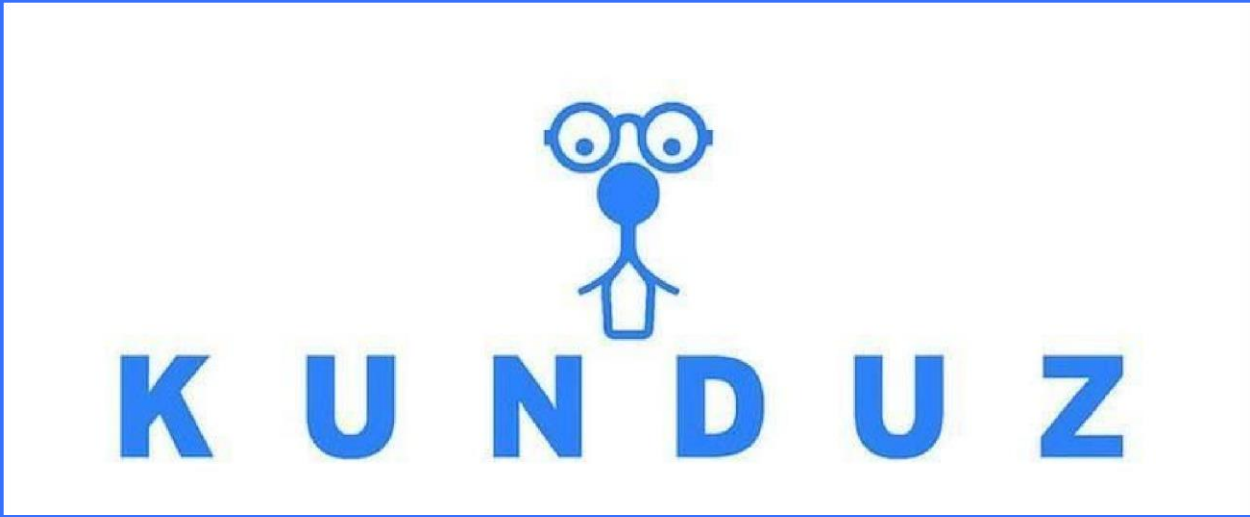


JEE Mains 6th September 2020 shift 2 Mathematics solutions prepared by KUNDUZ tutors

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Question number

Question and Answers

1

MATHEMATICS | JEE MAIN 2020 | DATE : 06/09/2020 | SHIFT-2

1) Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then $\det(B)$:

a) is zero b) is one c) lies in $(2,3)$ d) lies in $(1,2)$

Soln:

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$B = A + A^4 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$B = \begin{bmatrix} \cos 4\theta + \cos\theta & \sin 4\theta + \sin\theta \\ -(\sin\theta + \sin 4\theta) & \cos\theta + \cos 4\theta \end{bmatrix}$$

$$B = (\cos 4\theta + \cos\theta)^2 + (\sin 4\theta + \sin\theta)^2$$

$$B = 2 + 2\cos 3\theta$$

$$|B| = 2 + 2\cos\left(\frac{3\pi}{5}\right)$$

$$|B| = 2 - \left(\frac{\sqrt{5}-1}{2}\right) = \frac{5-\sqrt{5}}{2} \in (1,2)$$

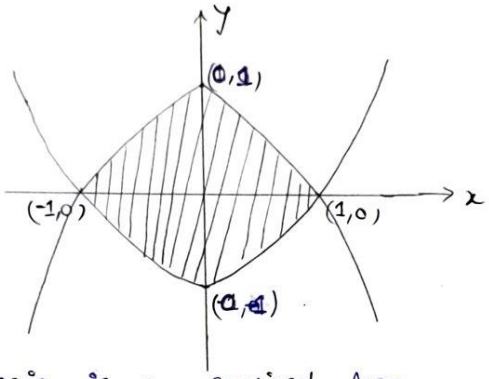
Hence option d) is correct.

