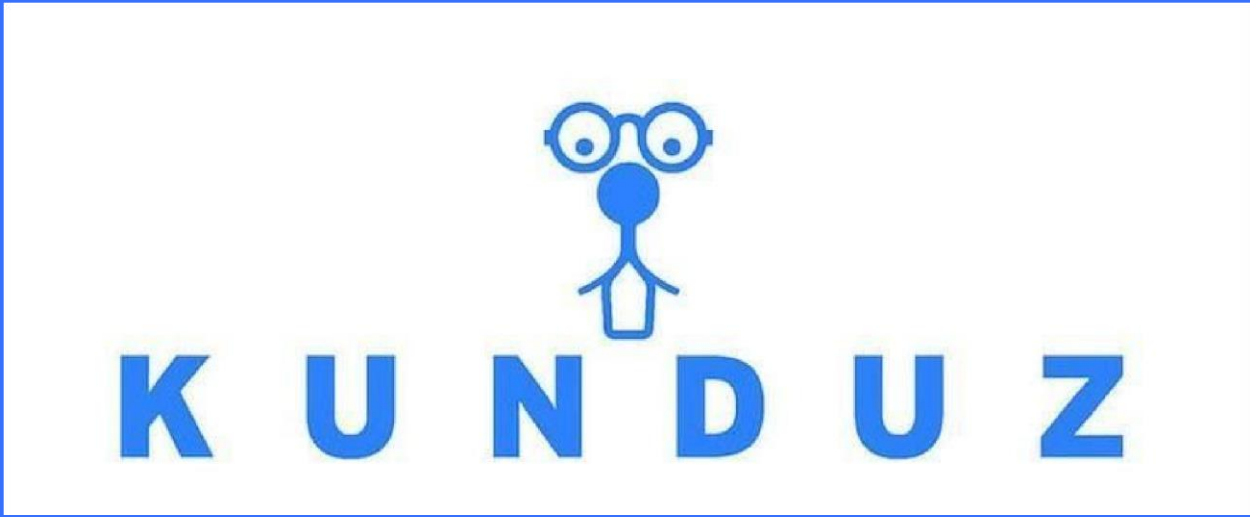


JEE Mains 5th September 2020 shift 1 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

MATHEMATICS | JEE MAIN 2020 | 05/09/2020 | SHIFT-1

1) A survey shows that 73% of the persons working in an office likes coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

a) 36 b) 63 c) 38 d) 54

Soln.

$n(C) = 73$

$n(T) = 65$

$n(C \cap T) = x$

$\Rightarrow n(C) + n(T) - n(C \cap T) \leq 100$

$\Rightarrow 73 + 65 - x \leq 100$

$\Rightarrow 138 - x \leq 100$

$\Rightarrow x \geq 38 \quad - (i)$

$n(C \cap T) \leq \min(n(C), n(T))$

$\Rightarrow x \leq 65 \quad - (2)$

from Eqn (i) and (2)

$38 \leq x \leq 65$

36 not lies in this interval.

Hence option a) is correct.

2

2) If the function $f(x) = \begin{cases} k_1(x-\pi)^2, & x \le \pi \\ k_2\cos x & x > \pi \end{cases}$ is twice differentiable then ordered pair (k_1, k_2) is equal to:

a) $\left(\frac{1}{2}, -1\right)$ b) $\left(\frac{1}{2}, 1\right)$ c) $(1, 0)$ d) $(1, 1)$

Sol^n:

$f(x)$ is differentiable then will also be continuous

then $f(\pi) = -1$

$f(\pi^+) = f(\pi)$

$-k_2 = -1$

$\Rightarrow k_2 = 1$

Now,

$$f'(x) = \begin{cases} 2k_1(x-\pi) & x \le \pi \\ -k_2\sin x & x > \pi \end{cases}$$

$f'(\pi^-) = f'(\pi^+) = 0$

$$f''(x) = \begin{cases} 2k_1 & x \le \pi \\ -k_2\cos x & x > \pi \end{cases}$$

$2k_1 = k_2$

$2k_1 = 1 \quad (\because k_2 = 1)$

$k_1 = \frac{1}{2}$

$(k_1, k_2) \equiv \left(\frac{1}{2}, 1\right)$

Hence option (b) is correct.

