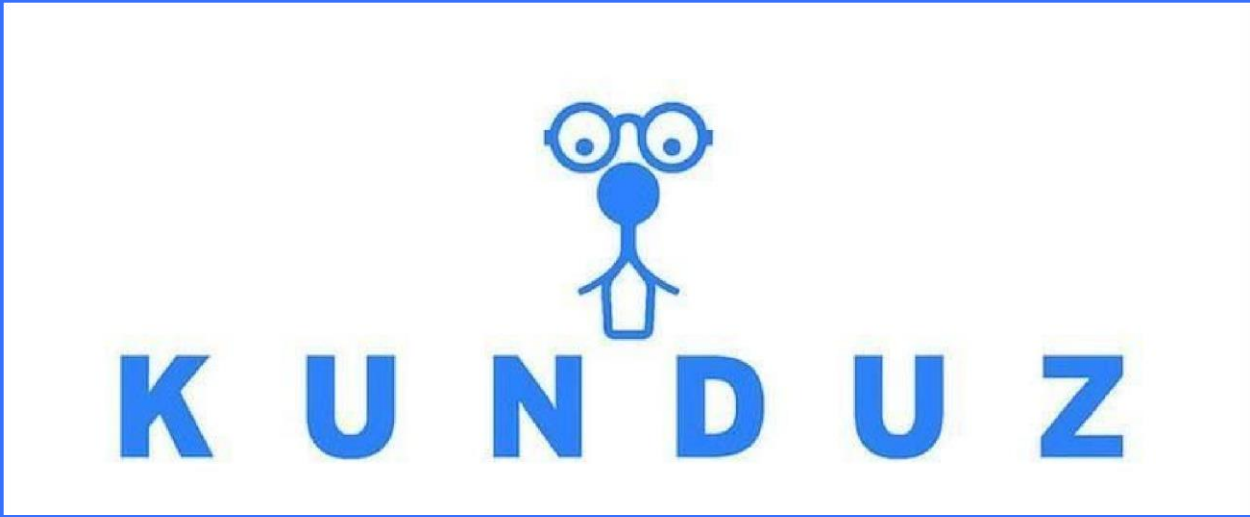


JEE Mains 6th September 2020 shift 1 Mathematics solutions prepared by KUNDUZ tutors

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Question number

1

Question and Answers

Mathematics (JEE MAIN 2020)

1) Let L_1 be a tangent to the Parabola $y^2 = 4(x+1)$ and L_2 be a tangent to the Parabola $y^2 = 8(x+2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the Straight Line:

a) $x+3=0$ b) $2x+1=0$ c) $x+2y=0$ d) $x+2=0$

Soln: $\because$ Tangent to $y^2 = 4ax$ is given by $ty = x + at^2$

Let tangent to $y^2 = 4(x+1)$ be

$L_1 : t_1 y = (x+1) + t_1^2$ — (i)

And tangent to $y^2 = 8(x+2)$ be

$L_2 : t_2 y = (x+2) + 2t_2^2$ — (ii)

$\because$ L_1 is Perpendicular to L_2

$\Rightarrow$ (Slope of L_1) $\times$ (Slope of L_2) $= -1$

$\frac{1}{t_1} \times \frac{1}{t_2} = -1$

$t_1 t_2 = -1$

Now,

(i) $\times t_2$ - (ii) $\times t_1$

$(t_1 t_2 y - (x+1)t_2 - t_1^2 t_2) - (t_1 t_2 y - (x+2)t_1 - 2t_2^2 t_1) = 0$

on solving

$x(t_1 - t_2) + 3(t_1 - t_2) = 0$

$\Rightarrow (x+3) = 0$

Hence option (a) is correct.