2

2) The Area (in sq units) of the Region enclosed by the curves $y = x^2 - 1$ and $y = (1 - x^2)$ is equal to:

a) $\frac{8}{3}$ b) $\frac{7}{2}$ c) $\frac{4}{3}$ d) $\frac{16}{3}$

Soln:



Shaded Region is our Required Area

Given Curves $y = x^2 - 1$ and $y = 1 - x^2$

Intersection of these points are $(\pm 1, 0)$

$$\text{Required Area } (A) = 4 \int_0^1 (1 - x^2)\,dx$$

$$= 4 \left(x - \frac{x^3}{3} \right)_0^1$$

$$= 4 \left(1 - \frac{1}{3} \right)$$

$$= 4 \times \frac{2}{3}$$

$$A = \frac{8}{3} \text{ sq units}$$

Hence option (a) is correct.

3

3) Let L denote the line in the xy-plane with x and y intercepts are 3 respectively. Then the image of the point $(-1,-4)$ in the line is:

a) $\left(\frac{8}{5}, \frac{29}{5}\right)$ b) $\left(\frac{11}{5}, \frac{28}{5}\right)$ c) $\left(\frac{29}{5}, \frac{8}{5}\right)$ d) $\left(\frac{29}{5}, \frac{11}{5}\right)$

Soln: Equation of line is

$$\frac{x}{3} + \frac{y}{1} = 1$$

$$x + 3y = 3$$

Let (x_1, y_1) be image then

$$\frac{x_1+1}{1} = \frac{y_1+4}{3} = -2\left(\frac{-1-12-3}{10}\right)$$

$$x_1+1 = \frac{y_1+4}{3} = \frac{16}{5}$$

$$\Rightarrow \quad x_1+1 = \frac{16}{5}, \quad y_1+4 = \frac{48}{5}$$

$$\Rightarrow \quad x_1 = \frac{11}{5}, \quad y_1 = \frac{28}{5}$$

Hence option b) is correct.

4

4.) If α and β are the roots of the equation $2x(2x+1)=1$ then β is equal to:

a) $2\alpha(\alpha-1)$ b) $2\alpha(\alpha+1)$ c) $2\alpha^2$ d) $-2\alpha(\alpha+1)$

Soln Given equation is $2x(2x+1)=1$

$$\Rightarrow 4x^2+2x-1=0 \quad -(1)$$

α, β are the roots of this equation

$$\therefore \alpha+\beta = -\frac{2}{4} = -\frac{1}{2} \Rightarrow \beta = \left(-\alpha-\frac{1}{2}\right)$$

$$\alpha\beta = -\frac{1}{4}$$

Put $\beta = \left(-\alpha-\frac{1}{2}\right)$

$$\alpha\left(-\alpha-\frac{1}{2}\right) = -\frac{1}{4} \Rightarrow \alpha^2 = \frac{1}{4}-\frac{\alpha}{2} \quad -(2)$$

Now,

$$-2\alpha(\alpha+1) = -2\alpha^2+2\alpha$$

$$-2\alpha(\alpha+1) = -2\left(\frac{1}{4}-\frac{\alpha}{2}\right)-2\alpha \quad [\text{from Eqn } (2)$$

$$-2\alpha(\alpha+1) = -\frac{1}{2}-\alpha$$

$$-2\alpha(\alpha+1) = \beta$$

Hence option d) is correct.

5

5) for a suitably chosen real number a, let a function $f: R-\{-a\} \to R$ be defined by $f(x) = \frac{a-x}{a+x}$. further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$ $(fof)(x) = x$ Then $f(-\frac{1}{2})$ is equal to:

a) 3 b) -3 c) $\frac{1}{3}$ d) $-\frac{1}{3}$

Sol^n:

$$fof(x) = \frac{a-f(x)}{a+f(x)} = x$$

$$\Rightarrow \frac{a-ax}{1+x} = f(x)$$

$$\Rightarrow \frac{a(1-x)}{(1+x)} = \frac{a-x}{a+x}$$

$$\Rightarrow a = 1$$

So we get $f(x) = \frac{1-x}{1+x}$

$$f(-\tfrac{1}{2}) = \frac{1-(-\frac{1}{2})}{1+(-\frac{1}{2})}$$

$$f(-\tfrac{1}{2}) = \frac{(\frac{3}{2})}{1-\frac{1}{2}}$$

$$f(-\tfrac{1}{2}) = \frac{3}{2} \times 2$$

$$\boxed{f(-\tfrac{1}{2}) = 3}$$

Hence option a) is correct.