3

3) If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4 - 2\sin 2\alpha}$ are the first three terms of an AP for some α, then sixth term of this AP is

a) 81 b) 65 c) 66 d) 78

Soln:

$\because$ a, b, c are in AP

$\Rightarrow 2b = a + c$

$$2(14) = 3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha}$$

$$28 = \frac{3^{2\sin 2\alpha}}{3} + \frac{3^4}{3^{2\sin 2\alpha}}$$

Let $3^{2\sin 2\alpha} = x$

$$28 = \frac{x}{3} + \frac{81}{x}$$

$$x^2 - 84x + 243 = 0$$

$$(x - 3)(x - 81) = 0$$

$$x = 3, 81$$

$$3^{2\sin 2\alpha} = 3, 3^4 \qquad (\because x = 3^{2\sin 2\alpha})$$

$$2\sin 2\alpha = 1, 4$$

$$\sin 2\alpha = \frac{1}{2}, 2$$

$\Rightarrow \sin 2\alpha = \frac{1}{2}$ $\left[\because \sin 2\alpha \neq 2 \quad \forall \alpha \in R\right.$

Now terms are 1, 14, 27, ...

$$T_6 = 1 + 5(13)$$

$$T_6 = 1 + 65$$

$$T_6 = 66$$

Hence option c) is correct.

4

4) If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}(\quad) + \tan^{-1}\left(\frac{1}{13}\right) + \ldots$, then $\tan(S)$ is equal to:

a) $-\frac{6}{5}$ b) $\frac{5}{11}$ c) $\frac{5}{6}$ d) $\frac{10}{11}$

Soln

$$S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \ldots \text{ upto 10 term}$$

$$S = \tan^{-1}\left(\frac{2-1}{1+1\cdot 2}\right) + \tan^{-1}\left(\frac{3-2}{1+2\cdot 3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{11-10}{1+11\cdot 10}\right)$$

$$S = \left(\tan^{-1}(2) - \tan^{-1}(1)\right) + \left(\tan^{-1}(3) - \tan^{-1}(2)\right) + \ldots + \left(\tan^{-1}11 - \tan^{-1}10\right)$$

$$\Rightarrow S = \tan^{-1}(11) - \tan^{-1}1$$

$$S = \tan^{-1}11 - \frac{\pi}{4}$$

$$\tan S = \tan\left(\tan^{-1}11 - \frac{\pi}{4}\right)$$

$$\tan S = \frac{\tan\left(\tan^{-1}11\right) - \left(\tan\frac{\pi}{4}\right)}{1 + \tan\left(\tan^{-1}11\right)\tan\frac{\pi}{4}}$$

$$\tan S = \frac{11-1}{1+11\times 1}$$

$$\tan S = \frac{10}{12}$$

$$\tan S = \frac{5}{6}.$$

Hence option (c) is correct.

5

5) If $2^{10} + 2^9 \cdot 3^1 + \ldots + 2\,3^9 + 3^{10} = S - 2^{11}$ then S is equal to:

a) 3^{11} b) $2\,3^{11}$ c) $\frac{3^{11}}{2} + 2^{10}$ d) $3^{11} - 2^{12}$

Sol^n:

Let $S' = 2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \ldots + 2 \cdot 3^9 + 3^{10}$

So it is G.P with

$a = 2^{10}$, $r = \frac{3}{2}$, $n = 11$

$$S' = 2^{10} \frac{\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\frac{3}{2} - 1}$$

[Sum of a G.P $S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

$r \rightarrow$ common ratio

$a \rightarrow$ first term

$$S' = 2^{11}\left(\frac{3^{11}}{2^{11}} - 1\right)$$

$$S' = 3^{11} - 2^{11}$$

$$S - 2^{11} = 3^{11} - 2^{11} \quad (\because S' = S - 2^{11} \text{ given})$$

$$S = 3^{11}$$

Hence option a) is correct.