2

2) If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, $x, y \in N$ where N is the set of all natural numbers then the value of $\frac{f(4)}{f(2)}$ is:

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{1}{9}$ d) $\frac{4}{9}$

Sol^n: Let $f(x) = a^x$ because a^x satisfies the property:

$$f(x+y) = f(x)f(y)$$

Now,

$$\sum_{x=1}^{\infty} f(x) = 2$$

$$\Rightarrow \sum_{x=1}^{\infty} a^x = 2$$

$$\Rightarrow a + a^2 + a^3 + \dots \infty \text{ terms} = 2$$

$$\frac{a}{1-a} = 2 \Rightarrow a = \frac{2}{3}$$

[$\because$ sum of an infinite G.P $= \frac{a}{1-r}$]

Now $f(x) = a^x = \left(\frac{2}{3}\right)^x$

$$f(4) = \left(\frac{2}{3}\right)^4$$

$$f(2) = \left(\frac{2}{3}\right)^2$$

$$\Rightarrow \frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Option d) is correct.

3

3) If $\sum_{i=1}^{n}(x_i-a)=n$ and $\sum_{i=1}^{n}(x_i-a)^2=na$, $(n,a>1)$ then the standard Deviation of n observations $x_1, x_2, x_3 \ldots x_n$ is

a) $a-1$ b) $\sqrt{n(a-1)}$ c) $n\sqrt{a-1}$ d) $\sqrt{a-1}$

Soln:

$$\text{Standard Deviation (S.D)} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Replace $x_i \to (x_i-a)$

$$SD = \sqrt{\frac{\sum (x_i-a)^2}{n} - \left(\frac{\sum (x_i-a)}{n}\right)^2}$$

Put the values of $\sum_{i=1}^{n}(x_i-a)$ & $\sum_{i=1}^{n}(x_i-a)^2$

$$SD = \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2}$$

$$S.D = \sqrt{a-1}$$

Hence option d) is correct.

4

4.) $\lim_{x \to 1} \left[\frac{\int_0^{(x-1)^2} t \cos t^2 \, dt}{(x-1)\sin(x-1)} \right] = ?$

a) is Equal to 1 b) does not Exist c) is Equal to $\frac{1}{2}$ d) is Equal to $-\frac{1}{2}$

Soln: Let $L = \lim_{x \to 1} \left(\frac{\int_0^{(x-1)^2} t \cos t^2 \, dt}{(x-1)\sin(x-1)} \right)$

By using L' Hopital Rule

$$L = \lim_{x \to 1} \frac{(x-1)^2 \cos[(x-1)^2]^2 \, 2(x-1)}{(x-1)\cos(x-1) + \sin(x-1)}$$

$$L = \lim_{x \to 1} \frac{2(x-1)^3 \cos(x-1)^4}{\sin(x-1) + (x-1)\cos(x-1)}$$

$$L = \lim_{x \to 1} \frac{2(x-1)^2 \cos(x-1)^4}{\frac{\sin(x-1)}{(x-1)} + \cos(x-1)}$$

$$L = \frac{0}{1+1}$$

$$L = 0$$

As No' option Matches with the given options. so this question would be a BONUS.

Ans. BONUS

5

5.) Which of the following points lies on the locus of the foot of the perpendicular drawn upon any tangent to the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci?

1) $(-2,\sqrt{3})$ 2) $(-1,\sqrt{2})$ 3) $(1,2)$ 4) $(-1,\sqrt{3})$

Sol^n:

$\because$ foot of perpendicular from focus upon any tangent lies on auxilliary circle.

Auxillary circle of Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2$

$\Rightarrow$ Auxillary circle of the given Ellipse would be $x^2+y^2=4$.

$\because$ $(-1,\sqrt{3})$ satisfies this Auxillary circle $x^2+y^2=4$

Hence option (4) is correct.

Ans. (4)

6

6.) Out of 11 consecutive natural Number if three numbers are selected at random (without Repetition) then the probability that they are in A.P with Positive common difference is:

a) $\frac{10}{99}$ b) $\frac{15}{101}$ c) $\frac{5}{33}$ d) $\frac{5}{101}$

Soln There are two cases Possible either first natural number is odd or Even.