6

6) If the tangent to the curve $y = f(x) = x\log_e x$, $(x>0)$ at a point $(c, f(c))$ is parallel to the line segment joining the points $(1,0)$ and (e,e), then c is equal to:

a) $\frac{e-1}{e}$ b) $\frac{1}{e-1}$ c) $e^{\left(\frac{1}{e-1}\right)}$ d) $e^{\left(\frac{1}{1-e}\right)}$

Soln.

$$f(x) = x\log_e x \quad , x>0$$

$$f'(x) = x \times \frac{1}{x} + \log_e x$$

$$f'(x) = 1 + \log_e x$$

$$f'(c) = 1 + \log_e c$$

$$\frac{f(e) - f(1)}{e-1} = 1 + \log_e c$$

$$\frac{e\log_e e - 1\log_e 1}{e-1} = 1 + \log_e c$$

$$\frac{e}{e-1} = 1 + \log_e c$$

$$\Rightarrow \quad \log_e c = \frac{e}{e-1} - 1$$

$$\Rightarrow \quad \log_e c = \frac{e - e + 1}{e-1}$$

$$\Rightarrow \quad \log_e c = \frac{1}{e-1}$$

$$\Rightarrow \quad c = e^{\left(\frac{1}{e-1}\right)}$$

Hence option (C) is correct.

7

7) If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals:

a) 3 b) 2 c) 1 d) 9

Soln

Given Binomial Expansion $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$

$$T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \left(-\frac{k}{x^2}\right)^r$$

$$T_{r+1} = {}^{10}C_r \, x^{\left(\frac{10-r}{2}\right)} \frac{(-k)^r}{x^{2r}}$$

$$T_{r+1} = {}^{10}C_r (-k)^r \, x^{5-\frac{5r}{2}}$$

for T_{r+1} to be constant $5 - \frac{5r}{2}$ should be zero

$$5 - \frac{5r}{2} = 0$$

$$\Rightarrow \frac{5r}{2} = 5$$

$$r = 2$$

$$\Rightarrow T_3 = {}^{10}C_2 k^2 = 405$$

$$\Rightarrow \frac{10(9)}{2} k^2 = 405$$

$$k^2 = 9$$

$$|k| = 3,$$

Hence option (a) is correct.

8

S) The Probabilities of three events A, B and C are given $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$ where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval:

a) $[0.36, 0.40]$ b) $[0.35, 0.36]$ c) $[0.25, 0.35]$ d) $[0.20, 0.25]$

Soln:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\Rightarrow \alpha = 1.4 - P(A \cap B) - \beta$$

$$\Rightarrow \alpha + \beta = 1.4 - P(A \cap B) \quad \text{— (1)}$$

Again $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\alpha = 1.2 - \beta \quad \text{— (2)}$$

Now, $0.85 \leq \alpha \leq 0.95$

$$\Rightarrow 0.85 \leq 1.2 - \beta \leq 0.95$$

$$\Rightarrow 0.85 - 1.2 \leq -\beta \leq 0.95 - 1.2$$

$$\Rightarrow 0.25 \leq \beta \leq 0.35$$

$$\beta \in [0.25, 0.35]$$

Hence option c) is correct.

