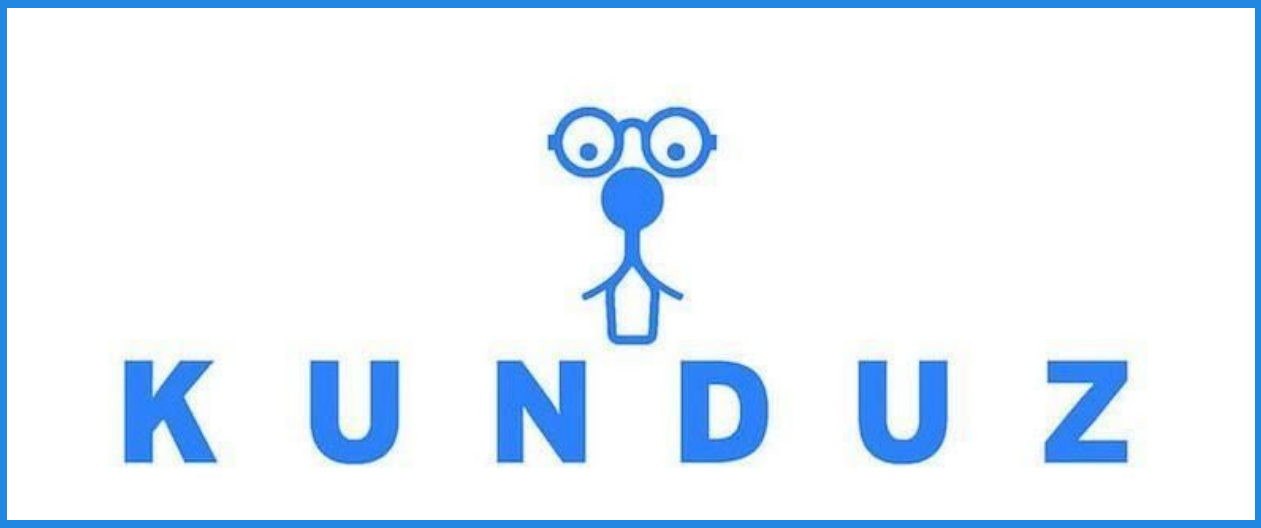
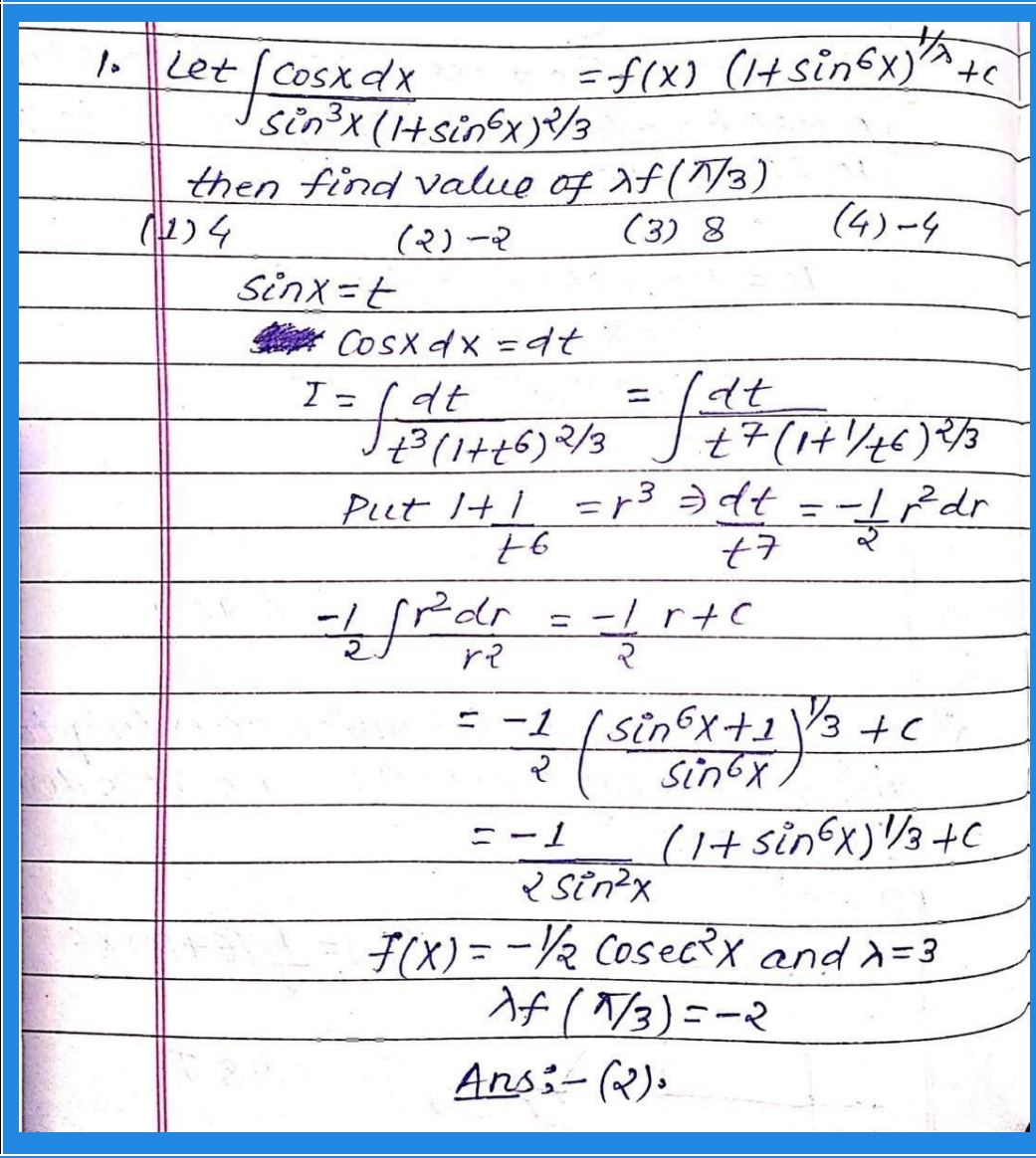


