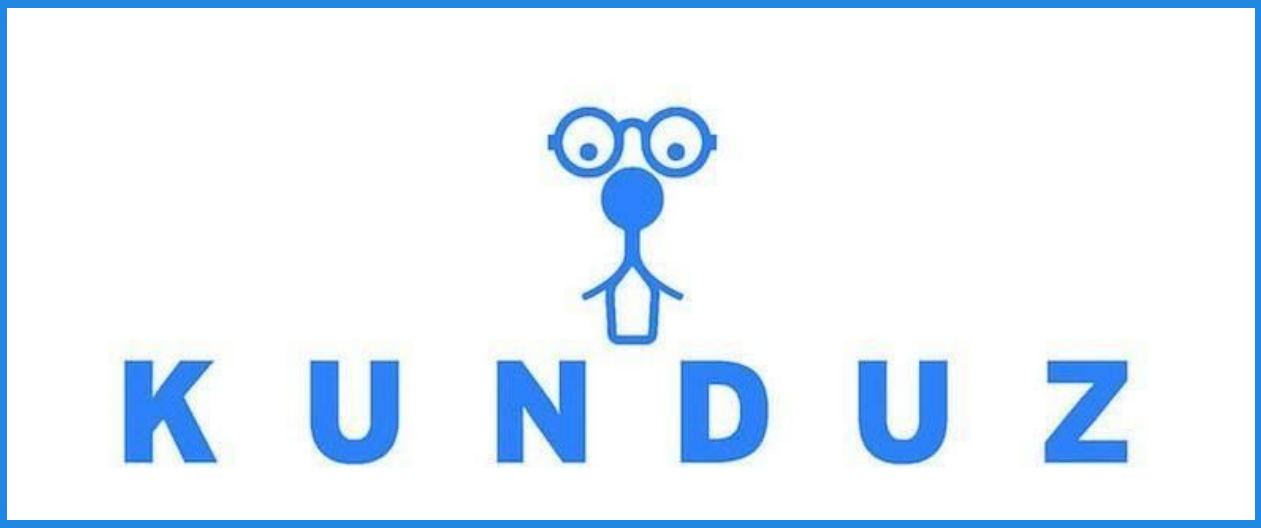
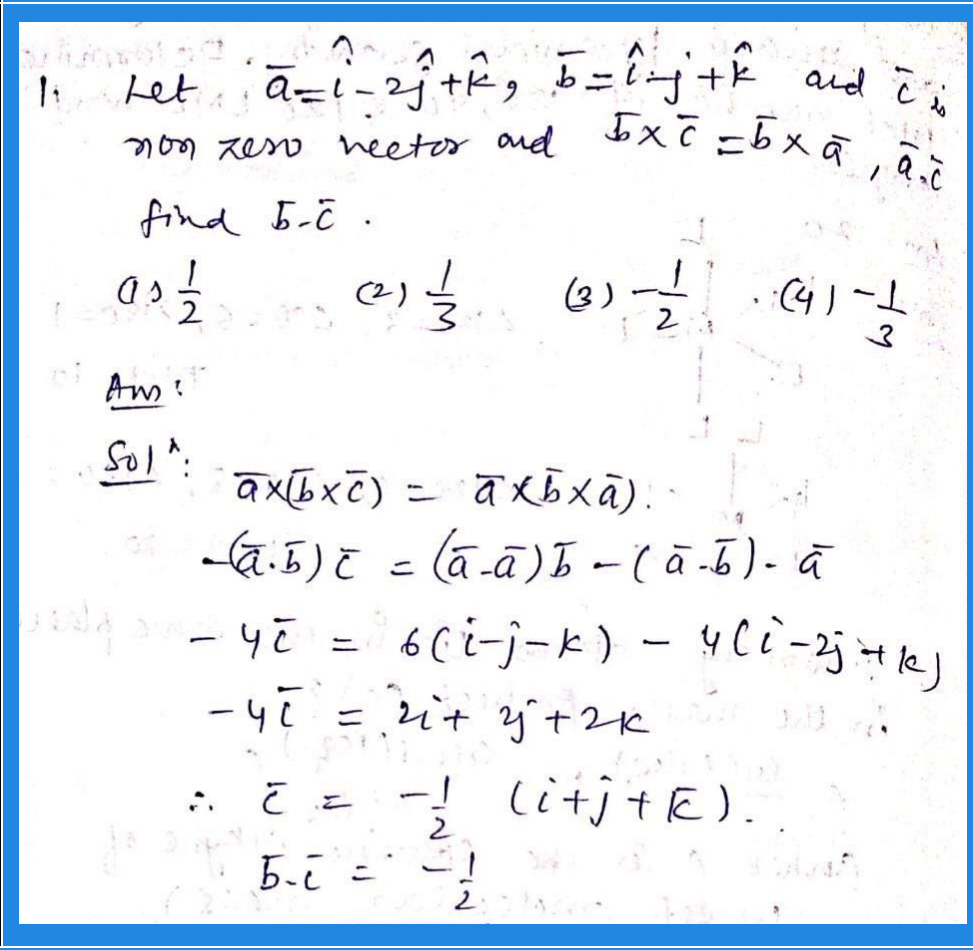