9

9) The integral $\int_1^2 e^x x^2 (2+\log_e x)\,dx$ equals :

a) $e(2e-1)$ b) $e(4e+1)$ c) $4e^2-1$ d) $e(4e-1)$

Soln:

Let $y = (ex)^2$

$\ln y = x[\ln ex]$

$\ln y = x[1+\ln x]$

$\frac{1}{y}\frac{dy}{dx} = 2+\ln x$

$\Rightarrow dy = (ex)^2(2+\ln x)\,dx$

Now,

$\int_1^2 e^x x^2 (2+\log_e x)\,dx$

$= \int_1^2 dy$

$= (y)_1^2$

$= [(ex)^x]_1^2$

$= 4e^2 - e$

$= e(4e-1)$

Hence option d) is correct.

10

10) The common difference of the A.P $b_1, b_2, b_3, \ldots b_m$ is 2 more than common difference of A.P $a_1, a_2, \ldots a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$ then b_1 is equal to:

a) 127 b) 81 c) -127 d) -81

Soln

Let $a_1, a_1+d, a_1+2d \ldots$ first A.P

$a_{40} = a_1 + 39d = -159$ — ①

$a_{100} = a_1 + 99d = -399$ — ②

from Equation ① and ②

$d = -4$

$a_1 = -3$

Now,

$b_{100} = a_{70}$

$\Rightarrow b_1 + 99D = a_1 + 69d$

$\Rightarrow b_1 + 99 \times (-2) = -3 + 69(-4)$ [According to Question D = d+2]

$\Rightarrow b_1 = -81$

Hence option d) is correct.

11

11) If the normal at an end of latus rectum of an ellipse passes through an extremity of the minor axis, the eccentricity e of the ellipse satisfies:

a) $e^4+2e^2-1=0$ b) $e^2+e-1=0$ c) $e^2+2e-1=0$ d) $e^4+e^2-1=0$

Soln

Equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Equation of Normal $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

At $(ae, \frac{b^2}{a})$

Normal: $\frac{a^2x}{ae}-\frac{b^2y}{(b^2/a)}=a^2-b^2$

If it passes through $(0,-b)$ it should satisfy the given equation

$$0-\frac{b^2(-b)}{b^2/a}=a^2-b^2$$

$$\frac{ab^3}{b^2}=a^2-b^2$$

$$ab=a^2-b^2$$

$$ab=a^2e^2$$

$$a^2b^2=a^4e^4 \qquad (\because b^2=a^2(1-e^2))$$

$$1-e^2=e^4$$

$$\Rightarrow e^4+e^2-1=0$$

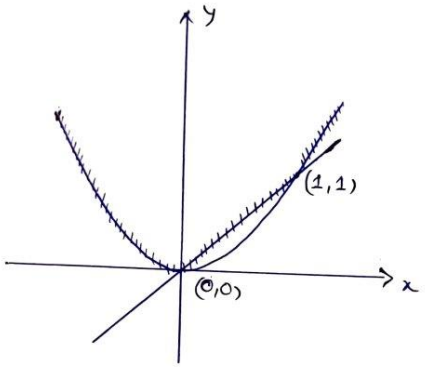
Hence option d) is correct.

12

12) Let $f: R \to R$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then:

a) ϕ (an empty set) b) $\{1\}$ c) $\{0\}$ d) $\{0,1\}$

Soln.



At $x=0$ function is not continuous so it will not differentiable at $x=0$.

At $x = 1$

$$f(1^+) \neq f(1^-)$$

Not Continuous $\Rightarrow$ Not Differentiable at $x=1$

$\{0,1\}$ are the values at which function is not differentiable.

Hence option d) is Correct.

13

13) The set of all real values λ for which the function $f(x)$
$f(x) = (1-\cos^2 x)\cdot(\lambda+\sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ has exactly one maxima
and exactly one minima, is:

a) $\left(-\frac{1}{2}, \frac{1}{2}\right)-\{0\}$ b) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ c) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ d) $\left(-\frac{3}{2}, \frac{3}{2}\right)-\{0\}$

Sol^n:

$$f(x) = (1-\cos^2 x)(\lambda+\sin x)$$

$$f(x) = \sin^2 x\,(\lambda+\sin x)$$

$$f'(x) = \sin x\cos x\,(2\lambda+3\sin x)$$

$$f'(x) = 0$$

$$\sin x\cos x\,(2\lambda+3\sin x) = 0$$

$\sin x = 0$ (one point)

$\sin x = -\frac{2\lambda}{3} \in (-1,1)-\{0\}$ — (i)

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right)-\{0\}$

Hence option d) is correct.

