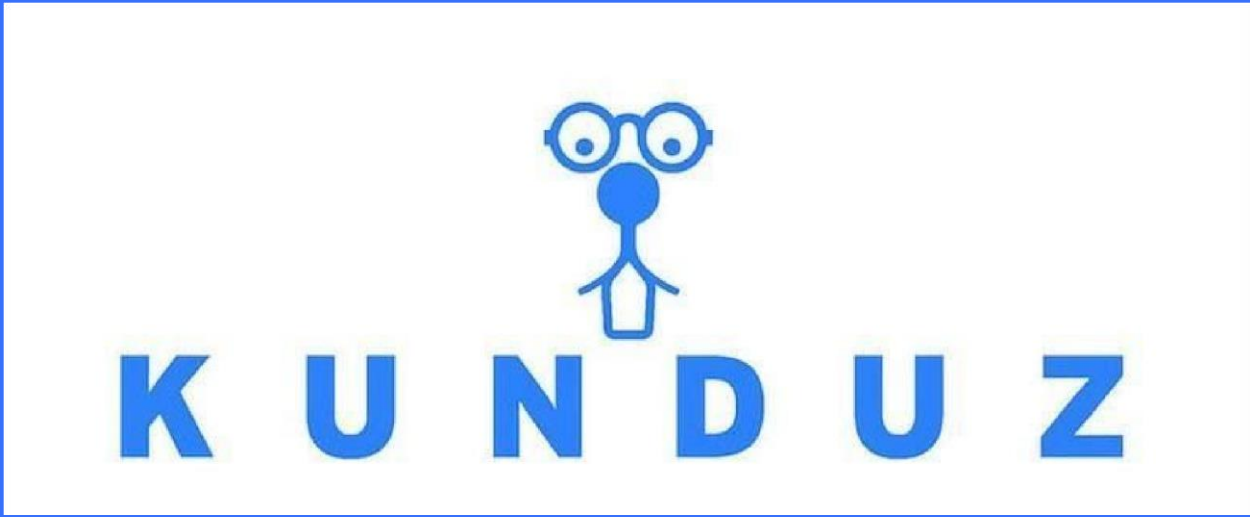


JEE Mains 5th September 2020 shift 2 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

MATHEMATICS | JEE MAIN 2020 | DATE: 05/09/2020 | SHIFT-2

1) If α and β are the roots of the Equations $7x^2-3x-2=0$ then the value of $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$ is Equal to:

a) $\frac{1}{24}$ b) $\frac{27}{32}$ c) $\frac{27}{16}$ d) $\frac{3}{8}$

Soln:

$$7x^2-3x-2=0$$

$$\alpha+\beta = -\left(\frac{-3}{7}\right) = \frac{3}{7}$$

$$\alpha\beta = -\frac{2}{7}$$

Now,

$$\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2} = \frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{(1-\alpha^2)(1-\beta^2)}$$

$$= \frac{(\alpha+\beta)(1-\alpha\beta)}{1+(\alpha\beta)^2-(\alpha^2+\beta^2)}$$

$$= \frac{(\alpha+\beta)(1-\alpha\beta)}{1+(\alpha\beta)^2-(\alpha+\beta)^2+2\alpha\beta}$$

$$= \frac{\left(\frac{3}{7}\right)\left(1+\frac{2}{7}\right)}{1+\left(\frac{2}{7}\right)^2-\left(\frac{3}{7}\right)^2+2\left(\frac{-2}{7}\right)}$$

$$= \frac{27}{16}.$$

Option (C) is correct.

2

2) The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is:

a) equivalent to $(p \vee q) \wedge (\neg p)$ b) a contradiction

c) a tautology d) equivalent to $(p \wedge q) \vee (\neg q)$

Soln:

p	q	$q \to p$	$p \vee q$	r: $p \to (q \to p)$	s: $p \to (p \vee q)$	$r \to s$
T	T	T	T	T	T	T
T	f	T	T	T	T	T
f	T	f	T	T	T	T
f	f	T	f	T	T	T

So By Above truth table given statement is equivalent to a tautology.