Jan 8 2020 shift 2 Mathematics prepared by KUNDUZ tutors

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Question no.	Questions and Answers
1	

1. Let $\bar{a}=\hat{i}-2\hat{j}+\hat{k}$, $\bar{b}=\hat{i}-\hat{j}+\hat{k}$ and $\bar{c}$ is non zero vector and $\bar{b}\times\bar{c}=\bar{b}\times\bar{a}$, $\bar{a}\cdot\bar{c}$ find $\bar{b}\cdot\bar{c}$.

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $-\frac{1}{2}$ (4) $-\frac{1}{3}$

Ans:

Sol^n:

$\bar{a}\times(\bar{b}\times\bar{c})=\bar{a}\times(\bar{b}\times\bar{a})$

$-(\bar{a}\cdot\bar{b})\bar{c}=(\bar{a}\cdot\bar{a})\bar{b}-(\bar{a}\cdot\bar{b})\cdot\bar{a}$

$-4\bar{c}=6(\hat{i}-\hat{j}-\hat{k})-4(\hat{i}-2\hat{j}+\hat{k})$

$-4\bar{c}=2\hat{i}+2\hat{j}+2\hat{k}$

$\therefore \bar{c}=-\frac{1}{2}(\hat{i}+\hat{j}+\hat{k})$.

$\bar{b}\cdot\bar{c}=-\frac{1}{2}$

2

2. Let coefficient of x^4 & x^2 in the expansion of $(x+\sqrt{x^2-1})^6+(x-\sqrt{x^2-1})^6$ is α and β then $\alpha-\beta$ is equal to :

(1) 48 (2) 60 (3) -132 (4) -60.

Ans: (3)

$$= 2\left[{}^6C_0 x^6 + {}^6C_2 x^4(x^2-1) + {}^6C_4 x^2(x^2-1)^2 + {}^6C_6 (x^2-1)^3\right]$$

$$= 2\left[x^6 + 15(x^6-x^4) + 15x^2(x^4-2x^2+1) + (-1+3x^2-3x^4+x^6)\right]$$

$$= 2(32x^6 - 48x^4 + 18x^2 - 1)$$

$\alpha = -96$ & $\beta = 36$ $\quad \alpha-\beta = -132$

3

3. Differential equation of $x^2 = 4b(y+b)$, where b is a parameter, is

(1) $x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x^2$

(2) $x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx} + x$.

(3) $x\left(\frac{dy}{dx}\right)^2 = y\left(\frac{dy}{dx}\right) + x^2$

(4) $x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx} + 2x^2$

Ans. (2)

Sol. $2x = 4by$, $\Rightarrow b = \frac{x}{2y}$,

$$x^2 = \frac{2xy}{y'}\cdot y + \frac{x^2}{y'^2}$$

4

4. Image of $(1,2,3)$ wrt a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$ then which of the following pts lie on the plane.

