)}x-\frac{4(-3)}{(-3+1)}y=0$

$$S: \quad x^2+y^2+3x-6y=0$$

$(-3,6)$ passes through the circle. Ans.

Option (4)

6

Q 6 In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is

(1) $\frac{30}{61}$ (2) $\frac{5}{6}$ (3) $\frac{5}{31}$ (4) $\frac{31}{61}$

Ans. A → Sum 6 → (1,5), (2,4), (3,3), (4,2), (5,1)

$P(A) = \frac{5}{36}$, $P(\bar{A}) = \frac{31}{36}$

A wins

B → Sum 7 → (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

$P(B) = \frac{6}{36}$, $P(\bar{B}) = \frac{30}{36}$

B wins

$P(A \text{ wins}) = P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A)$

$+ \cdots$

this is infinite G.P with common ratio

$r = P(\bar{A}) \times P(\bar{B})$

Probability of A wins $= \frac{P(A)}{1 - P(\bar{A})P(\bar{B})} = \frac{5/36}{1 - \frac{31}{36} \times \frac{30}{36}}$

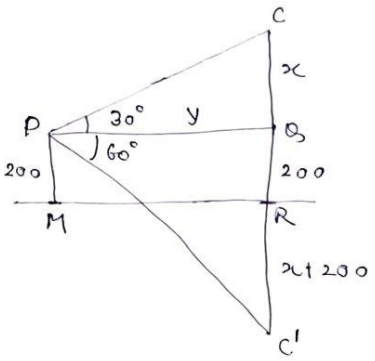
$= \frac{30}{61}$ ans. option (1)

7

Q 7 The angle of elevation of a cloud C from a point, 200 m above a still lake is 30°. If the angle of depression of the image of C in the ~~lake~~ lake from the point P is 60° PC (in m) is equal to

(1) 100 (2) $400\sqrt{3}$ (3) $200\sqrt{3}$ (4) 400

Ans.



$$\tan 30^\circ = \frac{x}{y} = \frac{1}{\sqrt{3}}$$

$$y = \sqrt{3}\,x$$

$$\begin{cases} QC = x, \quad PQ = y \\ PM = QR = 200 \\ RC = RC' = x + 200 \end{cases}$$

$$\tan 60^\circ = \frac{x+400}{y} = \sqrt{3}$$

$$\sqrt{3}\,y = x + 400 \Rightarrow \sqrt{3}\,(\sqrt{3}\,x) = x + 400$$

$$\Rightarrow 3x = x + 400 \Rightarrow 2x = 400$$

$$x = 200 \text{ m}$$

$$\sin 30^\circ = \frac{x}{PC} = \frac{1}{2} \Rightarrow \frac{200}{PC} = \frac{1}{2}$$

$$\Rightarrow PC = 400 \text{ m}$$ ans. option (4)

8

Q 8 The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1}x, & |x| \le 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$ is :

(1) Continuous on $R - \{-1\}$ and differentiable on $R - \{-1, 1\}$

(2) both Continuous and differentiable on $R - \{-1\}$

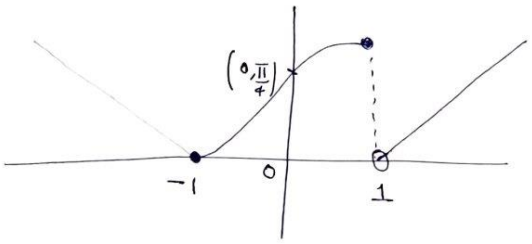
(3) both Continuous and differentiable on $R - \{1\}$

(4) Continuous on $R - \{1\}$ and differentiable on $R - \{-1, 1\}$

Ans.

$$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1}x, & |x| \le 1 \\ \frac{1}{2}(|x| - 1) & |x| > 1 \end{cases}$$

Graph of $f(x)$ is



$f(x)$ is continuous on $R - \{1\}$

$f(x)$ is differentiable on $R - \{-1, 1\}$

} Ans. option (4)

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9

Q 9 Suppose the vector x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2, and b_3 respectively. If

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and}$$

$$b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of } A \text{ is equal to:}$$