Hence option (c) is correct.

3

3) If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$ then which one of the following is true?

a) $4c^2 = 369$ b) $5m = 4$ c) $c^2 = 369$ d) $8m + 5 = 0$

Sol^n If $y = mx + c$ is tangent to $x^2 + y^2 = 36$

$c^2 = r^2(1 + m^2)$ where r is radius of circle

$\Rightarrow c^2 = 36(1 + m^2)$ — (1)

If $y = mx + c$ is tangent to $\frac{x^2}{100} - \frac{y^2}{64} = 1$

$c^2 = a^2m^2 - b^2$

$\Rightarrow c^2 = 100m^2 - 64$ — (2)

from Eq^n (1) and (2)

$36(1 + m^2) = 100m^2 - 64$

$36 + 36m^2 = 100m^2 - 64$

$100 = 64m^2$

$m^2 = \frac{100}{64}$

from Eq^n (1)

$c^2 = 36\left(1 + \frac{100}{64}\right)$

$c^2 = 36 \times \frac{164}{64}$

$c^2 = \frac{369}{4}$

$\Rightarrow 4c^2 = 369$

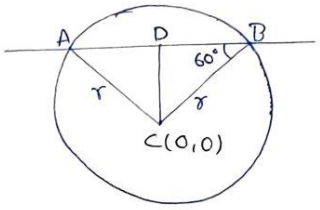
Hence option a) is correct.

4

4.) If the length of the chord of the circle $x^2+y^2=r^2$ $(r>0)$ along the line $y-2x=3$, then r^2 is equal to:

a) $\frac{9}{5}$ b) 12 c) $\frac{12}{5}$ d) $\frac{24}{5}$

soln:



$AB = r$ (given)

$\Rightarrow AD = \frac{r}{2}$

$CD = r\sin 60^\circ = \frac{\sqrt{3}}{2}r$

$\frac{|0+0-3|}{\sqrt{1^2+2^2}} = \frac{\sqrt{3}}{2}r$ [$\because$ CD is $\perp$ distance from (0,0) to $y-2x=3$.

$\Rightarrow r = 2\sqrt{\frac{3}{5}}$

$\Rightarrow r^2 = \frac{12}{5}$,

Hence option (c) is correct,

5

5) If $x = 1$ is a critical point of the function $f(x) = (3x^2+ax-2-a)e^x$ then:

a) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f.

b) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local minima of f.

c) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

d) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f

Soln:

$$f(x) = (3x^2+ax-2-a)e^x$$

$$f'(x) = (3x^2+ax-2-a)e^x + e^x(6x+a)$$

$$f'(x) = e^x(3x^2+(a+6)x+2) \quad \text{—— (i)}$$

$\because$ $x = 1$ is critical point

$$\Rightarrow f'(1) = 0$$

$$\Rightarrow 3+a+6-2 = 0$$

$$\Rightarrow a = -7$$

Now, from Eqn (i)

$$f'(x) = e^x(3x^2-x-2)$$

$$f'(x) = e^x(3x+2)(x-1)$$

At $x = -\frac{2}{3}$ $f''(x) < 0$: Local Maxima

At $x = 1$ $f''(x) > 0$: Local minima.

Hence option (c) is correct.

6

6) The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=\frac{1}{2}$ is:

a) $\frac{\sqrt{3}}{10}$ b) $\frac{\sqrt{3}}{12}$ c) $\frac{2\sqrt{3}}{5}$ d) $\frac{2\sqrt{3}}{3}$

Soln:

Let $y_1 = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta$

$$y_1 = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$y_1 = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