14

14) Consider the statement: "for an integer n, if n^3-1 is even, then n is odd". The contrapositive statement of this statement is:

a) for an integer n, if n is even, then n^3-1 is odd.
b) for an integer n, if n^3 is even, then n^3-1 is even.
c) for an integer n, if n^3-1 is not even, then n is not odd.
d) for an integer n, if n is odd, then n^3-1 is even.

Sol^n:

p: n^3-1 is even, q: n is odd

Contrapositive of $p \to q = \sim q \to \sim p$

$\Rightarrow$ "If n is not odd then n^3-1 is not even"

$\Rightarrow$ for an integer n, if n is even, then n^3-1 odd.

Hence option a) is correct.

15

15) The centre of the circle passing through the point $(0,1)$ and touching the Parabola $y=x^2$ at the point $(2,4)$ is:

a) $\left(\frac{3}{10}, \frac{16}{5}\right)$ b) $\left(\frac{-53}{10}, \frac{16}{5}\right)$ c) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ d) $\left(\frac{6}{5}, \frac{53}{10}\right)$

Sol^n:

$y = x^2$

Tangent at $(2,4)$

$(y-4) = \frac{dy}{dx}(x-2)$

$y-4 = 2(2)(x-2)$

$y-4 = 4x-8$

$4x-y-4=0$

Equation of circle $(x-2)^2+(y-4)^2+\lambda(4x-y-4)=0$

It passes through $(0,1)$

$\therefore 4+9+\lambda(0-1-4)=0$

$\Rightarrow \lambda = \frac{13}{5}$

$\therefore$ circle is $x^2-4x+4+y^2-8y+16+\frac{13}{5}(4x-y-4)=0$

$\Rightarrow x^2+y^2+\left(\frac{52}{5}-4\right)x-\left(8+\frac{13}{5}\right)y+20-\frac{52}{5}=0$

$\Rightarrow x^2+y^2+\frac{32}{5}x-\frac{53}{5}y+\frac{48}{5}=0$

$\because$ centre is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

Hence option c) is correct.

16

16) If $y=\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x$ is the solution of the differential equation, $\frac{dy}{dx}+P(x)y=\frac{2}{\pi}\operatorname{cosec}x$, $0<x<\frac{\pi}{2}$ then the function $P(x)$ is equal to:

a) $\tan x$ b) $\operatorname{cosec}x$ c) $\cot x$ d) $\sec x$

Sol^n:

$$y=\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x$$

$$\frac{dy}{dx}=\frac{2}{\pi}\operatorname{cosec}x-\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x$$

$$\frac{dy}{dx}+\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x=\frac{2}{\pi}\operatorname{cosec}x$$

$\because\quad y=\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x$

$$\Rightarrow \frac{dy}{dx}+y\cot x=\frac{2}{\pi}\operatorname{cosec}x$$

$$\Rightarrow P(x)=\cot x$$

Hence option c) is correct.

17

17) Let $z = x+iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the:

a) real axis b) line, $y=x$ c) line, $y=-x$ d) imaginary axis

sol^n

$$z^2 = i|z|^2$$

$\because z = x+iy$

$$(x+iy)^2 = i\left(\sqrt{x^2+y^2}\right)^2$$

$$(x^2 + i^2y^2 + 2xyi) = i(x^2+y^2)$$

$$x^2 - y^2 + i2xy = i(x^2-y^2)$$

Compare Real and Imaginary Parts

$x^2 - y^2 = 0 \Rightarrow y = \pm x$

$2xy = x^2 - y^2$

$\Rightarrow x = y$ (select)

Hence option (b) is correct.