Case 1: E, O, E, O, E, O, E, O, E, O, E

$\because$ a, b, c are in A.P

$\Rightarrow 2b = a+c \rightarrow$ Even [$\because$ b is a multiple of 2

So a and c should be either even or odd

$$P = \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_3} = \frac{5}{33}$$

Case 2: O, E, O, E, O, E, O, E, O, E, O

$\because$ a, b, c are in A.P

$\Rightarrow 2b = a+c$

$$P = \frac{{}^5C_2 + {}^6C_2}{{}^{11}C_3} = \frac{5}{33}$$

Total Probability $= \left(\frac{1}{2}\right)\left(\frac{5}{33}\right) + \left(\frac{1}{2}\right)\left(\frac{5}{33}\right) = \frac{5}{33}$

Hence option c) is correct.

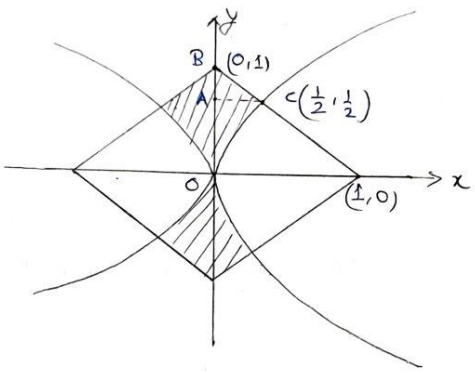
Ans. c)

7

7) The Area (in sq. units) of the region $A = \{(x,y) : |x|+|y| \le 1, 2y^2 \ge |x|\}$:

a) $\frac{5}{6}$ b) $\frac{7}{6}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$

Soln:



Shaded Region is our Required Area.

Area (A) = area (OAC) + $\triangle ABC$

$$A = \int_0^{1/2} 2y^2 dy + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$$

$$A = \left(\frac{2y^3}{3}\right)_0^{1/2} + \frac{1}{8} = \frac{2}{3}\times\frac{1}{8} + \frac{1}{8} = \frac{5}{24}$$

Now Required Area is = $4A = 4 \times \frac{5}{24} = \frac{5}{6}$"

option a) is Correct"

8

8.) Let m and M be respectively the minimum and maximum values of $\begin{vmatrix} \cos^2x & 1+\sin^2x & \sin 2x \\ 1+\cos^2x & \sin^2x & \sin 2x \\ \cos^2x & \sin^2x & 1+\sin 2x \end{vmatrix}$. Then the ordered pair (m, M) is Equal to-

a) $(-3, 3)$ b) $(1, 3)$ c) $(-3, -1)$ d) $(-4, -1)$

Soln:

Let $\Delta = \begin{vmatrix} \cos^2x & 1+\sin^2x & \sin 2x \\ 1+\cos^2x & \sin^2x & \sin 2x \\ \cos^2x & \sin^2x & 1+\sin 2x \end{vmatrix}$

$C_1 \to C_1 + C_2$

$$\Delta = \begin{vmatrix} \cos^2x+1+\sin^2x & 1+\sin^2x & \sin 2x \\ 1+\cos^2x+\sin^2x & \sin^2x & \sin 2x \\ \cos^2x+\sin^2x & \sin^2x & 1+\sin 2x \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 2 & 1+\sin^2x & \sin 2x \\ 2 & \sin^2x & \sin 2x \\ 1 & \sin^2x & 1+\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - 2R_3$ and $R_2 \to R_2 - 2R_3$

$$\Delta = \begin{vmatrix} 0 & \cos^2x & -(2+\sin 2x) \\ 0 & -\sin^2x & -(2+\sin 2x) \\ 1 & \sin^2x & (1+\sin 2x) \end{vmatrix}$$

$$\Delta = -2 - \sin 2x$$

Δ is minimum when $\sin 2x$ is maximum and vice-versa

$\Delta_{min} = -2 - 1 = -3$

$\Delta_{max.} = -2 - (-1) = -1$

$(m, M) \equiv (-3, -1)$

option c) is correct.