Let $y_2 = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\phi$

$$y_2 = \tan^{-1}\left(\frac{2\sin\phi\cos\phi}{1-2\sin^2\phi}\right)$$

$$y_2 = \tan^{-1}(\tan 2\phi) = 2\phi$$

$$y_2 = 2\sin^{-1}x$$

$$\frac{dy_1}{dy_2} = \frac{\left(\frac{dy_1}{dx}\right)}{\left(\frac{dy_2}{dx}\right)}$$

$$\frac{dy_1}{dy_2} = \frac{\frac{1}{2}\frac{1}{(1+x^2)}}{2\frac{1}{\sqrt{1-x^2}}} = \frac{\sqrt{1-x^2}}{4(1+x^2)}$$

$$\left(\frac{dy_1}{dy_2}\right)_{x=\frac{1}{2}} = \frac{\sqrt{1-\frac{1}{4}}}{4\left(1+\frac{1}{4}\right)} = \frac{\sqrt{3}}{10}$$

Hence option (a) is correct.

7

7) If the sum of the second, third and fourth terms of a positive term AP is 3 and the sum of its sixth, seventh and eight terms is 243 then the sum of the first 50 terms of this AP is:

a) $\frac{2}{13}(3^{50}-1)$ b) $\frac{1}{26}(3^{49}-1)$ c) $\frac{1}{13}(3^{50}-1)$ d) $\frac{1}{26}(3^{50}-1)$

Soln: Let $a, ar, ar^2, \ldots$ be GP

$T_2+T_3+T_4 = 3$ (given)

$ar + ar^2 + ar^3 = 3$

$ar(1+r+r^2) = 3$ — (i)

Also $T_6+T_7+T_8 = 243$ (given)

$ar^5+ar^6+ar^7 = 243$

$ar^5(1+r+r^2) = 243$ — (ii)

(ii) ÷ (i)

$$\frac{ar^5(1+r+r^2)}{ar(1+r+r^2)} = \frac{243}{3}$$

$r^4 = 81 \Rightarrow r = 3$

from Eqn (i)

$a = \frac{1}{13}$

$S_{50} = \frac{a(r^{50}-1)}{(r-1)}$

$S_{50} = \left(\frac{1}{13}\right)\left(\frac{3^{50}-1}{3-1}\right)$

$S_{50} = \frac{1}{26}(3^{50}-1)$

Hence option (d) is correct.

8

8) If the mean and standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively then a and b are the roots of the equations:

a) $x^2-20x+18=0$ b) $2x^2-20x+19=0$ c) $x^2-10x+19=0$ d) $x^2-10x+18=0$

Soln:

$$\text{Mean} = 5 \quad \text{(given)}$$

$$\frac{3+5+7+a+b}{5} = 5$$

$$\Rightarrow a+b+15 = 25$$

$$\Rightarrow a+b = 10$$

$$\text{Standard Deviation (SD)} = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$2 = \sqrt{\frac{3^2+5^2+7^2+a^2+b^2}{5} - 5^2}$$

$$4 = \frac{3^2+5^2+7^2+a^2+b^2}{5} - 25$$

$$4 = \frac{a^2+b^2+83}{5} - 25$$

$$\Rightarrow a^2+b^2 = 62$$

$$\because (a+b)^2 - 2ab = a^2+b^2$$

$$10^2 - 2ab = 62$$

$$ab = 19$$

So Eqn whose roots are a and b

$$x^2 - (a+b)x + ab = 0$$

$$x^2 - 10x + 19 = 0$$

Hence option (C) is correct.

9

9) If $\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|B(\theta)|+c$, where c is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

a) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$ b) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$ c) $\frac{2\sin\theta+1}{\sin\theta+3}$ d) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$

Sol: Let $I = \int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta$

$I = \int \frac{\cos\theta}{2\sin^2\theta+7\sin\theta+3}d\theta$

Put $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

$I = \frac{1}{2}\int \frac{dt}{t^2+\frac{7}{2}t+\frac{3}{2}}$

$I = \frac{1}{2}\int \frac{dt}{\left(t+\frac{7}{4}\right)^2-\left(\frac{5}{4}\right)^2}$

$I = \frac{1}{5}\ln\left|\frac{2t+1}{t+3}\right|+c$

$I = \frac{1}{5}\ln\left|\frac{2\sin\theta+1}{\sin\theta+3}\right|+c$

So $A = \frac{1}{5}$, $B(\theta) = \frac{2\sin\theta+1}{\sin\theta+3}$

$\frac{B(\theta)}{A} = \frac{\left(\frac{2\sin\theta+1}{\sin\theta+3}\right)}{\left(\frac{1}{5}\right)} = 5\left(\frac{2\sin\theta+1}{\sin\theta+3}\right)$

Hence option b) is correct.