(1) $\frac{3}{2}$ (2) 4 (3) $\frac{1}{2}$ (4) 2

Ans. let $X = [x_1 \quad x_2 \quad x_3] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$$B = [b_1 \quad b_2 \quad b_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$Ax = b$$

$$AX = B$$

$$|A||X| = |B| \Rightarrow |A| = \frac{|B|}{|X|}$$

$$|X| = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1(2-0) - 0(1-2) + 0(1-2)$$

$$|X| = 2$$

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$$|B| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$|B| = 1(4-0) - 0(0-0) + 0(0-0)$$

$$|B| = 4$$

$$|A| = \frac{|B|}{|X|} = \frac{4}{2}$$

$|A| = 2$ ans. Option (4)

10

Q 10. Let $a_1, a_2, \ldots, a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \cdots + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}), (S_{n-4}, a_{n-4}) is equal to:

(1) (2490, 248) (2) (2490, 249)

(3) (2480, 249) (4) (2480, 248)

Ans.

$$a_n = a_1 + (n-1)d$$

$$\Rightarrow 300 = 1 + (n-1)d$$

$$\Rightarrow d = \frac{299}{n-1} = \frac{13 \times 23}{n-1}$$ { common difference is an integer

$$\therefore n-1 = \pm 13, \pm 23, \pm 299, \pm 1$$

$$n = 14, -12, 24, -22, 300, -298, 2, 0$$

it is given that $15 \le n \le 50$

$$\therefore n = 24$$

$$d = \frac{299}{24-1} = \frac{13 \times 23}{23} = 13$$

$$S_{n-4} = S_{24-4} = S_{20} = \frac{20}{2}\left[2(1) + (20-1) \times 13\right]$$

$$= 2490$$

$$a_{n-4} = a_{24-4} = a_{20} = 1 + (20-1)13 = 248$$

$$(S_{n-4}, a_{n-4}) = (2490, 248)$$ Ans. option (4)

11

Q 11 Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If element of the set T is an exactly 20 of sets X_i's and exactly 6 sects Y_i's then n is equal to:

(1) 45 (2) 15 (3) 30 (4) 50

Ans.

$$\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = Z$$

each X_i contain 10 elements

then total element of $\bigcup_{i=1}^{50} X_i$ is 50×10

each Y_i contains 5 element

then total elemts of $\bigcup_{i=1}^{n} Y_i$ is $5 \times n$

$$\therefore \quad \frac{50 \times 10}{20} = \frac{5n}{6}$$

$$n = 30$$ ans. option (3)

12

Q 12. The area (in Sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $Y = x^2 - 1$ below the x-axis is :

(1) $\frac{1}{3\sqrt{3}}$ (2) $\frac{4}{3}$ (3) $\frac{4}{3\sqrt{3}}$ (4) $\frac{2}{3\sqrt{3}}$

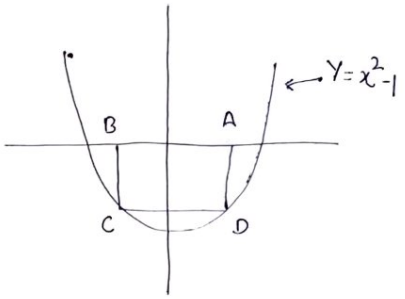
Ans. $A(\alpha, 0)$, $B(-\alpha, 0)$

$Y = x^2 - 1$

at $x = \alpha$

$Y = \alpha^2 - 1$

$D(\alpha, \alpha^2 - 1)$



Area of rectangle (ABCD) = AB × CD

$S = (2\alpha)(1 - \alpha^2) = 2\alpha - 2\alpha^3$

$\frac{dS}{d\alpha} = 2 - 6\alpha^2 = 0 \Rightarrow \alpha^2 = \frac{1}{3} \Rightarrow \alpha = \frac{1}{\sqrt{3}}$

$\frac{d^2S}{d\alpha^2} = -12\alpha < 0$ for $\alpha > 0$

Area $= 2\alpha - 2\alpha^3 = \frac{2}{\sqrt{3}} - \frac{2}{3\sqrt{3}}$

Area $= \frac{4}{3\sqrt{3}}$ Sq. units Ans. option (3)

13

Ques 13 Let $x = 4$ be a directrix to an ellipse whose center is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta)$, $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is

(1) $7x - 4y = 1$ (2) $4x - 2y = 1$

(3) $8x - 2y = 5$ (4) $4x - 3y = 2$

Ans. directrix $x = \frac{a}{e} = 4 \Rightarrow a = 4e$ $\left\{ e = \frac{1}{2} \right.$ eccentricity

$a = 4\left(\frac{1}{2}\right) \Rightarrow a = 2 \Rightarrow a^2 = 4$

$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \left(\frac{1}{2}\right)^2 = 1 - \frac{b^2}{(2)^2}$