9

9.) Let a, b, c, d and p non-zero real distinct numbers such that $(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) = 0$. Then:

Soln:

a) a, b, c, d are in A.P b) a, c, p are in G.P

c) a, b, c, d are in G.P d) a, c, p are in A.P

Soln:

$$(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) = 0$$

$$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) = 0$$

$$(ap-b)^2 + (bp-c)^2 + (cp-d)^2 = 0$$

$\because$ Squares of three distinct numbers will be zero only when the numbers are zero.

$$ap - b = 0 \Rightarrow p = \frac{b}{a}$$

$$bp - c = 0 \Rightarrow p = \frac{c}{b}$$

$$cp - d = 0 \Rightarrow p = \frac{d}{c}$$

$$\Rightarrow p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$\Rightarrow a, b, c, d$ are in G.P.

Hence option c) is correct.

10

10) If $\{p\}$ denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$ is Equal to:

a) $\frac{1}{8}$ b) $\frac{3}{8}$ c) $\frac{7}{8}$ d) $\frac{5}{8}$

Soln:

$$3^{200} = 9^{100} = (1+8)^{100}$$

$$3^{200} = {}^{100}C_0 \cdot 8^{100} + {}^{100}C_1 \cdot 8^{99} + \ldots + {}^{100}C_{100}$$

$$3^{200} = 8\left({}^{100}C_0 8^{99} + {}^{100}C_1 8^{98} + \ldots\right) + 1$$

$$3^{200} = 8\lambda + 1$$

where λ is $\left({}^{100}C_0 8^{99} + {}^{100}C_1 8^{98} + \ldots {}^{100}C_{99}\right)$

Now,

$$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{8\lambda+1}{8}\right\}$$

$$= \left\{\lambda + \frac{1}{8}\right\}$$

$$\left\{\frac{3^{200}}{8}\right\} = \frac{1}{8}$$

Hence option a) is correct.

11

11) The values of λ and μ for which the system of linear equations:

$$x+y+z=2$$
$$x+2y+3z=5$$
$$x+3y+\lambda z=\mu$$

has infinitely many solutions respectively:

a) 5 and 8 b) 4 and 9 c) 6 and 8 d) 5 and 7

Soln: A system of linear Equations has infinitely many solutions only when $\Delta, \Delta_1, \Delta_2, \Delta_3$ all are zero.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(2\lambda-9) - 1(\lambda-3) + 1(3-2) = 0$$

$$\Rightarrow \lambda = 5$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & \mu \end{vmatrix} = 0$$

$$\Rightarrow 1(2\mu-15) - 1(\mu-5) + 2(3-2) = 0$$

$$\Rightarrow 2\mu-15-\mu+5+2 = 0$$

$$\Rightarrow \mu = 8.$$

$\lambda = 5, \mu = 8$

Hence option (a) is correct.

12

12) The Region represented by $\{z = x+iy \in C : |z| - Re(z) \leq 1\}$ is also given by the inequality

a) $y^2 \geq 2(x+1)$ b) $y^2 \leq x + \frac{1}{2}$ c) $y^2 \leq 2x+1$ d) $y^2 \geq x+1$

Soln:

$$\because |z| - Re(z) \leq 1$$

$$\sqrt{x^2+y^2} - x \leq 1$$

$$\sqrt{x^2+y^2} \leq (x+1)$$

Squaring both sides

$$\Rightarrow x^2+y^2 \leq (x+1)^2$$

$$\Rightarrow x^2+y^2 \leq x^2+2x+1$$

$$\Rightarrow y^2 \leq 2x+1$$

Hence option (c) is correct.

13

13) The general solution of the differential equation

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$

a) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + c$

b) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + c$

c) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + c$

d) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + c$

Sol^n:

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$$

$$\Rightarrow \sqrt{(1+x^2)(1+y^2)} = -xy\frac{dy}{dx}$$

$$\int \frac{y}{\sqrt{1+y^2}}dy = -\int \frac{\sqrt{1+x^2}}{x}dx$$

$$\int \frac{2y}{2\sqrt{1+y^2}}dy = -\int \frac{(1+x^2)}{x(\sqrt{1+x^2})}dx$$

$$\sqrt{1+y^2} = -\int \frac{x}{\sqrt{1+x^2}}dx - \int \frac{x}{x^2\sqrt{1+x^2}}dx$$

$$\sqrt{1+y^2} = -\sqrt{1+x^2} + \frac{1}{2}\ln\left[\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right] + c$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\ln\left[\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right] + c$$

Hence option b) is correct.