10

10) The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is:

a) 6^5 b) $2^{15}i$ c) -2^{15} d) $-2^{15}i$

Soln:

$$\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} = \left(\frac{2\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)}\right)^{30}$$

$$= \frac{2^{30}}{2^{15}}\left(\frac{\cos 20\pi + i\sin 20\pi}{\cos\frac{15\pi}{2} - i\sin\frac{15\pi}{2}}\right)$$

$$= 2^{15}\left(\frac{1+i0}{0+i}\right)$$

$$= \frac{2^{15}}{i}$$

$$= \frac{2^{15}i}{i^2}$$

$$= -2^{15}i \qquad [\because i^2 = -1$$

Hence option d) is correct.

11

11) $$\lim_{x\to 0} \; x\left(\frac{e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)}-1}{\sqrt{1+x^2+x^4}-1}\right) = ?$$

a) does not Exist

b) is Equal to 1

c) is Equal to $\sqrt{e}$

d) is Equal to 0

Soln:

$$\lim_{x\to 0} \; x\left(\frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1}{\sqrt{1+x^2+x^4}-1}\right)$$

$$= \lim_{x\to 0} \frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1}{\frac{\sqrt{1+x^2+x^4}-1}{x}}$$

Put $\frac{\sqrt{1+x^2+x^4}-1}{x} = t$

Clearly when $x\to 0 \Rightarrow t\to 0$

$\therefore$ given limit is $\lim_{t\to 0} \frac{e^t-1}{t} = 1.$

Hence option b) is correct.

12

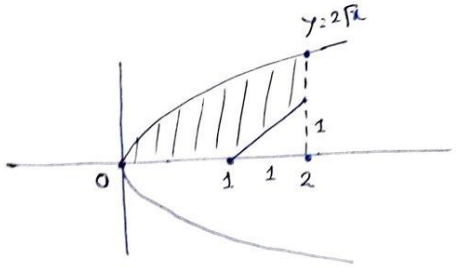
12) The Area (in sq. units) of the region $A = \{(x,y) : (x-1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$ where $[t]$ denotes the greatest integer function is:

a) $\frac{4}{3}\sqrt{2} - 1$ b) $\frac{8}{3}\sqrt{2} - 1$ c) $\frac{4}{3}\sqrt{2} + 1$ d) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

Soln:

$$y = [x](x-1)$$

$$= \begin{cases} 0 & 0 \le x < 1 \\ x-1 & 1 \le x < 2 \end{cases}$$



$$\text{Area} = \int_0^2 2\sqrt{x}\,dx - \frac{1}{2}(1)(1)$$

$$\text{Area} = 2\left(\frac{x^{3/2}}{3/2}\right)_0^2 - \frac{1}{2}$$

$$\text{Area} = 2\,\frac{2^{3/2}}{3/2} \cdot \frac{1}{2}$$

$$\text{Area} = \frac{4}{3}2\sqrt{2} - \frac{1}{2}$$

$$\text{Area} = \left(\frac{8\sqrt{2}}{3} - \frac{1}{2}\right)$$

Hence option (d) is correct.