(1) $(-1,1,-1)$ (2) $(-1,-1,-1)$

(3) $(-1,-1,1)$ (4) $(1,1,-1)$

Ans. (4):

dr of normal to plane. $\frac{10}{3}, \frac{10}{3}, \frac{10}{3}$ i.e. $1,1,1$

midpoint of P & Q $\left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$.

∴ eqn of plane $x+y+z=1$

5

5. $\lim\limits_{x\to 0} \frac{\int_0^x t\sin(10t)\,dt}{x}$ is equal to

(1) 1 (2) 10 (3) 5 (4) 0

Ans: (4)

Soln: Using L'Hospital

$\lim\limits_{x\to 0} \frac{x(\sin 10x)}{1} = 0.$

6

6. Let P be the set of points (x, y) such that $x^2 \le y \le -2x+3$. The area of the region bounded by points in set P is –

(1) $\frac{16}{3}$ (2) $\frac{32}{3}$ (3) $\frac{29}{3}$ (4) $\frac{20}{3}$

Ans: (2)

Sol^n: Point of intersection $y = x^2$ & $y = -2x+3$ is obtained by $x^2 + 2x - 3 = 0$

$\Rightarrow x = -3, 1$

So Area, $= \int_{-3}^{1} (3 - 2x - x^2)\, dx$

$= 3(4) - 2\left(\frac{1^2 - 3^2}{2}\right) - \left(\frac{1^3 + 3^3}{3}\right)$

$= 12 + 8 - \frac{28}{3} = \frac{32}{3}$

7

7. Let $f(x) = \frac{x[x]}{x^2+1} : (1,3) \to R$ then range of $f(x)$ is (where $[\cdot]$ denotes greatest integer function)

(1) $\left(0, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{7}{5}\right]$ (2) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(3) $\left(\frac{2}{5}, 1\right) \cup \left(1, \frac{4}{5}\right]$ (4) $\left(0, \frac{1}{3}\right] \cup \left(\frac{2}{5}, \frac{4}{5}\right]$

Ans. (2)

Sol^n. $f(x) = \begin{cases} \frac{x}{x^2+1}; & x \in (1, 2) \\ \frac{2x}{x^2+1}; & x \in (2, 3) \end{cases}$

$\therefore f(x)$ is a decreasing function.

$\Rightarrow y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

8

8. Let $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then value of $10A^{-1}$ is ___

(1) $4I - A$ (2) $6I - A$ (3) $A - 4I$ (4) $A - 6I$

Ans: (4)

Sol: Characteristic eqn of matrix A

$$\begin{vmatrix} 2-x & 2 \\ 9 & 4-x \end{vmatrix} = 0 \Rightarrow x^2 - 6x - 10 = 0$$

$\therefore A^2 - 6A - 10I = 0$

$10A^{-1} = A - 6I$

9

9. Solution set of $3^x(3^x-1)+2 = |3^x-1| + |3^x-2|$ is

(1) Singleton set (2) two elements

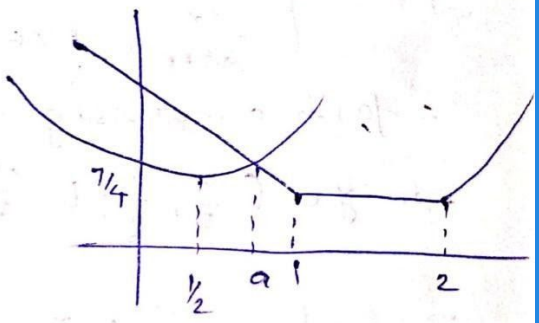
(3) at least four elements (4) infinite elements.

Ans (1).