6

6) If the common tangent to the Parabolas $y^2=4x$ and $x^2=4y$ also touches the circle $x^2+y^2=c^2$, then c is equal to:

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2\sqrt{2}}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$

Soln:

Given Parabolas are $y^2=4x$ and $x^2=4y$

Any tangent to $y^2=4x$ is $y=mx+\frac{1}{m}$

It is also tangent to $x^2=4y$

$\therefore \frac{1}{m} = -m^2$

$\Rightarrow m = -1$

Common tangent is $y=(x-1)$ it also touches $x^2+y^2=c^2$

$\therefore 1 = c^2(1+1)$

$\Rightarrow c^2 = \frac{1}{2}$

$\Rightarrow c = \frac{1}{\sqrt{2}}$

Hence option a) is correct.

7

7) If $y : y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y}\frac{dy}{dx}+e^x=0$ satisfying $y(0)=1$, then a value of $y(\log_e 13)$ is

a) -1 b) 0 c) 2 d) 1

Soln

Given $\left(\frac{5+e^x}{2+y}\right)\frac{dy}{dx}+e^x=0$

$$\Rightarrow \int\frac{dy}{2+y} = -\int\frac{e^x dx}{5+e^x}$$

$$\Rightarrow \ln(y+2) = -\ln(5+e^x)+\ln c$$

$$y+2 = \frac{c}{5+e^x}$$

$$y = \frac{c}{5+e^x}-2$$

$y(0)=1$ (given)

$$\frac{c}{5+1}-2=1$$

$$\frac{c}{6}=3$$

$$\Rightarrow c=18$$

Now eqn becomes $y=\frac{18}{5+e^x}-2$

$$y(\log_e 13) = \frac{18}{5+e^{\log_e 13}}-2$$

$$y(\log_e 13) = \frac{18}{5+13}-2$$

$$y(\log_e 13) = \frac{18}{18}-2$$

$$y(\log_e 13) = -1$$

Hence option a) is Correct.

8

Q) If $\int (e^{2x}+2e^{x}-e^{-x}-1)\, e^{(e^{x}+e^{-x})}\, dx = g(x)\, e^{(e^{x}+e^{-x})} + c$, where c is a constant of integration then $g(0)$ is equal to:

a) 1 b) e c) e^2 d) 2

Sol^n.

Let $I = \int (e^{2x}+2e^{x}-e^{-x}-1)\, e^{(e^{x}+e^{-x})}\, dx$

$I = \int (e^{2x}+e^{x}+1)\, e^{(e^{x}+e^{-x})}\, dx + \int (e^{x}-e^{-x})\, e^{(e^{x}+e^{-x})}\, dx$

$I = \int (e^{x}+1-e^{-x})\, e^{(e^{x}+e^{-x})}\, dx + e^{e^{x}-e^{-x}}$

Let $e^{x}+1+e^{-x} = t$

$(e^{x}-e^{-x}+1)\, dx = dt$

$I = \int e^{t}\, dt + e^{e^{x}-e^{-x}}$

$I = e^{t} + e^{e^{x}-e^{-x}}$

$I = e^{(e^{x}+e^{-x}+x)} + e^{e^{x}-e^{-x}}$ [$\because t = e^{x}+e^{-x}+x$

$I = e^{e^{x}+e^{-x}}\,(e^{x}+1)$

Now $g(x) = e^{x}+1$

$g(0) = 1+1$

$g(0) = 2$

Hence option d) is correct.