14

14) The Position of moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$ where a, b, c are real numbers greater than 1. The average speed of the car over the time interval $[t_1, t_2]$ is attained at the point:

a) $\frac{(t_1+t_2)}{2}$ b) $\frac{(t_2-t_1)}{2}$ c) $2a(t_1+t_2)+b$ d) $a(t_2-t_1)+b$

Soln:

$$V_{avg.} = f'(t) = \frac{f(t_2)-f(t_1)}{t_2-t_1}$$

$$f'(t) = \frac{(at_2^2+bt_2+c)-(at_1^2+bt_1+c)}{t_2-t_1}$$

$$f'(t) = \frac{a(t_2^2-t_1^2)+b(t_2-t_1)-c+c}{t_2-t_1}$$

$$f'(t) = \frac{a(t_2-t_1)(t_2+t_1)+b(t_2-t_1)}{t_2-t_1}$$

$$2at+b = a(t_2+t_1)+b$$

$$2at = a(t_2+t_1)$$

$$t = \frac{(t_1+t_2)}{2}$$

Hence option a) is correct.

15

15.) The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is Equivalent

a) $p \wedge \sim q$ b) $\sim p \wedge q$ c) $\sim p \wedge \sim q$ d) $\sim p \vee \sim q$

Soln:

Given Statement is $p \wedge (q \wedge \sim p)$

$\therefore$ Negation is $\sim (p \wedge (q \wedge \sim p))$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge p)$

$= (\sim p \wedge \sim q) \vee c$

$= \sim p \wedge \sim q$

Hence option c) is correct.

16

16.) Two families with three members each and one family with four members are to be seated in a row. In how many ways they can be seated so that the same family members are not separated?

a) $2!3!4!$ b) $3!(4!)^3$ c) $(3!)2.(4!)$ d) $(3!)^3(4!)$

Soln

Total Number of ways $= 3! \times 3! \times 3! \times 4!$

$= (3!)^3 \times 4!$

Hence option (d) is correct.

17

17) The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x+y+z+1=0$, $2x-y+z+3=0$ is:

a) $\frac{1}{2}$ b) 1 c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{\sqrt{2}}$

Soln Plane through line of intersection is $(x+y+z+1)+\lambda(2x-y+z+3)=0$

$$x(1+2\lambda)+y(1-\lambda)+z(1+\lambda)+(3\lambda+1)=0$$

It should be Parallel to given line

$$\Rightarrow \quad 0(1+2\lambda)-1(1-\lambda)+1(1+\lambda)=0$$

$$-1+\lambda+1+\lambda=0$$

$$\lambda=0$$

Plane Eqn $\rightarrow$ $x+y+z+1=0$

Shortest Distance = Perpendicular distance of $(1,-1,0)$ from this plane

$$= \frac{|1-1+0+1|}{\sqrt{1^2+1^2+1^2}}$$

$$= \frac{1}{\sqrt{3}}$$

Hence option c) is Correct.