Jan 8 2020 shift 1 Mathematics prepared by KUNDUZ tutors

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Question no.	Questions and Answers
1	

1. Let $\int \frac{\cos x\,dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda} + c$

then find value of $\lambda f(\pi/3)$

(1) 4 (2) −2 (3) 8 (4) −4

$\sin x = t$

$\cos x\,dx = dt$

$I = \int \frac{dt}{t^3(1+t^6)^{2/3}} = \int \frac{dt}{t^7(1+1/t^6)^{2/3}}$

Put $1+\frac{1}{t^6} = r^3 \Rightarrow \frac{dt}{t^7} = -\frac{1}{2}r^2 dr$

$-\frac{1}{2}\int \frac{r^2 dr}{r^2} = -\frac{1}{2}r + c$

$= -\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{1/3} + c$

$= \frac{-1}{2\sin^2 x}(1+\sin^6 x)^{1/3} + c$

$f(x) = -1/2\ \mathrm{cosec}^2 x$ and $\lambda = 3$

$\lambda f(\pi/3) = -2$

Ans:- (2).

2

2. If $y(x)$ is a solution of differential equation $\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0$, such that $y(1/2)=\sqrt{3}/2$ then

(1) $y(1/\sqrt{2})=-1/\sqrt{2}$ (2) $y(1/\sqrt{2})=\sqrt{3}/2$

(3) $y(1/\sqrt{2})=1/\sqrt{2}$ (4) $y(1/2)=1/2$

$$\frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0$$

$\Rightarrow \sin^{-1}y+\sin^{-1}x=c$

At $x=1/2, y=\sqrt{3}/2 \Rightarrow c=\pi/2 \Rightarrow \sin^{-1}y=\cos^{-1}x$

Hence $y\left(\frac{1}{\sqrt{2}}\right)=\sin\left(\cos^{-1}\frac{1}{\sqrt{2}}\right)$

$$=\frac{1}{\sqrt{2}}$$

3

3. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ is equal to

(1) e^{-2} (2) e^{2} (3) $e^{2/7}$ (4) $e^{3/7}$

$$L=\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}=e^{\lim_{x\to 0}\frac{1}{x^2}\left(\frac{3x^2+2}{7x^2+2}-1\right)}$$

$$=e^{\lim_{x\to 0}\frac{1}{x^2}\left\{\frac{-4x^2}{7x^2+2}\right\}}$$

$$=e^{-4/2}$$

$$=e^{-2}$$

Ans :- (1)

