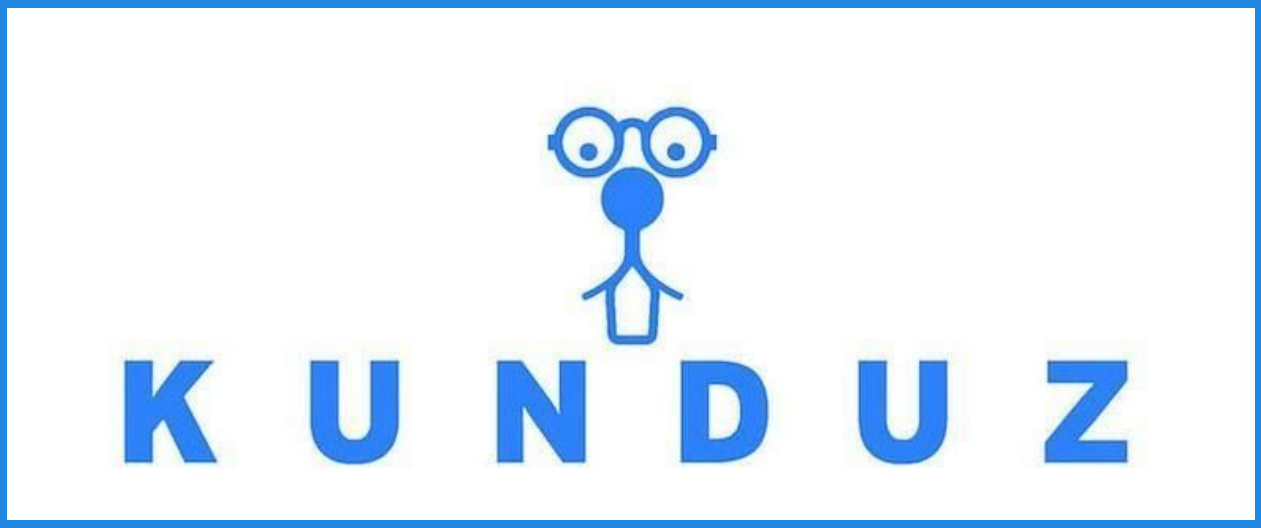
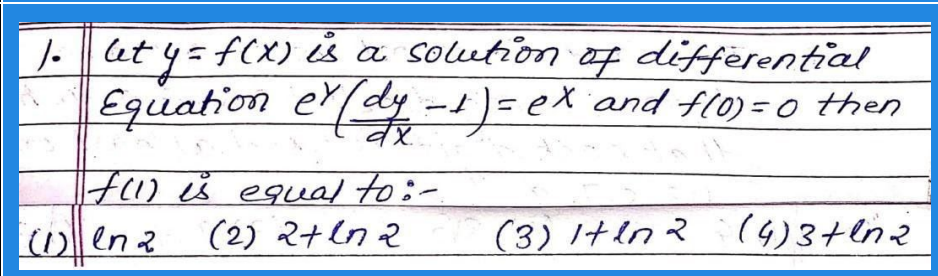
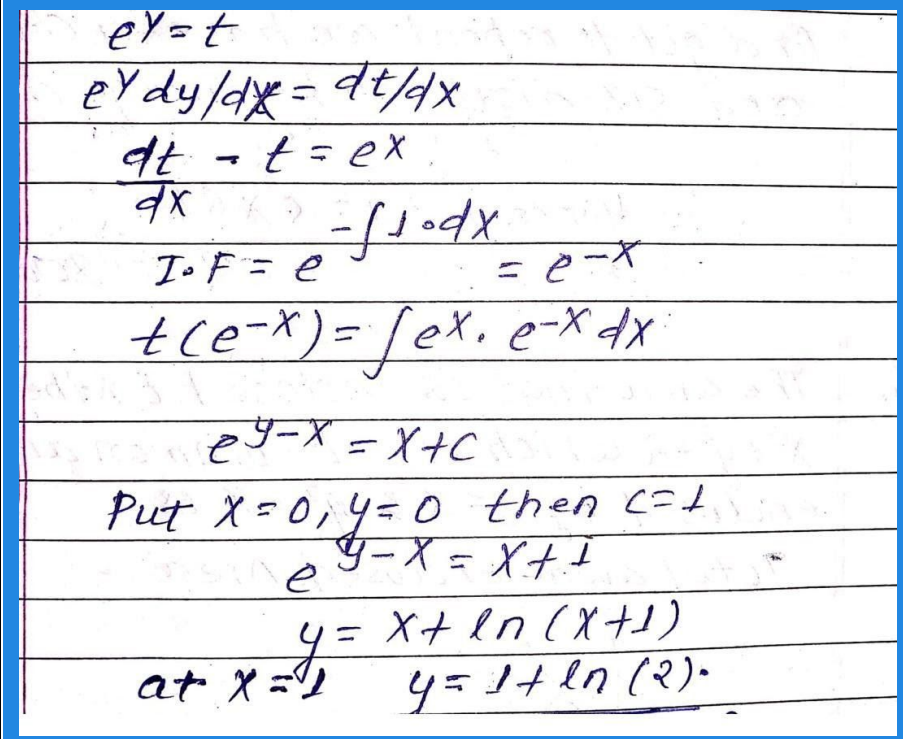