18

1Q) A Ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line $x = 1$ at the point A. The Ray gets reflected on the line $x = 1$ and meets x-axis at the Point B. Then the line AB passes through the point:

a) $\left(4, -\frac{\sqrt{3}}{2}\right)$ b) $(3, -\sqrt{3})$ c) $(4, -\sqrt{3})$ d) $\left(3, -\frac{1}{\sqrt{3}}\right)$

Soln:

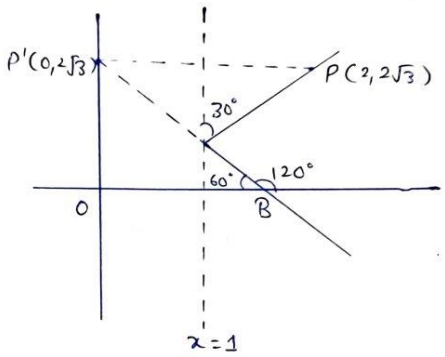


Image of Point $P(2, 2\sqrt{3})$ in $x = 1$ is $P'(0, 2\sqrt{3})$

Eqn of P'B is

$$y - 2\sqrt{3} = \tan 120^\circ (x - 0)$$

$$y - 2\sqrt{3} = -x\sqrt{3}$$

$$\sqrt{3}x + y = 2\sqrt{3}$$

$\because$ $(3, -\sqrt{3})$ satisfies this Eqn

Hence option b) is correct.

19

19) If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$, $I_2 = \alpha I_1$ then α is Equal to:

a) $\frac{5049}{5050}$ b) $\frac{5051}{5050}$ c) $\frac{5050}{5051}$ d) $\frac{5050}{5049}$

Sol^n

$\because I_2 = \alpha I_1$ (given)

$\alpha = \frac{I_2}{I_1}$

$\alpha = \frac{\int_0^1 (1-x^{50})^{101} dx}{\int_0^1 (1-x^{50})^{100} dx}$

$I_2 = \int_0^1 (1-x^{50})^{101} dx = \int_0^1 (1-x^{50})(1-x^{50})^{100} dx$

$I_2 = I_1 - \int_0^1 x \cdot x^{49} (1-x^{50})^{100} dx$

$I_2 = I_1 - \left[\frac{-x(1-x^{50})^{101}}{5050}\right]_0^1 - \int_0^1 \frac{(1-x^{50})^{101}}{5050} dx$

$I_2 = I_1 - \frac{I_2}{5050} \Rightarrow \frac{I_2}{I_1} = \frac{5050}{5051}$

$\alpha = \frac{I_2}{I_1} = \frac{5050}{5051}$

Hence option c) is correct.

20

20) If α and β are two roots of the equation $x^2-64x+256=0$ Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is:

a) 2 b) 4 c) 3 d) 1

Sol:

$\alpha+\beta = 64$

$\alpha\beta = 256$

Now $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{1/8} = \frac{\alpha^{3/8}}{\beta^{5/8}}+\frac{\beta^{3/8}}{\alpha^{5/8}}$

$= \frac{\alpha^{\frac{3}{8}+\frac{5}{8}}+\beta^{\frac{3}{8}+\frac{5}{8}}}{(\alpha\beta)^{5/8}}$

$= \frac{\alpha+\beta}{(\alpha\beta)^{5/8}}$

$= \frac{64}{(256)^{5/8}}$

$= \frac{64}{(2^8)^{5/8}}$

$= \frac{64}{32} = 2$

Hence option a) is correct.

21

21.) If $\vec{a}$ and $\vec{b}$ are unit vectors then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}|$ is

Sol^n: Let Angle between $\vec{a}$ and $\vec{b}$ be α.

$$\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}| = \sqrt{3}\sqrt{|a|^2+|b|^2+2|a||b|\cos\alpha} + \sqrt{|a|^2+|b|^2-2|a||b|\cos\alpha}$$

$$= \sqrt{3}\sqrt{2+2\cos\alpha} + \sqrt{2-2\cos\alpha}$$

$$= \sqrt{3}\sqrt{2\times 2\cos^2\frac{\alpha}{2}} + \sqrt{2\times 2\sin^2\frac{\alpha}{2}}$$

$$= 2\left(\sqrt{3}\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)$$

Maximum Value $= 2\times\sqrt{(\sqrt{3})^2+(1)^2}$

$= 2\times\sqrt{3+1}$

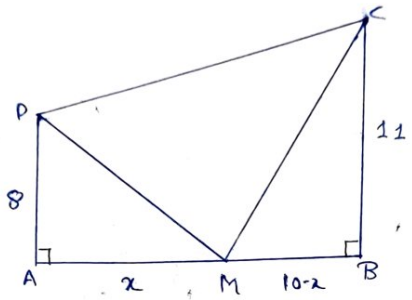
$= 2\times 2 = 4$,,

4 is the answer for this Problem.