18

18) A plane P meets the co-ordinate axes at A, B and C respectively. The centroid of ΔABC is given to be $(1,1,2)$. Then the equation of the line through this centroid and perpendicular to the plane P is:

a) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

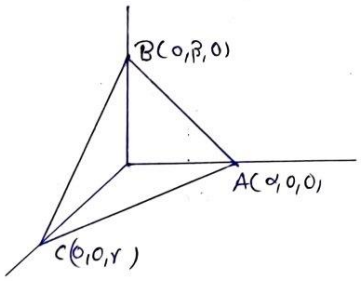
b) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

c) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

d) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

Sol^n: Let $A(\alpha,0,0)$, $B(0,\beta,0)$, $C(0,0,\gamma)$

then $G\left(\frac{\alpha}{3},\frac{\beta}{3},\frac{\gamma}{3}\right) \equiv (1,1,2)$



$\alpha = 3, \beta = 3, \gamma = 6$

Equation plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

$\Rightarrow \frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$\Rightarrow 2x + 2y + z = 6$

Required line $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$,

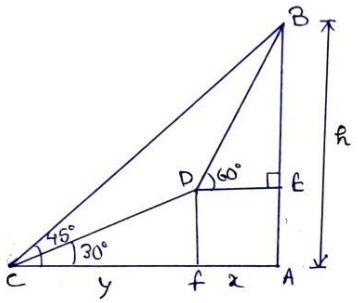
Hence option d) is correct.

19

19) The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then height (in km) of the summit from the ground is:

a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ b) $\frac{1}{\sqrt{3}+1}$ c) $\frac{1}{\sqrt{3}-1}$ d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Sol^n:



In ΔCDf

$\sin 30^\circ = \frac{x}{1}$ [CD = 1 km (given)]

$x = \frac{1}{2}$ — (1)

$\cos 30^\circ = \frac{y}{1} \Rightarrow \frac{\sqrt{3}}{2} = y$

Now in ΔABC

$\tan 45^\circ = \frac{h}{x+y}$

$\Rightarrow h = x+y$

$\Rightarrow x = h - \frac{\sqrt{3}}{2}$ — (2)

Now in ΔBDE

$\tan 60^\circ = \frac{h-2}{x}$

$\sqrt{3}x = h - \frac{1}{2} \Rightarrow \sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2} \Rightarrow (\sqrt{3}-1)h = 1 \Rightarrow h = \frac{1}{\sqrt{3}-1}$

Hence option (c) is correct.

20

20) for all twice differentiable functions $f: R \to R$ with $f(0) = f(1) = f'(0) = 0$,

a) $f''(x) = 0$ for some $x \in (0,1)$ b) $f''(x) = 0$, at every point $x \in (0,1)$

c) $f''(0) = 0$ d) $f''(x) \neq 0$, at every point $x \in (0,1)$

Soln Applying Rolle's theorem in $[0,1]$ for function $f(x)$

$f'(c) = 0 \quad c \in (0,1)$

Again Applying Rolle's theorem in $[0,c]$ for function $f'(x)$ is

$f''(c_1) = 0, \quad c_1 \in (0,c)$

Hence option a) is correct.

21

21) The Number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ___.

Soln Consonants are L, T, T, R

Vowels are E, E

Total Number of words (with or without meaning) from letters of word 'LETTER' $= \frac{6!}{2!2!} = \frac{720}{4} = 180$

Total Number of words (with or without meaning) from letters of word 'LETTER' if vowels are together $= \frac{5!}{2!} = \frac{120}{2} = 60$

$\therefore$ Required $= 180 - 60$

$= 120.$

Ans. 120

22

22) Consider the data on x taking the values $0, 2, 4, 8, \ldots 2^n$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to —