13

13) If the system of linear equations

$x+y+3z=0$

$x+3y+k^2z=0$

$3x+y+3z=0$

has a non-zero solution (x,y,z) for some $k \in R$ then $x+\frac{y}{z}$ is:

a) -9 b) -3 c) 9 d) 3

Sol^n for non-zero solutions Δ must be zero

$$\Delta = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$$1(9-k^2) - 1(3-3k^2) + 3(1-9) = 0$$

$$(9-k^2) - 3+3k^2 - 24 = 0$$

$$9-27+2k^2 = 0$$

$$2k^2 = 18$$

$$k^2 = 9$$

Now, Eq^n's are

$x+y+3z=0$ —①

$x+3y+9z=0$ —②

$3x+y+3z=0$ —③

① − ③

$$(x+y+3z) - (3x+y+3z) = 0$$

$$x = 0$$

from Eqn ③

$$y+3z=0 \Rightarrow \frac{y}{z} = -3$$

So $x+\frac{y}{z} = 0-3 = -3$,,

Hence option (b) is Correct.

14

14) If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

a) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\left(\frac{\pi}{8}\right)$ b) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

c) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$ d) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\left(\frac{\pi}{8}\right)$

Soln:

$$L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$$

$$L = \sin\left(\frac{\pi}{16}+\frac{\pi}{8}\right)\sin\left(\frac{\pi}{16}-\frac{\pi}{8}\right)$$

$$L = \sin\left(\frac{3\pi}{16}\right)\sin\left(-\frac{\pi}{16}\right)$$

$$L = \frac{1}{2}\left[2\sin\left(\frac{3\pi}{16}\right)\sin\left(-\frac{\pi}{16}\right)\right]$$

$$L = \frac{1}{2}\left[\cos\left(\frac{3\pi}{16}+\frac{\pi}{16}\right) - \cos\left(\frac{3\pi}{16}-\frac{\pi}{16}\right)\right]$$

$$L = \frac{1}{2}\left[\frac{1}{\sqrt{2}} - \cos\left(\frac{\pi}{8}\right)\right]$$

$$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$$

$$M = \cos\left(\frac{\pi}{16}+\frac{\pi}{8}\right)\cos\left(\frac{\pi}{16}-\frac{\pi}{8}\right)$$

$$M = \frac{1}{2}\left[2\cos\left(\frac{3\pi}{16}\right)\cos\left(\frac{\pi}{16}\right)\right]$$

$$M = \frac{1}{2}\left[\cos\left(\frac{3\pi}{16}+\frac{\pi}{16}\right) + \cos\left(\frac{3\pi}{16}-\frac{\pi}{16}\right)\right]$$

$$M = \frac{1}{2}\left[\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right]$$

$$M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\left(\frac{\pi}{8}\right).$$

Hence option b) is correct.

15

15) If for some $\alpha \in R$, the lines

$L_1 : \frac{x+1}{2} = \frac{y-3}{-1} = \frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

a) $(2,-10,-2)$ b) $(10,2,2)$ c) $(-2,10,2)$ d) $(10,-2,-2)$

Sol^n: $\because$ Lines are coplanar

so, $$\begin{vmatrix} \alpha & 5-\alpha & 1 \\ 2 & -1 & 1 \\ 2-1 & 1-(-2) & 1-(-1) \end{vmatrix} = 0$$

$$\begin{vmatrix} \alpha & 5-\alpha & 1 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$$

$\Rightarrow \alpha(-2-3) - (5-\alpha)(4-1) + 1(6+1) = 0$

$\Rightarrow -5\alpha - 3(5-\alpha) + 7 = 0$

$\Rightarrow -5\alpha - 15 + 3\alpha + 7 = 0$

$-2\alpha - 8 = 0$

$2\alpha = -8$

$\alpha = -4$

$\Rightarrow L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Now, $(2,-10,-2)$ satisfies this line.

Hence option (a) is correct.

16

16) If $a+x = b+y = c+z+1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to.

a) $y(a-b)$ b) 0 c) $y(b-a)$ d) $y(a-c)$

Soln:

$x+a = y+b+1 = c+z$ (given) —— (i)

Now, $\begin{vmatrix} x & a+y & a+x \\ y & b+y & b+y \\ z & c+y & c+z \end{vmatrix}$

$C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix}$

$C_2 \to C_2 - C_3$

$\Rightarrow \begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix}$

$\Rightarrow y \begin{vmatrix} x & 1 & a \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$y \begin{vmatrix} x & 1 & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix} = y \begin{vmatrix} x & 1 & a \\ a-b & 0 & -(a-b) \\ z-x & 0 & c-a \end{vmatrix} = y(a-b) \begin{vmatrix} x & 1 & a \\ 1 & 0 & -1 \\ z-x & 0 & c-a \end{vmatrix}$

$= -y(a-b)(c-a+z-x)$

$= -y(a-b)(-1)$ [from Eqn (i)]

$= y(a-b)$

Hence option a) is correct.