# Jan 7 2020 shift 1 Mathematics prepared by KUNDUZ tutors

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| Question no. | Questions and Answers |
|---|---|
| 1 |   |



$e^y = t$

$e^y \, dy/dx = dt/dx$

$\frac{dt}{dx} - t = e^x$

$I.F = e^{-\int 1 \cdot dx} = e^{-x}$

$t(e^{-x}) = \int e^x \cdot e^{-x} dx$

$e^{y-x} = x + c$

Put $x = 0, y = 0$ then $c = 1$

$e^{y-x} = x + 1$

$y = x + \ln(x+1)$

at $x = 1$ $y = 1 + \ln(2)$.

**2**

2. If $\alpha$ is roots of equation $x^2+x+1=0$ and $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha\end{bmatrix}$ then $A^{31}$ equal to

(1) A (2) $A^2$ (3) $A^3$ (4) $A^4$

$$A^2=\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

$\Rightarrow A^4 = I$

$\Rightarrow A^{31} = A^{28} \times A^3$

$= A^3$

**3**

3. The six digit numbers that can be formed using digits 1, 3, 5, 7, 9 such that each digit is used atleast once.

1, 3, 5, 7, 9

for digit to repeat we have $^5C_1$ choice and six digit can be arrange in $\frac{6!}{2!}$

Hence total $= 5 \times \frac{6!}{2!} = 1800$

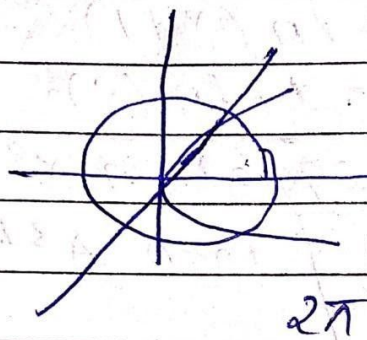
**4**

4. The area that is enclosed in the circle $x^2+y^2=2$ which is not common area enclosed by $y=x$ & $y^2=x$ is

(1) $\frac{1}{12}(24\pi-1)$ (2) $\frac{1}{6}(12\pi-1)$

(3) $\frac{1}{12}(6\pi-1)$ (4) $\frac{1}{12}(12\pi-1)$

Total area - Enclosed Area



$$2\pi - \int_0^1 (\sqrt{x} - x)\,dx$$

$$= 2\pi - \left[\frac{2x^{3/2}}{2} - \frac{x^2}{2}\right]_0^1$$

$$= 2\pi - \left(\frac{2}{3} - \frac{1}{2}\right)$$

$$= 2\pi - 1/6$$

$$= \frac{12\pi - 1}{6}$$

**5**

5. If sum of all the coefficients of even powers in $(1 - x + x^2 - x^3 \cdots x^{2n})(1 + x + x^2 + x^3 \cdots + x^{2n})$ is 61 then $n$ is equal to

(1) 30 (2) 32 (3) 28 (4) 36

$(1-x+x^2-\cdots)(1+x+x^2+\cdots)=a_0+a_1x+a_2x^2+\cdots$

Put $x=1$

$1(2n+1)=a_0+a_1+a_2+\cdots+a_{2n}$

Put $x=-1$

$(2n+1)\times 1=a_0-a_1+a_2+\cdots+a_{2n}$

$4n+2=2(a_0+a_2+\cdots)$

$=2\times 61$

$2n+1=61$

$\Rightarrow n=30$

**6**

6. If variance of first $n$ natural number is 10 & Variance of first $m$ even natural number is 16 then the value of $m+n$ is

$\text{Var}(1,2,\cdots,n)=10 \Rightarrow \frac{1^2+2^2+\cdots+n^2}{n}-\left(\frac{1+2+\cdots+n}{n}\right)^2$