Soln: let $3^x = t$

$t(t-1)+2 = |t-1|+|t-2|$

$t^2-t+2 = |t-1|+|t-2|$.



are positive solutions

$t = a$,

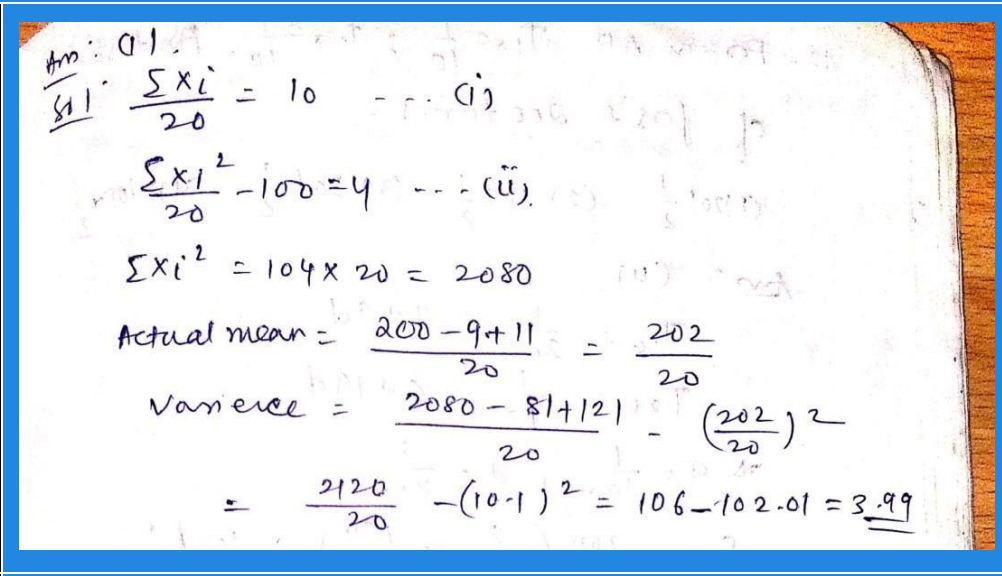
$3^x = a$

$\boxed{x = \log_3 a.}$ → so singleton set.

10

10. Mean and variance of 20 observation are 10 & 4. If it was found, that in place of 11, 9 was taken by mistake, find correct variance.

(1) 3.99 (2) 3.98 (3) 4.01 (4) 4.02



Ans: 01.

Sol: $\frac{\sum x_i}{20} = 10$ ... (i)

$\frac{\sum x_i^2}{20} - 100 = 4$ ... (ii)