17

17) Let $y = y(x)$ be the solution of the Differential Equation

$$\cos x \frac{dy}{dx} + 2y\sin x = \sin 2x, \quad x \in \left(0, \frac{\pi}{2}\right)$$

If $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{4}\right)$ is equal to:

a) $\frac{1}{\sqrt{2}} - 1$ b) $2 - \sqrt{2}$ c) $\sqrt{2} - 2$ d) $2 + \sqrt{2}$

Soln Given Differential Equation

$$\cos x \frac{dy}{dx} + 2y\sin x = \sin 2x$$

$$\frac{dy}{dx} + 2y\tan x = 2\sin x$$

$$I.f = e^{\int 2\tan x\, dx} = \sec^2 x$$

$$\Rightarrow \sec^2 x \frac{dy}{dx} + 2y\tan x \sec^2 x = 2\sin x \sec^2 x$$

$$\Rightarrow \int d(y\sec^2 x) = \int 2\sin x \sec^2 x\, dx$$

$$\Rightarrow y\sec^2 x = 2\sec x + c \quad - (i)$$

$$y\left(\frac{\pi}{3}\right) = 0 \quad \text{given}$$

$$\Rightarrow \frac{2\sec\left(\frac{\pi}{3}\right) + c}{\sec^2\left(\frac{\pi}{3}\right)} = 0$$

$$c = -4$$

Now $y\sec^2 x = 2\sec x - 4$

$$y\left(\frac{\pi}{4}\right) = \sqrt{2} - 2$$

Hence option (c) is correct.

18

18) There are 3 sections in a Question Paper and each question contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from Each Section. Then the number of ways in which the candidates can question is:

a) 3000 b) 2250 c) 2255 d) 1500

Sol:

Section-A (5Q)	Section-B (5Q)	Section-C (5Q)
A_1, A_2, A_3, A_4, A_5	B_1, B_2, B_3, B_4, B_5	C_1, C_2, C_3, C_4, C_5

$A_1 A_2 A_3 B_1 C_1 \Rightarrow {}^3C_1 \times {}^5C_3 \times {}^5C_1 \times {}^5C_1 = 750$

$A_1 A_2 B_1 B_2 C_1 \Rightarrow {}^3C_2 \times {}^5C_2 \times {}^5C_2 \times {}^5C_1 = 1500$

Total ways = 750 + 1500

= 2250.

Hence option b) is Correct.

19

19) If the sum of the first 20 terms of the series $\log_{7^{1/2}} x + \log_{7^{1/3}} x + \ldots$ 20 times is 460, then x is equal to:

a) e^2 b) $7^{48/21}$ c) 7^2 d) $7^{1/2}$

Soln Given $\log_{7^{1/2}} x + \log_{7^{1/3}} x + \ldots 20 \text{ times} = 460$

$$2\log_7 x + 3\log_7 x + \ldots + 20 \text{ times} = 460$$

$$(2+3+4+\ldots+21)\log_7 x = 460$$

$$\frac{20}{2}(2+21)\log_7 x = 460$$

$$23\log_7 x = 46$$

$$\log_7 x = 2$$

$$x = 7^2$$

$$x = 49$$

Hence option c) is Correct.