Sol^n

x_i (observation)	f_i (frequency)
0	${}^nC_0$
2	${}^nC_1$
2^2	${}^nC_2$
⋮	⋮
2^n	${}^nC_n$

$$\text{Mean}(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\frac{728}{2^n} = \frac{0 \times {}^nC_0 + 2 \times {}^nC_1 + 2^2 \times {}^nC_2 \ldots + 2^n \times {}^nC_n}{{}^nC_0 + {}^nC_1 + \ldots + {}^nC_n}$$

$$\frac{728}{2^n} = \frac{3^n - 1}{2^n}$$

$$\Rightarrow 728 = 3^n - 1$$

$$\Rightarrow 3^n = 729$$

$$\Rightarrow 3^n = 3^6$$

$$n = 6,$$

Ans 6,

23

23) Suppose that a function $f: R \to R$ satisfies $f(x+y) = f(x)f(y)$ $\forall\ x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to—

Sol^n.

Let $f(x) = a^x$ because it satisfies the property $f(x+y) = f(x)f(y)$

$f(1) = a = 3$ (given)

Now, $\sum_{i=1}^{n} f(i) = 363$

$\sum_{i=1}^{n} 3^i = 363$

$\Rightarrow 3 + 3^2 + \cdots 3^n = 363$

$\Rightarrow \frac{3(3^n - 1)}{2} = 363$

$\Rightarrow 3^n - 1 = 242$

$\Rightarrow 3^n = 243$

$3^n = 3^5$

$n = 5$,

Ans. 5.

24

24) If $\bar{x}$ and $\bar{y}$ be two non-zero vectors such that $|\bar{x}+\bar{y}|=|\bar{x}|$ and $2\bar{x}+\lambda\bar{y}$ is perpendicular to $\bar{y}$ then the value of λ is —

Soln:

$$|\vec{x}+\vec{y}| = |\vec{x}|$$

$$|\vec{x}+\vec{y}|^2 = |\vec{x}|^2$$

$$\Rightarrow |\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}\cdot\vec{y} = |\vec{x}|^2$$

$$|\vec{y}|^2 + 2\vec{x}\cdot\vec{y} = 0 \quad \text{— (i)}$$

and $(2\vec{x}+\lambda\vec{y})\cdot\vec{y} = 0$

$$\Rightarrow \lambda|\vec{y}|^2 + 2\vec{x}\cdot\vec{y} = 0 \quad \text{— (ii)}$$

(ii) − (i)

$$(\lambda|\vec{y}|^2 + 2\vec{x}\cdot\vec{y}) - (|\vec{y}|^2 + 2\vec{x}\cdot\vec{y}) = 0$$

$$\lambda|\vec{y}|^2 - |\vec{y}|^2 = 0$$

$$|\vec{y}|^2(\lambda-1) = 0$$

$$\Rightarrow \lambda = 1.$$

Ans. 1.

25

25) The sum of distinct values of λ for which the system of equations

$$(\lambda-1)x + (3\lambda+1)y + 2\lambda z = 0$$
$$(\lambda-1)x + (4\lambda-2)y + (\lambda+3)z = 0$$
$$2x + (3\lambda+1)y + 3(\lambda-1)z = 0$$

had non-zero solutions is ___.

Sol^n

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ 0 & \lambda-3 & -\lambda+3 \\ 3-\lambda & 0 & \lambda-3 \end{vmatrix} = 0$$

$C_1 \to C_1 + C_3$

$$\begin{vmatrix} 3\lambda-1 & 3\lambda+1 & 2\lambda \\ 3-\lambda & \lambda-3 & 3-\lambda \\ 0 & 0 & \lambda-3 \end{vmatrix} = 0$$

$\Rightarrow (\lambda-3)^2(6\lambda) = 0$

$\Rightarrow \lambda = 0, 3$

Sum of values of $\lambda = 0+3 = 3$.

Ans. 3.

Enrolling yourself for coaching classes has long been considered the first step towards cracking JEE. However, this trend is starting to slip. Recent studies show that, for the students who cleared the JEE Main/Advance, the number of students who had joined a coaching class is lower than the number of students who managed the feat with self-study.

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