$=10$

$\Rightarrow \frac{(n+1)(2n+1)}{6}-\left(\frac{n+1}{2}\right)^2=10$

$\Rightarrow n^2-1=120 \Rightarrow n=11$

$\text{Var}(2,4,6,\cdots,2m)=16 \Rightarrow \text{Var}(1,2,\cdots,m)=4$

$\Rightarrow m^2-1=48 \Rightarrow m=7$

$m+n=18$

**7**

7. Evaluate $\lim_{x \to 2} \frac{3^x + 3^{x-1} - 12}{3^{-x/2} - 3^{1-x}}$

Put $3^{x/2} = t$

$\Rightarrow \lim_{t \to 3} \frac{4t^2/3 - 12}{-3/t^2 + 1/t} = \lim_{t \to 3} \frac{4(t^2-9)t^2}{3(-3+t)}$

$= \lim_{t \to 3} \frac{4t^2(3+t)}{3} = \frac{4 \times 9 \times 6}{3}$

$= 72$

**8**

8. If $f(x)$ is continuous and differentiable in $x \in [-7, 0]$ and $f'(x) \leq 2 \; \forall \; x \in [-7, 0]$, also $f(-7) = -3$ then range of $f(-1) + f(0)$.

(1) $[-5, -7]$ (2) $(-\infty, 6]$ (3) $(-\infty, 20]$

(4) $[-5, 3]$

Lets use LMVT for $x \in [-7, -1]$

$\frac{f(-1) - f(-7)}{(-1+7)} \leq 2$

$\Rightarrow \frac{f(-1)+3}{6} \leq 2 \Rightarrow f(-1) \leq 9$

Also use LMVT for $x \in [-7, 0]$

$\frac{f(0) - f(-7)}{0+7} \leq 2$

$\Rightarrow \frac{f(0)+3}{7} \leq 2 \Rightarrow f(0) \leq 11$

$\therefore f(0) + f(-1) \leq 20.$

**9**

9. If $y = mx + 4$ is common tangent to parabolas $y^2 = 4x$ & $x^2 = 2by$. The value of $b$ is

(1) $-64$ (2) $-32$ (3) $-128$ (4) $16$

$y = mx + 4$ -- (i)

$y^2 = 4x$ tangent

$y = mx + a/m \Rightarrow y = mx + 1/m$ -- (ii)

from (i) & (ii)

$4 = 1/m \Rightarrow m = 1/4$

So line $y = 1/4\,x + 4$ is also tangent $x^2 = 2by$

$x^2 = 2b\left(\frac{x+16}{4}\right)$

$\Rightarrow 2x^2 - bx - 16b = 0 \Rightarrow D = 0$

$\Rightarrow b^2 + 32 \times 4b = 0$

$b = -128,\ b \neq 0$ (not possible)

So $b = -128$.

**10**

10. If $\alpha$ & $\beta$ are the roots of equation $(k+1)\tan^2 x - \sqrt{2}\lambda \tan x = 1 - k$ & $\tan^2(\alpha+\beta) = 50$. find value of $\lambda$.

(1) 10 (2) 5 (3) 7 (4) 12

$(k+1)\tan^2 x - \sqrt{2}\lambda \tan x + (k-1) = 0$

$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$

$\tan\alpha \cdot \tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = \frac{\sqrt{2}\lambda/k+1}{1-\frac{k-1}{k+1}} = (k-1) = 0 \Rightarrow$ so $k=1$

$= \lambda/\sqrt{2}$

$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$

$\lambda = 10$

**11**

11. Find image of point (2,1,6) in the plane containing points (2,1,0), (6,3,3) & (5,2,2)

(1) (6,5,−2) (2) (6,−5,2) (3) (2,−3,4) (4) (2,−5,6)

Planes is $x+y-2z=3$

$\Rightarrow \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = -2\left(\frac{2+1-12-3}{6}\right)$

$\Rightarrow (x,y,z) = (6,5,-2)$

**12**

12. Let $(x)^k + (y)^k = (ak)$ where $a, k > 0$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{1/3} = 0$ then find k.