20

20) Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point $(1,0)$?

a) $(-2,4)$ b) $(2,2)$ c) $(-2,6)$ d) $(2,6)$

Soln:

Given $x^4e^y + 2\sqrt{y+1} = 3$

Differentiate w.r.t x

$$e^y y' x^4 + 4x^3 e^y + 2y' \frac{1}{2\sqrt{y+1}} = 0$$

Put $x=1, y=0$

$$y' + 4 + y' = 0$$

$$\Rightarrow y = -2$$

Equation of tangent at $(1,0)$ is

$$y - 0 = -2(x-1)$$

$$y = -2x + 2$$

$$2x + y = 2$$

or $2x + y - 2 = 0$

$\therefore (-2,6)$ satisfies this equation.

Hence option (c) is correct.

21

21) In a bombing attack there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs that must be dropped to ensure that there is at least 99% chance of completely destroying the target is ___

Soln

Let Probability of hitting the target = p

$\Rightarrow p = \frac{1}{2}$

Let n be the number of bombs

According to given condition

$$1 - ({}^{n}C_0 p^0 (1-p)^n + {}^{n}C_1 p^1 (1-p)^{n-1}) \geq \frac{99}{100}$$

$$\Rightarrow \quad 2^n \geq (n+1)\,100$$

If $n = 10 \Rightarrow 2^{10} \geq 1100$ (Reject)

If $n = 11 \Rightarrow 2^{11} \geq 1200$ (Select)

11 will be the Answer for this Question.

Ans. 11.

22

22) Let vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}|=2$, $|\vec{b}|=4$ and $|\vec{c}|=4$. If the Projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ perpendicular to $\vec{c}$, then the value of $|\vec{a}-\vec{b}-\vec{c}|$ is —

Soln

$\vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$ (given)

$|\vec{a}+\vec{b}-\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{c} - \vec{a}\cdot\vec{c})$

$|\vec{a}+\vec{b}-\vec{c}|^2 = 4 + 16 + 16 + 2(\vec{a}\cdot\vec{b} - 0 - \vec{a}\cdot\vec{b})$

$|\vec{a}+\vec{b}-\vec{c}|^2 = 36$

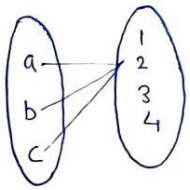
$\Rightarrow |\vec{a}+\vec{b}-\vec{c}| = 6$

Ans 6

23

23) Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of Elements in the set $C = \{f : A \to B \mid 2 \in f(A) \text{ and } f \text{ is not one to one}\}$ is —

Soln



only 2 in the range → 1 function

one element out of 1,3,4 is in the range with '2'

Number of ways $= {}^3C_1 \cdot \frac{3!}{2!\,1!} \cdot 2! = 18$

(select one from 1,3,4 and distribute among a,b,c)

Total function $= 1 + 18 = 19$,

Ans 19

24

24) If the lines $x+y=a$ and $x-y=b$ touch the curve $y=x^2-3x+2$ at the points where the curve intersects the x-axis then $\frac{a}{b}$ is Equal to ___.

Sol^n

$y=x^2-3x+2$, $x+y=a$, $x-y=b$

$2x_1-0=4$

$\Rightarrow x_1=2$

Put $x_1=2$ in Eq^n $y=x^2-3x+2$

$x_1=4-6+2=0$

$(2,0)$

$\Rightarrow b=2$

Now,

$2x_2-3=-1$

$2x_2=-1+3$

$x_2=1$

Put $x_2=1$ in Eq^n $y=x^2-3x+2$

$y_2=(1)^2-3(1)+2$

$y_2=0$

$(1,0)$

$\Rightarrow a=1$

Hence, $\frac{a}{b}=\frac{1}{2}=0.5$

Ans 0.5,

25

25) The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^6$ in powers of x, is ___

Sol^n Given $(1+x+x^2+x^3)^6$

$(1+x)^6(1+x^2)^6$

$(6C_0 + 6C_1x + \dots)\left(6C_0 + 6C_1x^2 + 6C_2x^4 + \dots\right)$

So, coefficient of x^4 $= 6C_0 \times 6C_2 + 6C_4 \times 6C_0 + 6C_2 \times 6C_1$

$= 1 \times 15 + 15 \times 1 + 15 \times 6$

$= 15 + 15 + 90$

$= 120.$

Ans. 120.

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