9

9) If α is the positive root of the equation $p(x) = x^2 - x - 2$ then

$$\lim_{x \to \alpha} \frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$$ is equal to

a) $\frac{3}{2}$ b) $\frac{3}{\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{2}$

Sol:

$$p(x) = 0$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + 1(x-2) = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, -1$$

$\because \alpha$ is positive $\Rightarrow \alpha = 2$

Now,

$$\lim_{x \to 2} \frac{\sqrt{1-\cos(x^2-x-2)}}{(x-2)}$$

$$\Rightarrow \lim_{x \to 2} \frac{\sqrt{2\sin^2\left(\frac{x^2-x-2}{2}\right)}}{(x-2)}$$

$$\lim_{x \to 2} \sqrt{2} \frac{\left|\sin\left(\frac{x^2-x-2}{2}\right)\right|}{x-2} \quad \left[\because \sin\left(\frac{x^2-x-2}{2}\right) > 0\right]$$

$$\lim_{x \to 2} \sqrt{2} \frac{\sin\left(\frac{x^2-x-2}{2}\right)\left(\frac{x^2-x-2}{2}\right)}{\left(\frac{x^2-x-2}{2}\right)(x-2)}$$

$$\lim_{x \to 2} \frac{1}{\sqrt{2}} \frac{(x-2)(x+1)}{(x-2)} = \frac{3}{\sqrt{2}}$$

Hence option b) is correct.

10

10) If the coordinates of two points A and B are $(\sqrt{7},0)$ and $(-\sqrt{7},0)$ respectively and P is any point on the conic $9x^2+16y^2=144$ then PA+PB is equal to:

a) 8 b) 16 c) 9 d) 6

Soln

for Ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

$a=4,\ b=3$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

A and B are foci

then $PA+PB=2a$

$PA+PB=2(4)=8.$

Hence option a) is correct.

11

11) The product of the roots of the equation $9x^2-18|x|+5=0$

a) $\frac{25}{81}$ b) $\frac{5}{27}$ c) $\frac{25}{9}$ d) $\frac{5}{9}$

Soln

$9x^2-18|x|+5=0$

Let $x^2=|x|^2=t$

$9t^2-18t+5=0$

$9t^2-15t-3t+5=0$

$3t(3t-5)-1(3t-5)=0$

$(3t-5)(3t-1)=0$

$t=\frac{1}{3},\frac{5}{3}$

$|x|=\frac{1}{3},\frac{5}{3}$

Product of roots $=\left(\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(-\frac{5}{3}\right)=\frac{25}{81}$

Hence option a) is correct.

12

12) If the volume of a parallelopiped whose conterminous edges are given by the vectors $\vec{a} = \hat{i}+\hat{j}+n\hat{k}$, $\vec{b} = 2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c} = \hat{i}+n\hat{j}+3\hat{k}$ $n>0$ is 158 cubic units, then:

a) $n=7$ b) $n=9$ c) $\vec{b}\cdot\vec{c} = 10$ d) $\vec{a}\cdot\vec{c} = 17$

Sol^n:

Volume of Parallelopiped $V = |[\vec{a}\,\vec{b}\,\vec{c}]|$

$$V = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix}$$

$$V = 1(12+n^2) - 1(6+n) + n(2n-4)$$

$$V = 12+n^2-6-n+2n^2-4n$$

$$V = 3n^2-5n+6$$

$$158 = 3n^2-5n+6$$

$$\Rightarrow 3n^2-5n-152 = 0$$

$$n = 8, -\frac{19}{3}$$

$\Rightarrow n = 8$ because $n>0$

Now,

$$\vec{b}\cdot\vec{c} = 2+4n-3n = 2+n = 10$$

$$\vec{a}\cdot\vec{c} = 1+n+3n = 1+4n = 33$$

Hence option (c) is correct.