(1) 1/3 (2) 2/3 (3) 4/3 (4) 2

$$k \cdot x^{k-1} + k \cdot y^{k-1} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \left(\frac{-x}{y}\right)^{k-1}$$

$$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0.$$

$$k-1 = -1/3$$

$$k = 1 - 1/3 = 2/3$$

**13**

13. If $g(x) = x^2 + x - 1$ & $g(f(x)) = 4x^2 - 10x + 5$ then find $f(5/4)$

(1) $1/2$ (2) $-1/2$ (3) $-1/3$ (4) $1/3$

$$g(f(x)) = f^2(x) + f(x) - 1$$

$$= g(f(5/4)) = 4\left(\frac{5}{4}\right)^2 - 10 \cdot 5/4 + 5$$

$$= -\frac{5}{4}$$

$$-5/4 = f^2(5/4) + f(5/4) - 1$$

$$f^2(5/4) + f(5/4) + 1/4 = 0.$$

$$\left(f(5/4) + 1/2\right)^2 = 0$$

$$f\left(\frac{5}{4}\right) = \frac{-1}{2}$$

**14**

14. if $z = x+iy$ and real part $\left(\frac{z-1}{2z+i}\right) = 1$ then locus of $z$ is

(1) straight line with slope 2
(2) straight line with slope $-1/2$
(3) circle with diameter $\sqrt{5}/2$
(4) circle with diameter $1/2$

$z = x+iy$

$$\left(\frac{z-1}{2z+i}\right) = \frac{(x-1)+iy}{2(x+iy)+i} = \frac{(x-1)+iy}{2x+(2y+1)i} \times \frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$Re\left(\frac{z-1}{2z+i}\right) = \frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2} = 1$$

$$\Rightarrow 2x^2+2y^2-2x+y = 4x^2+4y^2+4y+1$$

$$\Rightarrow 2x^2+2y^2+2x+3y+1=0$$

$$\Rightarrow x^2+y^2+x+\frac{3}{2}y+\frac{1}{2}=0$$

$$r = \sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}} = \frac{\sqrt{5}}{4}$$

circle with diameter $\sqrt{5}/2$

**15**

15. if distance between the foci of an ellipse is 6 & distance between its directrices is 12, then length of its latus rectum is

(1) 4 (2) $3\sqrt{2}$ (3) 9 (4) $2\sqrt{2}$

$2ae = 6 \Rightarrow \frac{2a}{e} = 12$

$\Rightarrow ae = 3 \quad \& \quad a/e = 6$

$\Rightarrow a^2 = 18$

$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9$

Latus Rectum $= \frac{2b^2}{a} = \frac{2 \times 9}{3\sqrt{2}}$

$= 3\sqrt{2}$

**16**

16. If $y = \sqrt{\frac{2(\tan\alpha + \cot\alpha)}{1+\tan^2\alpha} + \frac{1}{\sin^2\alpha}}$ when $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$ then find $dy/d\alpha$ at $\alpha = 5\pi/6$

6. (1) 4 (2) 2 (3) 3 (4) -4

$y = \sqrt{\frac{2\cos^2\alpha}{\sin\alpha\cos\alpha} + \frac{1}{\sin^2\alpha}} = \sqrt{2\cot\alpha + \operatorname{cosec}^2\alpha}$

$= |1 + \cot\alpha|$

$= -1 - \cot\alpha$

$\frac{dy}{d\alpha} = \operatorname{cosec}^2\alpha$

$\Rightarrow \left(\frac{dy}{d\alpha}\right)$ at $\alpha = \frac{5\pi}{6}$ will be 4.

**17**

17. If A(1,1), B(6,5), C(3/2,2) and vertices of ΔABC. A Point P is such that the area of ΔPAB, ΔPAC, ΔPBC are equal also Q(−7/6, −1/3), then length of PQ is P will be Centroid of ΔABC.

(1) 2 (2) 3 (3) 4 (4) 5

P will be Centroid of ΔABC.

$$P\left(\frac{17}{6}, \frac{8}{3}\right)$$

$$PQ = \sqrt{\left(\frac{24}{6}\right)^2 + \left(\frac{9}{3}\right)^2}$$

$$= 5$$

**18**

18. $(P \to q) \wedge (q \to \sim P)$ is equivalent to

(1) $\sim P$ (2) $P$ (3) $P \wedge q$ (4) $P \vee q$

| P | q | P→q | ~P | q→~P | (P→q)∧(q→~P) |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | F | T | T | T | T |

~P

**19**

19. find greatest value of k for which $49^k + 1$ is factor of $1 + 49 + 49^2 \cdots (49)^{125}$