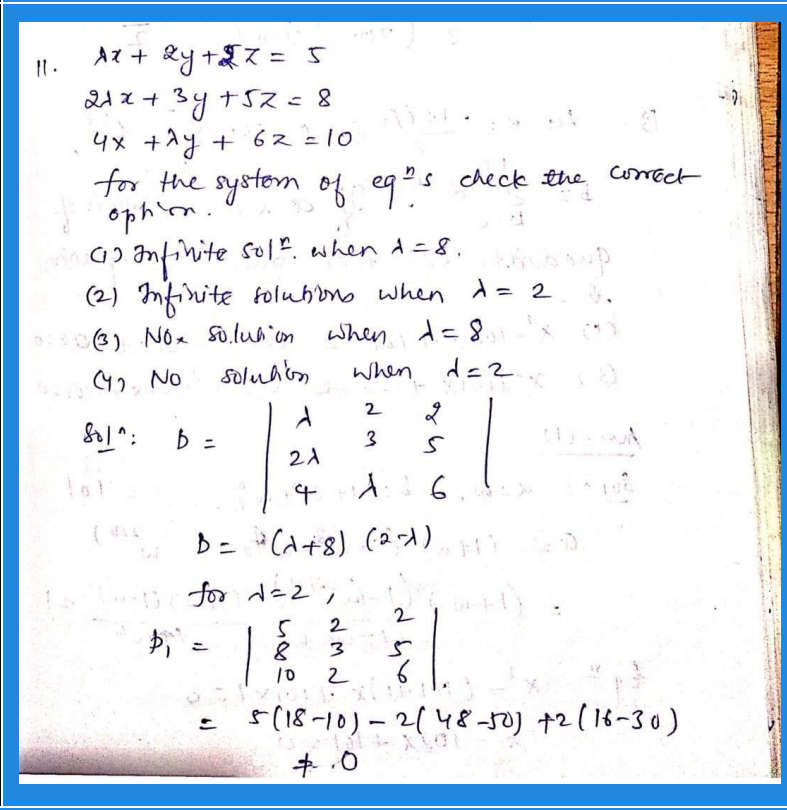
$\sum x_i^2 = 104 \times 20 = 2080$

Actual mean $= \frac{200 - 9 + 11}{20} = \frac{202}{20}$

Variance $= \frac{2080 - 81 + 121}{20} - \left(\frac{202}{20}\right)^2$

$= \frac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99$

11



11. $\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$

for the system of eq^{n}s check the correct option.

(1) Infinite sol^n. when $\lambda = 8$.

(2) Infinite solutions when $\lambda = 2$

(3) No solution when $\lambda = 8$

(4) No solution when $\lambda = 2$

Sol^n: $D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$

$D = (\lambda + 8)(2 - \lambda)$

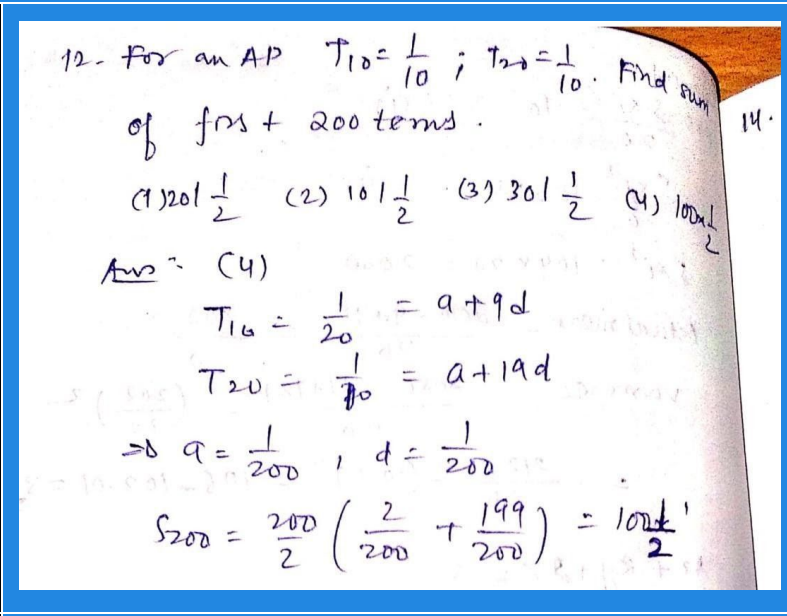
for $\lambda = 2$,

$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$.

$= 5(18 - 10) - 2(48 - 50) + 2(16 - 30)$

$\neq 0$

12



12. For an AP $T_{10} = \frac{1}{10}$; $T_{20} = \frac{1}{20}$. Find sum of first 200 terms.

(1) $201\frac{1}{2}$ (2) $101\frac{1}{2}$ (3) $301\frac{1}{2}$ (4) $100\frac{1}{2}$

Ans: (4)