13

15) If the point P on the curve $4x^2+5y^2=20$ is farthest from the point $Q(0,-4)$ then PQ^2 is equal to:

a) 29 b) 48 c) 21 d) 36

Sol^n:

Eqn $4x^2+5y^2=20$

$\Rightarrow \frac{x^2}{5}+\frac{y^2}{4}=1$

$P(\sqrt{5}\cos\theta, 2\sin\theta)$

$(PQ)^2 = (\sqrt{5}\cos\theta-0)^2+(2\sin\theta+4)^2$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta+2)^2$

$(PQ)^2 = 5\cos^2\theta + 4(\sin^2\theta+4+4\sin\theta)$

$(PQ)^2 = 5\cos^2\theta + 4\sin^2\theta + 16 + 16\sin\theta$

$(PQ)^2 = 25 - (\sin\theta-8)^2$

$(PQ)^2 = 25-49$

$(PQ)^2_{max} = 36$ $\because (\sin\theta-8)\in[49, 81]$

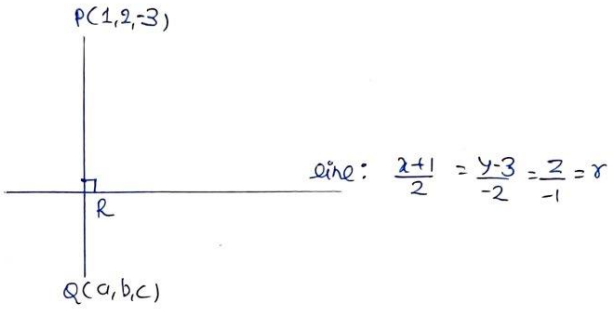
Hence option d) is correct.

14

14) If (a,b,c) is the image of the point $(1,2,-3)$ in the line $\frac{x+1}{2} = \frac{y+3}{-2} = \frac{z}{-1}$ then $a+b+c$ is equal to:

a) 3 b) -1 c) 2 d) 1

Soln



$R(-1+2r, 3-2r, -r)$

Direction Ratios of PR are $(2-2r, -1+2r, -3+r)$

Then $2(2-2r) + 2(-1+2r) + 1(-3+r) = 0$

$$9 - 9r = 0$$

$$9r = 9$$

$$\Rightarrow r = 1$$

$\Rightarrow R(1,1,-1)$

then $a+1 = 2$ $b+2 = 2$ $c-3 = -2$

$a = 1$ $b = 0$ $c = 1$

$\Rightarrow a+b+c = 1+0+1 = 2$.

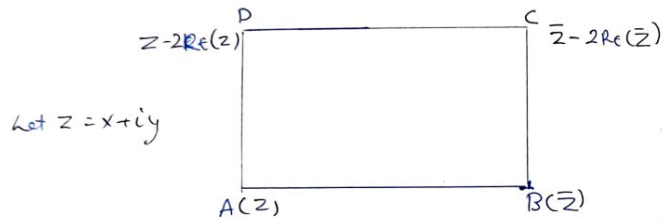
Hence option (c) is correct.

15

15) If four complex numbers $z, \bar{z}, \bar{z}-2Re(\bar{z})$ and $z-2Re(\bar{z})$ represent the vertices of a square of side 4 units in the argand plane then $|z|$ is equal to:

a) 4 b) 2 c) $4\sqrt{2}$ d) $2\sqrt{2}$

Sol^n



Let $z = x+iy$

Length of side = 4

then $|z-\bar{z}| = 4$

$$|(x+iy)-(x-iy)| = 4$$

$$|2iy| = 4$$

$$|y| = 2$$

Also $|z-(z-2Re(z)|| = 4$

$$|2x| = 4$$

$$\Rightarrow |x| = 2$$

$$|z| = \sqrt{x^2+y^2}$$

$$|z| = \sqrt{2^2+2^2}$$

$$|z| = 2\sqrt{2}$$

Hence option d) is correct.

16

16.) If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair (m,M) is equal to:

a) $(-4,4)$ b) $(0,2\sqrt{2})$ c) $(-4,0)$ d) $(0,4)$

Soln

$$f(\theta)=\begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$$f(\theta)=\begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & -2 & -2 \end{vmatrix}$$

$f(\theta) = 4(\cos^2\theta-\sin^2\theta)$

$f(\theta) = 4\cos 2\theta \qquad \theta \in \left[\frac{\pi}{4},\frac{\pi}{2}\right]$

$f(\theta)_{max.} = M = 0$

$f(\theta)_{min} = m = -4$

$(-4,0)$ is the required pair.