4

4. In a bag there are 5 red balls, 3 white balls & 4 black balls. four balls are drawn from the bag. Find the number of ways in which at most 3 red balls are selected

(1) 450 (2) 360 (3) 490 (4) 510

No. of ways $= \overbrace{{}^7C_4}^{0\text{Red}} + \overbrace{{}^5C_1 \cdot {}^7C_3}^{1\text{Red}} + \overbrace{{}^5C_2 \cdot {}^7C_2}^{2\text{Red}} + \overbrace{{}^5C_3 \cdot {}^7C_1}^{3\text{Red}}$

$= 35 + 175 + 210 + 70$

$= 490$

Ans :- (3)

5

5. Let $f(x) = \{(\sin(\tan^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1$ where $|x| > 1$ and $\frac{dy}{dx} = \frac{1}{2}\frac{d(\sin^{-1}f(x))}{dx}$.

If $y(\sqrt{3}) = \pi/6$ then $y(-\sqrt{3}) =$

(1) $\frac{5\pi}{6}$ (2) $\frac{-\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

$2y = \sin^{-1}f(x) + c = \sin^{-1}(\sin(2\tan^{-1}x)) + c$

$\Rightarrow 2\left(\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + c$

$\frac{\pi}{3} = \frac{\pi}{3} + c \quad \therefore c = 0.$

for $x = -\sqrt{3}$, $2y = \sin^{-1}\left(\sin\left(\frac{-2\pi}{6}\right)\right) + 0.$

$\Rightarrow 2y = -\pi/3$

$y = -\pi/6$

Ans :- (2)

6

6. If $2^{1-x} + 2^{1+x}$, $f(x)$, $3^x + 3^{-x}$ are in A.P. then maximum value of $f(x)$ is

(1) 1 (2) 2 (3) 3 (4) 4

$$f(x) = \left(\frac{2^{1-x} + 2^{1+x} + 3^x + 3^{-x}}{2}\right)$$

using AM $\geq$ GM

$$f(x) \geq 3$$

Ans:- (3)

7

7. Which of the following is tautology

(1) $(P \wedge (P \to q)) \to q$ (2) $q \to P \wedge (P \to q)$

(3) $P \vee (P \wedge q)$ (4) $(P \wedge (P \vee q)$

P	q	$P \to q$	$P \wedge (P \to q)$	$(P \wedge (P \to q)) \to q$	$q \to P \wedge (P \to q)$	$P \wedge q$	$P \vee (P \wedge q)$
T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
F	T	F	F	T	F	F	F
F	F	F	F	T	T	F	F

$P \vee q$	$P \wedge (P \vee q)$
T	T
T	T
T	F
F	F

Ans :- (1)

8

8. A is a 3X3 matrix whose elements are from the set $\{-1,0,1\}$. Find the number of matrices A such that $tr(AA^T)=3$ where tr(A) is sum of diagonal elements of matrix A.

(1) 572 (2) 612 (3) 672 (4) 682

Let $A=[a_{ij}]_{3\times3}$

$tr(AA^T)=3$

$a_{11}^2+a_{12}^2+a_{13}^2+a_{21}^2+\cdots+a_{33}^2=3$.

Possible cases

$0,0,0,0,0,0,1,1,1 \longrightarrow 1$

$0,0,0,0,0,0,-1,-1,-1 \longrightarrow 1$

$0,0,0,0,0,0,1,1,-1 \longrightarrow 3$

$0,0,0,0,0,0,-1,1,-1 \longrightarrow 3$

$^9C_6 \times 8 = 84\times8$

$= 672$

Ans :- (3)

9

9. Mean and standard deviations of 10 observations are 20 and 2 respectively. if p $(p\neq0)$ is multiplied to each observation and then q $(q\neq0)$ is subtracted then new mean and standard deviation becomes half of original value. Then find q.