(1) 63 (2) 65 (3) 2 (4) 5

| | |
|---|---|
| | $\frac{(49)^{125}-1}{49-1} = \frac{[(49)^{63}+1][(49)^{63}-1]}{48}$ <br> $= 63$ |
| **20** | 20. If $f(x) = \|2-\|x-3\|\|$ is non differentiable in $x \in S$. Then value of $\sum_{x \in S} f(f(x))$ is <br> $\because f(x)$ is non-differentiable at $x = 1, 3, 5$ <br> $\sum f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))$ <br> $= 1 + 1 + 1$ <br> $= 3$ |
| **21** | 21. If system of equations <br> $2x + 2ay + az = 0$ <br> $2x + 3by + bz = 0$ <br> $2x + 4cy + cz = 0$ have non trivial solution <br> $\therefore$ (1) $a + b + c = 0$ <br> (2) $a, b, c$ are in A.P <br> (3) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P <br> (4) $a, b, c$ in G.P |

for non trivial solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$$(3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$$

$$ab + bc = 2ac$$

$a, b, c$ in H.P

So, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ in A.P.

**22**

22. If sum of 5 consecutive terms of an A.P is 25 & product of these terms is 2520 if one of the term is $\frac{1}{2}$ then value of greatest term is

(1) $21/2$ (2) 16 (3) 5 (4) 7

Let terms be $a-2d, a-d, a, a+d, a+2d$

Sum $= 25 \Rightarrow 5a = 25 \Rightarrow a = 5$

$(5-2d)(5-d)5(5+d)(5+2d) = 2520$

$\Rightarrow 4d^4 - 125d^2 + 121 = 0$

$\Rightarrow (d^2-1)(4d^2-121) = 0$

$d = \pm 1 \quad d = \pm \frac{11}{2}$

$d = \pm 1$ does not give $-1/2$ as terms

so $d = 11/2$

Largest term $= 5 + 2d$

$= 5 + 11 = 16$.

**23**

23. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\vec{a}$ lies in plane of $\vec{B}$ & $\vec{C}$

$\vec{B} = \hat{i} + \hat{j}$ & $\vec{C} = \hat{i} - \hat{j} + 4\hat{k}$

(1) $\vec{a} \cdot \vec{k} + 2 = 0$ ✓ (2) $\vec{a} \cdot \vec{k} + 4 = 0$

(3) $\vec{a} \cdot \vec{k} - 2 = 0$ (4) $\vec{a} \cdot \vec{k} + 5 = 0$

$\vec{a} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{i}+\hat{j}+4\hat{k}}{3\sqrt{2}}\right) = \frac{\lambda}{3\sqrt{2}}\left[4\hat{i}+2\hat{j}+4\hat{k}\right]$

$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{a} = \frac{\mu}{3\sqrt{2}}(2\hat{i} + 4\hat{j} - 4\hat{k})$

$\frac{4\mu}{3\sqrt{2}} = 2 \Rightarrow \mu = \frac{3\sqrt{2}}{2}$

✓ $\vec{a} \cdot \vec{k} + 2 = 0$

$\Rightarrow -2 + 2 = 0$

**24**

24. Given $f(a+b+1-x) = f(x)\ \forall\ x \in R$ then the value of $\frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx$ is equal to

(1) $\int_{a+1}^{b+1} f(x)dx$ (2) $\int_{a+1}^{b+1} f(x+1)dx$

(3) $\int_{a-1}^{b-1} f(x)dx$ (4) $\int_{a-1}^{b-1} f(x+1)dx$

$$I = \frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx \quad --(1)$$

$x \to a+b-x$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(a+b-x)+f(a+b+1-x)]dx$$

$$I = \frac{1}{a+b}\int_a^b (a+b-x)[f(x+1)+f(x)]dx \quad --(2)$$

(1) + (2)

$$2I = \int_a^b [f(x+1)+f(x)]dx$$

$$2I = \int_a^b f(x+1)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

**25**

25. An unbiased coin is thrown 5 times. Let X be a random variable and k be the value assigned to X for k = 3, 4, 5 times Head occurs consecutively & otherwise the value of X is assigned −1. What is value of expectation?

(1) 1/8 (2) −1/8 (3) 3/8 (4) −3/8

| K | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(K) | 1/32 | 12/32 | 11/32 | 5/32 | 2/32 | 1/32 |

k = no. of times head occur consecutively

Now expectation

$$= \sum X \, P(k)$$

$$= (-1)\times\frac{1}{32} + (-1)\times\frac{12}{32} + (-1)\times\frac{11}{32} + 3\times\frac{5}{32} + 4\times\frac{2}{32} + 5\times\frac{1}{32}$$

$$= \frac{1}{8}.$$

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