Hence option (c) is correct.

17

17) The negation of the Boolean expression $x \leftrightarrow \neg y$ is equivalent to:

a) $(\neg x \wedge y) \vee (\neg x \wedge \neg y)$ b) $(x \wedge y) \vee (\neg x \wedge \neg y)$

c) $(x \wedge \neg y) \vee (\neg x \wedge y)$ d) $(x \wedge y) \wedge (\neg x \vee \neg y)$

Sol^n:

Negation of $x \leftrightarrow \neg y$

$\equiv \neg(x \leftrightarrow \neg y)$

$\equiv x \leftrightarrow \neg(\neg y)$

$\equiv x \leftrightarrow y$

$\equiv (x \wedge y) \vee (\neg x \wedge \neg y)$

Hence option (b) is correct.

18

(18) Let $\lambda \in R$ The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$
$$x_1 - 6x_2 + x_3 = 2$$
$$\lambda x_1 - 10x_2 + 4x_3 = 8$$ is inconsistent for:

a) Every value of λ b) Exactly two values of λ

c) exactly one (+ve) value of λ d) exactly one negative value of λ.

Sol^n

for system linear Equations have no solution

$\Delta = 0$ and at least one of $\Delta_1, \Delta_2, \Delta_3$ should be non-zero.

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Now,

$$\Delta = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix}$$

$$\Delta = 2(-64+10) + 4(4-\lambda) + \lambda(-10+6\lambda)$$

$$\Delta = 2(-54) + 16 - 4\lambda - 10\lambda + 6\lambda^2$$

$$\Delta = -108 + 16 - 4\lambda - 10\lambda + 6\lambda^2$$

$$\Delta = 6\lambda^2 - 92 - 14\lambda = 0$$

$$3\lambda^2 - 7\lambda - 46 = 0$$

$$(\lambda - 3)(3\lambda + 2) = 0$$

$$\lambda = 3, -\frac{2}{3}$$

$$\Delta_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix}$$

$$\Delta_1 = 1(-24+10) + 4(8-3) + \lambda(-20+18)$$

$$\Delta_1 = 2(3-\lambda) \neq 0$$

$$\Rightarrow \lambda \neq 3$$

Hence $\lambda = -\frac{2}{3}$

Hence option d) is correct.

19

19) The mean and variance of 7 observations are 8 and 16 respectively. If five observations are 2, 4, 10, 12, 14 then the absolute difference of the remaining two observations is:

a) 1 b) 2 c) 3 d) 4

Soln:

$$\bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow 42+x+y = 56$$

$$\Rightarrow x+y = 14 \quad -(1)$$

Now,

$$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$16 = \frac{4+16+100+144+196+x^2+y^2}{7} - (8)^2$$

$$16+64 = \frac{460+x^2+y^2}{7}$$

$$\Rightarrow 560 = 460 + x^2+y^2$$

$$\Rightarrow x^2+y^2 = 100 \quad -(2)$$

from Eqn (1) & (2)

$$xy = 48$$

Now,

$$(x-y)^2 = (x+y)^2 - 4xy$$

$$(x-y)^2 = (14)^2 - 4(48)$$

$$(x-y)^2 = 4$$

$$|x-y| = 2$$

Hence option (b) is correct.

20

20) The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{1+e^{\sin x}}$ is:

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) π

Sol:

$$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}\,dx \quad \text{— (1)}$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}}\,dx \qquad \left[\because \int_{-a}^{a} f(x)\,dx = \int_{-a}^{a} f(-x)\,dx\right.$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}}\,dx - (2)$$

(1) + (2)

$$2I = \int_{-\pi/2}^{\pi/2} \left(\frac{1}{1+e^{\sin x}} + \frac{e^{\sin x}}{1+e^{\sin x}}\right) dx$$

$$2I = \int_{-\pi/2}^{\pi/2} dx$$

$$2I = [x]_{-\pi/2}^{\pi/2}$$

$$2I = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$I = \frac{\pi}{2}$$

Hence option b) is correct.