(1) −10 (2) −20 (3) −5 (4) 10

If each observation is multiplied with p and then q is subtracted.

Now mean $\bar{x}_1 = p\bar{x} - q$

$\Rightarrow 10 = p(20) - q \quad \cdots (1)$

and new standard deviations

$\sigma_2 = |p|\sigma_1 \Rightarrow 1 = |p|(2)$

$\Rightarrow |p| = 1/2 \Rightarrow p = \pm 1/2$

if $p = 1/2$

then $q = 0$

If $p = -1/2$

$q = -20$

So, $-(2)$

10

10. If maximum value of $19cp$ is a, $20cq$ is b, $21cr$ is c, then relation between a, b, c is

(1) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$ (2) $\frac{a}{22} = \frac{b}{11} = \frac{c}{42}$

(3) $\frac{a}{22} = \frac{b}{42} = \frac{c}{11}$ (4) $\frac{a}{21} = \frac{b}{11} = \frac{c}{22}$

We know ^nC_r is max at middle term

$a = {}^{19}C_p = {}^{19}C_{10} = {}^{19}C_9$

$b = {}^{20}C_9 = {}^{20}C_{10}$

$c = {}^{21}C_{\text{[illegible]}} = {}^{21}C_{10} = {}^{21}C_{11}$

$$\frac{a}{{}^{19}C_9} = \frac{b}{{}^{20}C_{10}} = \frac{c}{{}^{21}C_{11}}$$

$$\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

Ans :- (1)

11

11. Let $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{6}$ where A and B are independent events then

(1) $P\left(\frac{A}{B}\right) = \frac{1}{6}$ (2) $P\left(\frac{A}{B'}\right) = \frac{1}{3}$

(3) $P\left(\frac{A}{B'}\right) = \frac{2}{3}$ (4) $P\left(\frac{A}{B}\right) = \frac{5}{6}$

A & B are independent events so

$P\left(\frac{A}{B'}\right) = \frac{1}{3}$

Ans :- (2).

12

12. Let $f(x) = \dfrac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$ then inverse of $f(x)$ is

(1) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$ (2) $\frac{1}{2}\log_e\left(\frac{1-x}{1+x}\right)$

(3) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$ (4) $\frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$

$$y = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$$

$$\frac{1+y}{1-y} = \frac{8^{2x}}{8^{-2x}}$$

$$8^{4x} = \frac{1+y}{1-y}$$

$$4x = \log_8\left(\frac{1+y}{1-y}\right)$$

$$f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$$

13

13. Roots of the equation $x^2 + bx + 45 = 0, b \in R$ lie on the curve $|z+1| = 2\sqrt{10}$, where z is a complex number then

(1) $b^2 + b = 12$ (2) $b^2 - b = 30$

(3) $b^2 - b = 36$ (4) $b^2 + b = 30$

	Let $z=\alpha+i\beta$ be roots of the equation $2a=-b$ & $\alpha^2+\beta^2=45$ $(\alpha+1)^2+\beta^2=90$ $(\alpha+1)^2-\alpha^2=-5$ $2\alpha+1=-5$ $2\alpha=-6$ $b=6$ $b^2-b=30$
14	14. for $f(x)=\ln\left(\frac{x^2+\alpha}{7x}\right)$, Rolle's theorem is applicable on $[3,4]$, the value of $f''(c)$ is equal to (1) $1/12$ (2) $-1/12$ (3) $1/6$ (4) $-1/6$ $f(3)=f(4) \Rightarrow \alpha=12$ $f'(x)=\frac{x^2-12}{x(x^2+12)}$ $\therefore f'(c)=0$ $\therefore c=\sqrt{12}$ $\therefore f''(c)=\frac{1}{12}$

15

15. Let $f(x) = x\cos^{-1}(\sin(-|x|)),\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then

(1) $f'(0) = -\pi/2$

(2) $f'(x)$ is not defined at $x = 0$.