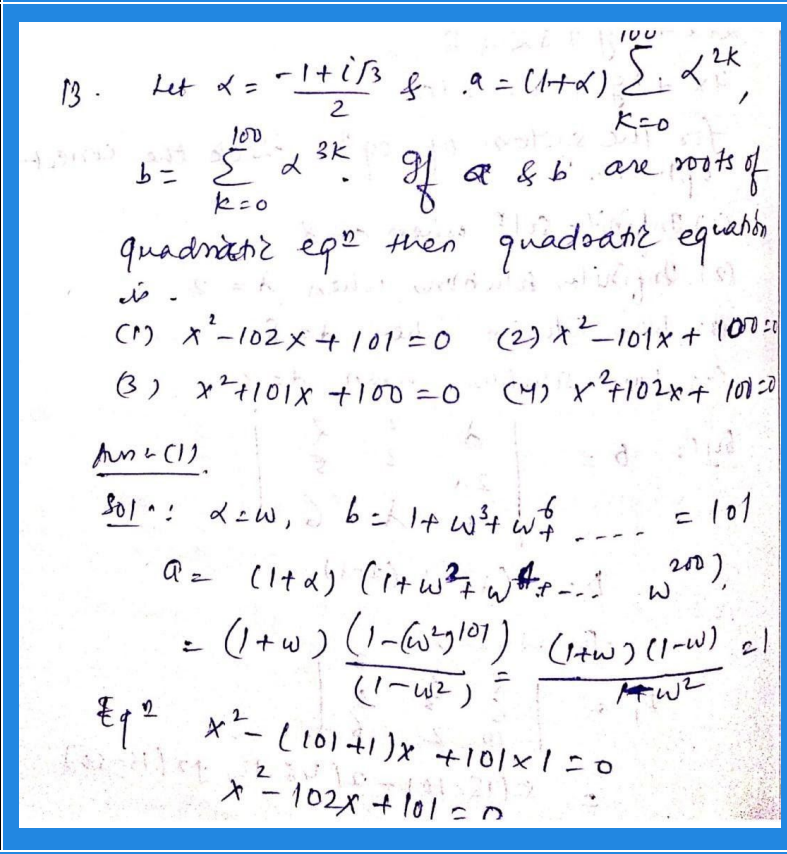
$T_{10} = \frac{1}{20} = a + 9d$

$T_{20} = \frac{1}{10} = a + 19d$

$\Rightarrow a = \frac{1}{200}$, $d = \frac{1}{200}$

$S_{200} = \frac{200}{2}\left(\frac{2}{200} + \frac{199}{200}\right) = 100\frac{1}{2}$

13



13. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$ & $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$,

$b = \sum_{k=0}^{100} \alpha^{3k}$. If a & b are roots of quadratic eqn then quadratic equation is.

(1) $x^2 - 102x + 101 = 0$ (2) $x^2 - 101x + 100 = 0$

(3) $x^2 + 101x + 100 = 0$ (4) $x^2 + 102x + 101 = 0$

Ans: (1)

Sol: $\alpha = \omega$, $b = 1 + \omega^3 + \omega^6 + \ldots = 101$

$a = (1+\alpha)(1+\omega^2+\omega^4+\ldots+\omega^{200})$,

$= (1+\omega)\frac{(1-(\omega^2)^{101})}{(1-\omega^2)} = \frac{(1+\omega)(1-\omega)}{1+\omega^2} = 1$

Eqn $x^2 - (101+1)x + 101 \times 1 = 0$

$x^2 - 102x + 101 = 0$

14

14. Let $f(x)$ is a three degree polynomial for which $f'(-1)=0$, $f''(1)=0$, $f(-1)=10$, $f(1)=-6$ then local minima of $f(x)$ exists at.

(1) $x=3$ (2) $x=2$ (3) $x=1$, (4) $x=-1$

Ans: (1)

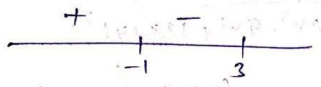
Sol: Let $f(x) = ax^3+bx^2+cx+d$

$$a=\frac{1}{4} \quad b=-\frac{3}{4} \quad c=-\frac{9}{4} \quad d=\frac{35}{4}$$

$$\Rightarrow f(x) = a(x^3-3x^2-9x)+d$$

$$f'(x) = \frac{3}{4}(x^2-2x+3)$$

$\Rightarrow f'(x)=0$ at $x=3,-1$



local minimal at $x=3$

15

15. Let A & B are two events such that $P(\text{exactly one}) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{2}$ then $P(A \cap B) =$