22

22) Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8m, BC = 11m and AB = 10m, then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is.

Soln



In $\triangle MAD$

$$MD^2 = AM^2 + AD^2 = x^2 + 64$$

In $\triangle MBC$

$$MC^2 = BM^2 + BC^2 = (10-x)^2 + 121$$

Let $D = (MD^2 + MC^2) = (x^2+64) + (10-x)^2 + 121$

$$D = x^2 + 64 + 100 + x^2 - 20x + 121$$

$$D = 285 - 20x + 2x^2$$

$$\frac{dD}{dx} = -20 + 4x = 0 \Rightarrow x = 5$$

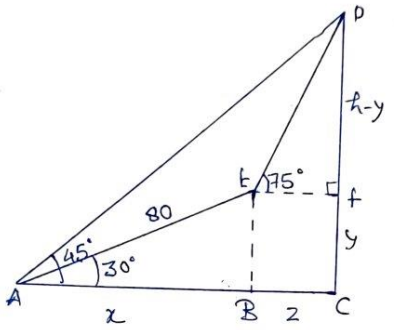
At $x = 5$ $MD^2 + MC^2$ is Minimum.

5 is the Answer for this Problem.

23

23) The Angle of Elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top up a slope inclined at angle of 30° to the horizontal plane the angle of Elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is.

Soln



from the figure

$$x = 80\cos 30^\circ = 80 \times \frac{\sqrt{3}}{2} = 40\sqrt{3}$$

$$y = 80\sin 30^\circ = 80 \times \frac{1}{2} = 40$$

In $\triangle ACB$

$$\tan 45^\circ = \frac{(h-y)+y}{x+z} = \frac{h}{x+z}$$

$$1 = \frac{h}{x+z}$$

$$x+z = h$$

$$40\sqrt{3}+z = h \quad \text{— (1)}$$

Similarly In $\triangle EFD$

$$\tan 75^\circ = \frac{h-y}{z} \Rightarrow (2+\sqrt{3}) = \frac{h-40}{z} \quad \text{— (2)}$$

on Solving Eqn (1) & (2)

$$\boxed{h = 80m}$$

∴ 80m is the correct answer for this question.

24

24.) Set A has m elements and set B has n elements. If total number of subsets is 112 more than the total number of subsets of B, then the value of (m×n) is ____.

Soln

$$2^m = 2^n + 112 \quad \text{(given)}$$

$$2^m - 2^n = 112$$

$$2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$$

On comparing we get

$$n = 4$$

$$m - n = 3$$

$$\Rightarrow m = 7$$

So $(m \times n) = (7 \times 4) = 28$

28 is the correct Answer for this Question.

25

25) $f: R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0 & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists is ____

Soln:

$$f'(x) = \begin{cases} 5x^4 \sin\left(\frac{1}{x}\right) - x^3 \cos\frac{1}{x} + 10x & x < 0 \\ 0 & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x & x > 0 \end{cases}$$

$$f''(x) = \begin{cases} 20x^3 \cdot \sin\left(\frac{1}{x}\right) - 5x^2 \cos\left(\frac{1}{x}\right) - 3x^2 \cos\left(\frac{1}{x}\right) - x \sin\left(\frac{1}{x}\right) + 10 & x < 0 \\ 0 & x = 0 \\ 20x^3 \cdot \cos\left(\frac{1}{x}\right) + 5x^2 \sin\left(\frac{1}{x}\right) + 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) + 2\lambda & x > 0 \end{cases}$$

Now for $f''(x)$ to be exists at $x = 0$

$$f''(0^+) = f''(0^-)$$

$$2\lambda = 10$$

$$\Rightarrow \boxed{\lambda = 5}$$

5 is the correct Answer for this question:

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