$\Rightarrow \frac{1}{4} = 1 - \frac{b^2}{4} \Rightarrow 1 = 4 - b^2$

$\Rightarrow b^2 = 3$

equation of the ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

P lies on ellipse, then $\{ P(1, \beta)$

$$\frac{1}{4} + \frac{\beta^2}{3} = 1 \Rightarrow \frac{\beta^2}{3} = \frac{3}{4}$$

$$\beta = \frac{3}{2} \Rightarrow P\left(1, \frac{3}{2}\right)$$

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$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\Rightarrow \frac{2x}{4} + \frac{2y}{3}\frac{dy}{dx} = 0 \quad \Rightarrow \frac{dy}{dx} = -\frac{3x}{4y}$$

$$\Rightarrow \left.\frac{dy}{dx}\right)_{P(1,3/2)} = -\frac{3(1)}{4(3/2)} = -\frac{1}{2}$$

Slope of normal $= -\left(\frac{1}{dy/dx}\right) = -\left(\frac{1}{(-1/2)}\right)$

$= 2$

equation of normal on ellipse at $P\left(1, \frac{3}{2}\right)$

$$y - 3/2 = 2(x-1)$$

$$\Rightarrow 2y - 3 = 4x - 4$$

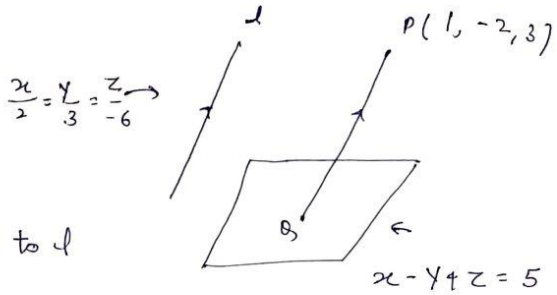
$$\Rightarrow 4x - 2y = 1 \quad \underline{\text{ans.}} \text{ option (2)}$$

14

Ques 14 The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is :

(1) $\frac{1}{7}$ (2) 7 (3) 1 (4) $\frac{7}{5}$

Ans.



PQ is ~~pdt~~ parallel to l

$\therefore$ equation of PQ

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$

Let $Q \equiv (x = 2\lambda + 1,\ y = 3\lambda - 2,\ z = 3 - 6\lambda)$

Q lies in the plane $x - y + z = 5$

$\therefore (2\lambda + 1) - (3\lambda - 2) + (3 - 6\lambda) = 5$

$-7\lambda = -1 \Rightarrow \lambda = 1/7$

$\therefore Q \left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$

$$PQ = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(-3 + \frac{15}{7}\right)^2}$$

$$PQ = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \sqrt{\frac{49}{49}}$$

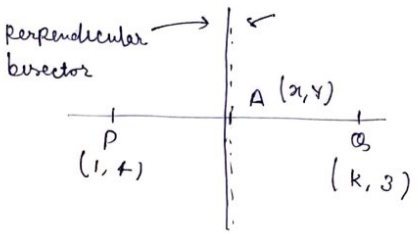
$PQ = 1$ ans. option (3)

15

Q 15 If the perpendicular bisector of the line segment joining the points $P(1,4)$ and $Q(k,3)$ has y-intercept equal to -4, then a value of k is;

(1) $\sqrt{15}$ (2) -4 (3) -2 (4) $\sqrt{14}$

Ans.



A is on perpendicular bisector

$PA = QA$

$$\sqrt{(x-1)^2+(y-4)^2} = \sqrt{(x-k)^2+(y-3)^2}$$

$$\Rightarrow x^2-2x+1+y^2-8y+16 = x^2+k^2-2kx+y^2+9-6y$$

$$\Rightarrow (2k-2)x - 2y + 8 - k^2 = 0 \rightarrow \text{equation of perpendicular bisector}$$

y-intercept is -4

$\Rightarrow x = 0, \quad y = -4$ ~~But~~

put $(0,-4)$ in the equation of perpendicular bisector

$$\Rightarrow (2k-2)(0) - 2(-4) + 8 - k^2 = 0$$

$$k^2 = 16 \quad \Rightarrow \quad k = \pm 4$$

$k = -4, 4$

$k = -4$ ans. option (2)

16

Q 16 The integral $\int_{\pi/6}^{\pi/3} \tan^3(x)\sin^2(3x)\left(2\sec^2 x \sin^2 3x + 3\tan x \sin 6x\right)dx$

is equal to :