(3) $f'(x)$ is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and $f'(x)$ is decreasing in $(0, \pi/2)$

(4) $f'(x)$ is decreasing in $(-\pi/2, 0)$ and $f'(x)$ is increasing in $(0, \pi/2)$

$$f(x) = x(\pi - \cos^{-1}(\sin|x|))$$

$$= x\left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right)$$

$$= x\left(\frac{\pi}{2} + |x|\right)$$

$$f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right) & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right) & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \pi/2 + 2x & x \geq 0 \\ \frac{\pi}{2} - 2x & x < 0 \end{cases}$$

$f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$. and decreasing in $\left(-\frac{\pi}{2}, 0\right)$

Ans :- (4).

16

16. Let P be a point on $x^2 = 4y$. The segment joining A(0,−1) and P is divided by point Q in the ratio 1:2 then locus of Point Q is

(1) $9x^2 = 3y + 2$ (2) $9x^2 = 12y + 8$

(3) $9y^2 = 12x + 8$ (4) $9y^2 = 3x + 2$

Let point P be $(2t, t^2)$ and Q be (h, k)

$$h = \frac{2t}{3}, \quad k = -\frac{2 + t^2}{3}$$

Hence locus is $3k + 2 = \left(\frac{3h}{2}\right)^2$

$$\Rightarrow 9x^2 = 12y + 8$$

Ans :- (2)

17

17. Ellipse $2x^2 + y^2 = 1$ and $y = mx$ meet at point A in first quadrant. Normal to the ellipse at P meets x-axis at $\left(\frac{-1}{3\sqrt{2}}, 0\right)$ and y-axis at $(0, \beta)$ then $|\beta|$ is

(1) $2/\sqrt{3}$ (2) $2\sqrt{2}/3$ (3) $\sqrt{2}/3$ (4) $2/3$

Let P be (x_1, y_1)

Equation of Normal at P is $\frac{x}{2x_1} - \frac{y}{y_1} = -1/2$

If passes through $\left(\frac{-1}{3\sqrt{2}}, 0\right) \Rightarrow \frac{-1}{6\sqrt{2}\,x_1} = -\frac{1}{2}$

$\Rightarrow x_1 = \frac{1}{3\sqrt{2}}$ so $y_1 = \frac{2\sqrt{2}}{3}$ (as P lies 1st Quadrant)

So, $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$ Ans:-(3)

18

18. If $y^2 = ax$ and $x^2 = ay$ intersect at A & B. Area bounded by both curves is bisected by line $x = b$ (given $a > b > 0$). Area of triangle formed by line AB, $x = b$ and x-axis is $1/2$. Then

(1) $a^6 - 12a^3 - 4 = 0$ (2) $a^6 + 12a^3 - 4 = 0$

(3) $a^6 - 12a^3 + 4 = 0$ (4) $a^6 + 12a^3 + 4 = 0$.

$$\int_0^b \left(\sqrt{a}\,x^{1/2} - \frac{x^2}{a}\right) dx = \frac{a^2}{6}$$

$$\Rightarrow \frac{2}{3}\sqrt{a}\,b^{3/2} - \frac{b^3}{3a} = \frac{a^2}{6} \quad -(i)$$

Also, area of $\Delta OQR = 1/2$

$$\frac{1}{2}b^2 = \frac{1}{2} \Rightarrow b = 1$$

Put in (i)

$$4a\sqrt{a} - 2 = a^3$$

$$\Rightarrow a^6 + 4a^3 + 4 = 16a^3$$

$$\Rightarrow a^6 - 12a^3 + 4 = 0.$$

Ans:- (3)

19

19. Let ABC is a triangle whose vertices are $A(1,-1), B(0,2), C(x', y')$ and area of ΔABC is 5 and $C(x', y')$ lie on $3x+y-4\lambda=0$, then

$$D=\frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix}$$

$$-2(1-x')+(y'+x')=\pm 10$$

$$-2+2x'+y'+x'=\pm 10$$

$$3x'+y'=12 \qquad 3x'+y'=-8$$

$$\lambda=3,-2$$

Ans:- (1)