21

21) The number of words with or without meaning that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two words are distinct and two letters are alike is ___.

Sol^n

SYLLABUS

S-2, L-2, A, B, Y, U

Total ways = $^2C_1 \times {}^5C_2 \times \frac{4!}{2!} = 2 \times 10 \times \frac{24}{2} = 240$

Ans. 240

22

22.) The natural number m for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540 is ___.

Sol^n:

$$T_{r+1} = {}^{22}C_r (x^m)^{22-r} (x^{-2})^r$$

$$T_{r+1} = {}^{22}C_r \, x^{m(22-r)-2r}$$

To find coefficient of x $\quad m(22-r) - 2r = 1$

$$22m - 1 = r(m+2)$$

$$r = \frac{22m-1}{m+2}$$

$$r = \frac{22m+44-45}{m+2}$$

$$r = 22 - \frac{3 \times 3 \times 5}{m+2}$$

So possible values of $m = 1, 3, 7, 13, 43$

But $^{20}C_r = 1540$

only possible value of m is 13.

Ans. 13

23

23.) If the line $2x-y+2=0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x-2y+\alpha=0$ and $6x-3y+\beta=0$ respectively, then the sum of all possible values of α and β is —.

Sol^n:

$$2x-y+2=0 \quad — (1)$$

$$4x-2y+\alpha=0 \Rightarrow 2x-y+\frac{\alpha}{2}=0 \quad — (2)$$

$$6x-3y+\beta=0 \Rightarrow 2x-y+\frac{\beta}{3}=0 \quad — (3)$$

$$d_1 = \frac{\left|\frac{\alpha}{2}-2\right|}{\sqrt{2^2+1^2}} = \frac{1}{\sqrt{5}}$$

$$\Rightarrow |\alpha-6| = 2$$

$$\Rightarrow \alpha-6 = 2, -2$$

$$\alpha = 8, 4$$

$$d_2 = \frac{\left|\frac{\beta}{3}-2\right|}{\sqrt{2^2+1^2}} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow |\beta-6| = 6$$

$$\Rightarrow \beta-6 = 6, -6$$

$$\Rightarrow \beta = 12, 0$$

Sum of all values of $\alpha+\beta = 8+4+12+0 = 30$.

Ans. 30

24

24) Let $f(x) = x\left[\frac{x}{2}\right]$, for $-10 < x < 10$ where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to:

Soln

$\because \quad -10 < x < 10$

$-5 < \frac{x}{2} < 5$

$\Rightarrow \quad \left[\frac{x}{2}\right] = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$

Hence function is discontinuous at $-4, -3, -2, -1, 1, 2, 3, 4$

Number of values $= 8$

Ans. 8

25

25) four fair dice are thrown independently 27 times. The the expected number of times, at least two dice show up a three or a five is:

Soln

$$P(\text{at least 2 show 3 or 5}) = {}^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {}^4C_4\left(\frac{2}{6}\right)^4$$

$$= \frac{384+128+16}{6^4}$$

$$= \frac{11}{27}$$

$n = 27$

$\therefore$ Expectation of number of times $= np$

$= 27 \times \frac{11}{27}$

$= 11$

Ans. 11

Enrolling yourself for coaching classes has long been considered the first step towards cracking JEE. However, this trend is starting to slip. Recent studies show that, for the students who cleared the JEE Main/Advance, the number of students who had joined a coaching class is lower than the number of students who managed the feat with self-study.

Preparation for JEE Main/Advance requires a lot of hard work and determination. Be ready to even go through many sleepless nights, missed birthdays and extracurricular to finish the vast JEE syllabus. But simply completing the syllabus is not enough! If you want to have an edge over your counterparts you need to step up your game. This means practicing a variety of problems and revising diligently while following a meticulous study plan that recognizes your strengths and takes care of all your weaknesses. To get your doubts solved or questions checked there is no better place than Kunduz.

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