(1) $-\frac{1}{9}$ (2) $\frac{9}{2}$ (3) $-\frac{1}{18}$ (4) $\frac{7}{18}$

Ans. $I = \int_{\frac{\pi}{6}}^{\pi/3} \tan^3 x \sin^2 3x \left(2\sec^2 x \sin^2 3x + 3\tan x \sin 6x\right) dx$

$I = \int_{\frac{\pi}{6}}^{\pi/3} \frac{2}{2}\left(2\tan^3 x \sec^2 x \sin^4 3x + \left(2\sin^3 3x \cos 3x \cdot 3\right)\tan^4 x\right) dx$

$I = \frac{1}{2}\int_{\frac{\pi}{6}}^{\pi/3} \left(\left(4\tan^3 x \sec^2 x\right)\cdot \sin^4 3x + \left(4\sin^3 3x \cdot \cos 3x \cdot 3\right)\tan^4 x\right) dx$

$I = \frac{1}{2}\int_{\pi/6}^{\pi/3} \left\{\sin^4 3x \cdot \frac{d}{dx}(\tan^4 x) + \tan^4 x \frac{d}{dx}(\sin^4 3x)\right\} dx$

$I = \frac{1}{2}\int_{\pi/6}^{\pi/3} \left\{\frac{d}{dx}\left(\tan^4 x \sin^4 3x\right)\right\} dx$

$I = \frac{1}{2}\left[\tan^4 x \cdot \sin^4 3x\right]_{\pi/6}^{\pi/3}$

$I = \frac{1}{2}\left[\tan^4\frac{\pi}{3}\sin^4 3\times\frac{\pi}{3} - \tan^4\frac{\pi}{6}\cdot\sin^4 3\times\frac{\pi}{6}\right]$

$I = \frac{1}{2}\left[0 - \frac{1}{9}\right] \Rightarrow I = -\frac{1}{18}$ ans. option (3)

17

Q 17 The minimum value of $2^{\sin x} + 2^{\cos x}$ is :

(1) $2^{-1+\frac{1}{\sqrt{2}}}$ (2) $2^{-1+\sqrt{2}}$ (3) $2^{1-\sqrt{2}}$ (4) $2^{1-\frac{1}{\sqrt{2}}}$

Ans. A.M. $\geq$ G.M.

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \left(2^{\sin x} \cdot 2^{\cos x}\right)^{\frac{1}{2}}$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2 \cdot \left(2^{\frac{\sin x + \cos x}{2}}\right)$$

$$\left\{ -\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2} \right\}$$

for minimum value $\sin x + \cos x = -\sqrt{2}$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2\left(2^{-\frac{\sqrt{2}}{2}}\right)$$

$$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1-\frac{1}{\sqrt{2}}}$$

then minimum value of $2^{\sin x} + 2^{\cos x}$ is $2^{1-\frac{1}{\sqrt{2}}}$

ans. option (4)

18

Ques 18 If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then $a+b$ is equal to:

Ans. $\alpha = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$$\alpha + 1 = 1 - \frac{1}{2} + \frac{i\sqrt{3}}{2}$$

$$\Rightarrow \alpha + 1 = \frac{1}{2} + \frac{i\sqrt{3}}{2} \quad \text{---- (1)}$$

$$\alpha^2 = \alpha \cdot \alpha = \left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) \quad \{ i^{02} = -1$$

$$\Rightarrow \alpha^2 = \frac{1}{4} - \frac{i\sqrt{3}}{4} - \frac{i\sqrt{3}}{2} + \frac{i^2 3}{4}$$

$$\Rightarrow \alpha^2 = -\frac{1}{2} - \frac{i\sqrt{3}}{2} \quad \text{--- (2)}$$

from (1) and (2)

$$1 + \alpha = -\alpha^2$$

$$\alpha^3 = \alpha^2 \cdot \alpha = \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$\Rightarrow \alpha^3 = \frac{1}{4} - \frac{i\sqrt{3}}{4} + \frac{i\sqrt{3}}{4} - \frac{i^2 3}{4} \quad \{ i^2 = -1$$

$$\alpha^3 = \frac{1}{4} + \frac{3}{4} \Rightarrow \alpha^3 = 1$$

$$(2+\alpha)^4 = a + b\alpha$$

$$\Rightarrow (4 + \alpha^2 + 4\alpha)^2 = a + b\alpha \quad \{ 1 + \alpha = -\alpha^2$$

$$\Rightarrow (4(1+\alpha) + \alpha^2)^2 = a + b\alpha$$

$$\Rightarrow (-4\alpha^2 + \alpha^2)^2 = a + b\alpha$$

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$\Rightarrow (-3\alpha^2)^2 = a + b\alpha$

$\Rightarrow 9\alpha^4 = a + b\alpha$

$\Rightarrow 9\alpha \cdot \alpha^3 = a + b\alpha \quad \{ \alpha^3 = 1$

$\Rightarrow 0 + 9\alpha = a + b\alpha$

$a = 0 \quad , b = 9$

$a + b = 0 + 9 = 9$ ans.