(1) $\frac{1}{10}$ (2) $\frac{2}{9}$ (3) $\frac{1}{8}$ (4) $\frac{1}{12}$

Sol: $P(\text{exactly one}) = \frac{2}{5}$

$$P(A)+P(B)-2P(A\cap B) = \frac{2}{5}$$

$$P(A\cap B) = \frac{1}{2}$$

$$P(A)+P(B)-P(A\cap B) = \frac{1}{2}$$

$$P(A\cap B) = \frac{1}{2}-\frac{2}{5} = \frac{5-4}{10} = \frac{1}{10}$$

16

16. Let $I = \int_1^2 \frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$ (2) $\frac{1}{3} < I^2 < \frac{1}{2}$

(3) $\frac{1}{9} < I < \frac{1}{8}$ (4) $\frac{1}{3} < I < \frac{1}{2}$

Ans: (1)

Sol^n: $f(x) = \frac{1}{\sqrt{2x^3-9x^2+12x+4}}$

$f'(x) = \frac{-1}{2} \frac{(6x^2-18x+12)}{(2x^3-9x^2+12x+4)^{3/2}}$

$= \frac{-6(x-1)(x-2)}{2(2x^3-9x^2+12x+4)^{3/2}}$

$f(1) = \frac{1}{3}$ $f(2) = \frac{1}{\sqrt{8}}$

$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$

17

17. Normal at (2, 2) to curve $x^2+2xy-3y^2=0$ is L. Then perpendicular distance from origin to line L is

(1) $4\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Sol^n: $x^2+2xy-3y^2=0$

$x^2+3xy-xy-3y^2=0$

$(x-y)(x+3y)=0$

$x-y=0$ $x+3y=0$

(2,2) satisfies $x-y=0$

Normal $x+y=\lambda \Rightarrow 2+2=\lambda \Rightarrow x+y=4$

perpendicular distance from origin $= \left|\frac{0+0-4}{\sqrt{2}}\right| = \frac{4}{2\sqrt{2}}$

18

18. Which of the following is tautology.

(1) $\sim(p \vee \sim q) \rightarrow (p \vee q)$

(2) $(\sim p \vee q) \rightarrow (p \vee q)$

(3) $\sim(p \wedge \sim q) \rightarrow (p \vee q)$

(4) $\sim(p \vee \sim q) \rightarrow (p \wedge q)$

Ans: (1)

Sol^n: $\sim(p \vee \sim q) \rightarrow (p \vee q)$

$\sim\{(p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\sim(\sim p \wedge f)$

19

19. A hyperbola has vertices $(\pm 6, 0)$ and $P(10, 16)$ lies on it, then the equation of normal at P is.

(1) $2x + 5y = 100$ (2) $2x + 5y = 10$

(3) $2x - 5y = 100$ (4) $5x + 2y = 100$

Ans: (1)

Sol^n: vertex at $(\pm 6, 0)$. $a = 6$

Let the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Putting point $P(10, 16)$ on the hyperbola

$\frac{100}{36} - \frac{256}{b^2} = 1 \qquad b^2 = 144$

$\therefore$ hyperbola is $\frac{x^2}{36} - \frac{y^2}{144} = 1$

Eq^n of normal $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$

On putting $2x + 5y = 100$

20

20. If $y=mx+c$ is a tangent to the circle $(x-3)^2+y^2=1$ and also the perpendicular to the tangent to the circle $x^2+y^2=1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ then

(1) $c^2+6c+7=0$ (2) $c^2-6c+7=0$

(3) $c^2+6c-7=0$ (4) $c^2-6c-7=0$

Ans: (1)

Soln: Slope of tangent to $x^2+y^2=1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