20

20. The system of equation $3x+4y+5z=\mu$

$x+2y+3z=1$

$4x+4y+4z=8$

is inconsistent, then $(5,\mu)$ can be

(1) (4,6) (2) (3,4) (3) (4,3) (4) (1,0)

Ans:- (3)

$$D=\begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix} \quad (R_3 \to R_3-2R_1+3R_2)$$

$$=\begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix}=0$$

Now let $P_3=4x+4y+4z-5=0$ of system has solutions it will have infinite solution,

So $P_3 \equiv \alpha P_1+\beta P_2$

Hence $3\alpha+\beta=4$ & $4\alpha+2\beta=4 \Rightarrow \alpha=2, \beta=-2$

$2\mu \neq 5+2$ system inconsistent.

21

21. Shortest distance between the lines $\frac{x-3}{1} = \frac{y-8}{4} = \frac{z-3}{22}$, $\frac{x+3}{1} = \frac{y+7}{1} = \frac{z-6}{7}$ is

(1) $3\sqrt{30}$ (2) $2\sqrt{30}$ (3) $\sqrt{30}$ (4) $4\sqrt{30}$

$\overrightarrow{AB} = 6\hat{i} + 15\hat{j} + 3\hat{k}$

$\vec{p} = \hat{i} + 4\hat{j} + 22\hat{k}$

$\vec{q} = \hat{i} + \hat{j} + 7\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 22 \\ 1 & 1 & 7 \end{vmatrix} = 6\hat{i} + 15\hat{j} - 3\hat{k}$$

$$S.D = \frac{|\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|} = \frac{|36 + 225 + 9|}{\sqrt{36 + 225 + 9}}$$

$$= 3\sqrt{30}$$

22

22. If volume of parallelopiped whose there coterminous edges are $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} + \hat{k}$ & $\vec{w} = \hat{i} + \hat{j} + 3\hat{k}$ is 1 cubic unit then cosine of angle between $\vec{u}$ and $\vec{v}$ is

(1) $\frac{7}{3\sqrt{10}}$ (2) $\frac{7}{6\sqrt{3}}$ (3) $\frac{5}{3\sqrt{3}}$ (4) $\frac{5}{7}$

$$\pm 1 = \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix}$$

$\Rightarrow -\lambda + 3 = \pm 1$

$\Rightarrow \lambda = 2$ or $\lambda = 4$

for $\lambda = 4$

$$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$$

Ans :- (1)

23

23. Find the sum $\sum_{k=1}^{20} (1+2+3+\cdots+k)$

$= \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

So, sum $= \frac{20 \times 21}{2}$

$= \sum_{k=1}^{20} \frac{k(k+1)}{2}$

$= \frac{1}{2} \sum_{k=1}^{20} (k^2+k)$

$= \frac{1}{2}\left[\frac{20(21)(41)}{6} + \frac{20(21)}{2}\right]$

$= \frac{1}{2}\left[\frac{420 \times 41}{6} + \frac{20 \times 21}{2}\right]$

$= 1540$

24

24. If normal at P on the curve $y^2 - 3x^2 + y + 10 = 0$ passes through the point $(0, 3/2)$ then slope of tangent at P is n. The value of $|n|$ is equal to

$P \equiv (x_1, y_1)$

$2yy' - 6x + y' = 0 \Rightarrow y' = \left(\frac{6x_1}{1+2y_1}\right)$

$\left(\frac{3/2 - y_1}{-x_1}\right) = -\left(\frac{1+2y_1}{6x_1}\right)$

$9 - 6y_1 = 1 + 2y_1$

$\Rightarrow y_1 = 1$

$\therefore x_1 = \pm 2$

$\therefore$ Slope of tangent $= \left(\frac{\pm 12}{3}\right)$

$= \pm 4.$

$\therefore |n| = 4.$

25

25. If $2x^2 + (a-10)x + \frac{33}{2} = 2a$, $a \in z^+$ has real roots, then minimum value of a is equal to

$D \geq 0$

$(a-10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0.$

$(a-10)^2 - 4(33 - 4a) \geq 0.$

$a^2 - 4a - 32 \geq 0.$

$\Rightarrow a \in (-\infty, -4] \cup [8, \infty)$

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