19

Ques 19 let $\lambda \neq 0$ be in R, If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are roots of the eq $3x^2 - 10x + 27\lambda = 0$ then $\frac{\beta\gamma}{\lambda}$ is equal to:

(1) 27 (2) 36 (3) 9 (4) 18

Ans α and β are roots of $x^2 - x + 2\lambda = 0$

$\alpha + \beta = 1$, $\alpha\beta = 2\lambda$

and $\alpha^2 - \alpha + 2\lambda = 0$ ---(1)

α and γ are roots of $3x^2 - 10x + 27\lambda = 0$

$\alpha + \gamma = \frac{10}{3}$ $\alpha\gamma = 9\lambda$

and $3\alpha^2 - 10\alpha + 27\lambda$ ---(2)

from equation (1) and (2)

$3\alpha^2 - 10\alpha + 27\alpha = 0$

$-3\alpha^2 \mp 3\alpha \pm 6\lambda = 0$

$21\lambda = 7\alpha \Rightarrow 3\lambda = \alpha$

by equation (1)

$9\lambda^2 - 3\lambda + 2\lambda = 0 \Rightarrow 9\lambda^2 - \lambda = 0$

$\lambda = 0, \frac{1}{9}$ $\{\lambda \neq 0\}$

$\alpha = 3\lambda \Rightarrow \alpha = \frac{1}{3}$

$\gamma\beta = 2\lambda \Rightarrow \beta = \frac{2}{3}$

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$$\therefore \quad \frac{\beta r}{\lambda} = ?$$

$$\alpha r = 9\lambda \Rightarrow r = 3$$

$$\frac{\beta r}{\lambda} = \frac{{}^{2}/_{3} \cdot 3}{1/9}$$

$$\frac{\beta r}{\lambda} = 18 \text{ ans.} \quad \text{option (4)}$$

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20

Ques 20 If the system of equations

$$x + y + z = 2$$

$$2x + 4y - z = 6$$

$3x + 2y + \lambda z = \mu$ has infinitely many solutions, then :

(1) $2\lambda - \mu = 5$ (2) $\lambda + 2\mu = 14$ (3) $2\lambda + \mu = 14$

(4) $\lambda - 2\mu = -5$

Ans. for infinitely many solutions

$$D = D_1 = D_2 = D_3 = 0$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(4\lambda - (-2)) - 1(2\lambda - (-3)) + 1(4 - 12) = 0$$

$$\Rightarrow 4\lambda + 2 - 2\lambda - 3 + 4 - 12 = 0$$

$$2\lambda = 9 \quad \Rightarrow \lambda = 9/2$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & \mu \end{vmatrix} = 0$$

$$\Rightarrow 1(4\mu - 12) - 1(2\mu - 18) + 2(4 - 12) = 0$$

$$\mu = 5$$

$2\lambda + \mu = 14$ ans. Option (3)

21

Ques 21 Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ____

Ans. $x^2 + y^2 = 9$ PQ is diameter

Let $P(3\cos\theta, 3\sin\theta)$ and $Q(-3\cos\theta, -3\sin\theta)$

α and β are perpendicular distance from P and Q on line $x + y - 2 = 0$

$$\alpha = \frac{|3\cos\theta + 3\sin\theta - 2|}{\sqrt{2}}$$

$$\beta = \frac{|-3\cos\theta - 3\sin\theta - 2|}{\sqrt{2}}$$

$$\alpha\beta = \frac{|(3\cos\theta + 3\sin\theta - 2)\cdot(3\cos\theta + 3\sin\theta + 2)|}{2}$$

$$\alpha\beta = \frac{|(3\cos\theta + 3\sin\theta)^2 - (2)^2|}{2}$$

$$\alpha\beta = \frac{|9(1 + 2\sin 2\theta) - 4|}{2}$$

for $\alpha\beta_{max}$, $\sin 2\theta = 1$

$\{-1 \le \sin\theta \le 1$
$\{-1 \le \sin 2\theta \le 1$

$$\alpha\beta_{max} = \frac{|18 - 4|}{2} = \frac{14}{2}$$

$$\alpha\beta_{max} = 7 \text{ ans.}$$

22

Q 22 If the variance of the following frequency distribution:

Class:	10 – 20	20 – 30	30 – 40
Frequency:	2	x	2

is 50, then x is equal to:

Ans.

x_i	15	25	35
f_i	2	x	2

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2\times 15 + x\times 25 + 2\times 35}{2+x+2}$$

$$\bar{X} = \frac{100 + x\cdot 25}{x+4} = \frac{25(4+x)}{x+4}$$

$$\bar{X} = 25$$

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{X})^2 = 50$$

$$\Rightarrow \frac{2\times(15)^2 + x(25)^2 + 2(35)^2}{2+x+2} - (25)^2 = 50$$

$$\Rightarrow \frac{450 + 625x + 2450}{x+4} - 625 = 50$$

$$\Rightarrow \frac{2900 + 625x}{x+4} = 675$$

$$\Rightarrow 2900 + 625x = 675x + 2700$$

$$\Rightarrow 50x = 200$$

$$x = 4 \quad \text{ans.}$$

23

Ques 23 A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct, in the no of way in which a candidate answers all six questions such that exactly four of the answers are correct, is - - -

Ans. No. of ways of giving wrong answer of a question is = 3

required no. of ways $= {}^{6}C_{4}\,(1)^{4}\,(3)^{2}$

$= \frac{6 \times 5}{1 \times 2} \times 1 \times 9$

$= 15 \times 9$

$= 135$ ans.

24

Q 24 Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\leq x$ respectively of a real number x. If $\int_0^n \{x\}\,dx$, $\int_0^n [x]\,dx$ and $10(n^2-n)$, $\{n \in N, n>1\}$ are three consecutive terms of a G.P. then n is equal to.

Ans. $$\int_0^n \{x\}\,dx = n\int_0^1 x\,dx = n\left(\frac{x^2}{2}\right)_0^1$$

$$= n\left(\frac{1}{2}-0\right) = \frac{n}{2}$$

$$\int_0^n [x]\,dx = \int_0^n (x-\{x\})\,dx$$

$$= \int_0^n x\,dx - \int_0^n \{x\}\,dx$$

$$= \left(\frac{x^2}{2}\right)_0^n - \frac{n}{2} = \left(\frac{n^2}{2}-0\right) - \frac{n}{2}$$

$$= \frac{n^2}{2} - \frac{n}{2} = \frac{n^2-n}{2}$$

Now $\frac{n}{2}$, $\frac{n^2-n}{2}$ and $10(n^2-n)$ are in G.P.

$$\frac{(n^2-n)^2}{4} = \frac{n}{2}\cdot 10(n^2-n)$$

$$\Rightarrow \frac{n^2(n-1)^2}{4} = 5n^2(n-1)$$

$$\Rightarrow n-1 = 20 \Rightarrow n = 21$$ ans.

25

Ques 25 If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of

$|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to:

Ans. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k} = 2\hat{i} + \hat{j} + 2\hat{k}$

$$\hat{i} \times (\vec{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\vec{a} - (\vec{a} \cdot \hat{i})\hat{i}$$

$$\hat{i} \times (\vec{a} \times \hat{i}) = x\hat{i} + y\hat{j} + z\hat{k} - x\hat{i}$$

$$\hat{i} \times (\vec{a} \times \hat{i}) = y\hat{j} + z\hat{k}$$

$x = 2$
$y = 1$
$k = 2$

Similarly $\hat{j} \times (\vec{a} \times \hat{j}) = x\hat{i} + z\hat{k}$

$\hat{k} \times (\vec{a} \times \hat{k}) = x\hat{i} + y\hat{j}$

$$|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$$

$$= |y\hat{j} + z\hat{k}|^2 + |x\hat{i} + z\hat{k}|^2 + |x\hat{i} + y\hat{j}|^2$$

$$= y^2 + z^2 + x^2 + z^2 + x^2 + y^2$$

$$= 2(x^2 + y^2 + z^2)$$

$$= 2((2)^2 + (1)^2 + (2)^2)$$

$$= 2(4 + 1 + 4)$$

$$= 2 \times 9$$

$$= 18 \text{ ans.}$$

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