$$x^2+y^2=1$$

$$2x+2yy'=0$$

$$y'=\frac{-x}{y}=-1$$

$y=mx+c$ is tangent of $x^2+y^2=1$

so, $m=1$

$$y=x+c$$

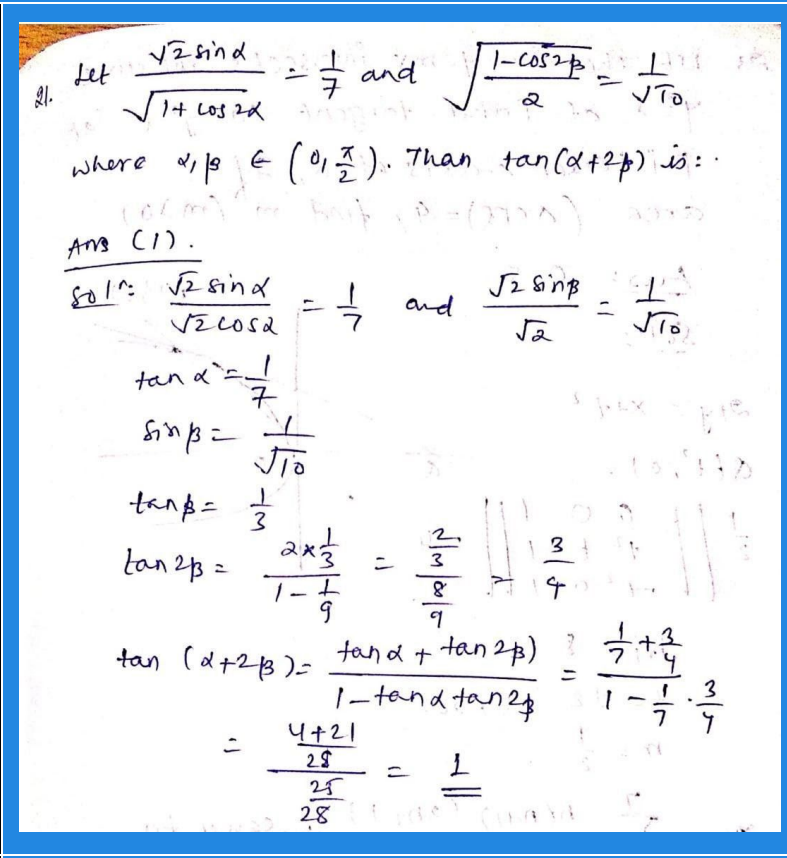
Now distance of $y=x+c$ from $(3,0)$ is

$$\left|\frac{c+3}{\sqrt{2}}\right|=1$$

$$c^2+6c+9=2$$

$$\boxed{c^2+6c+7=0}$$

21



21. Let $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$

where $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$. Than $\tan(\alpha+2\beta)$ is:

Ans (1).

Sol^n: $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\tan\alpha = \frac{1}{7}$

$\sin\beta = \frac{1}{\sqrt{10}}$

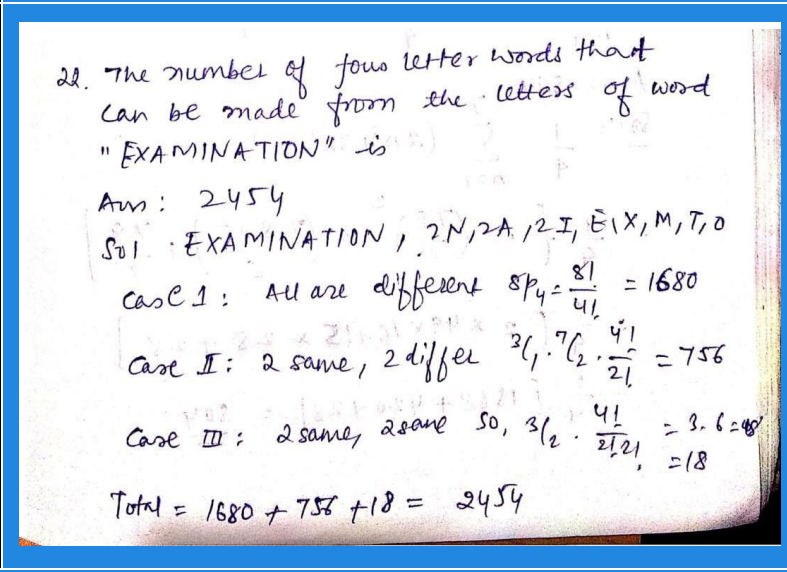
$\tan\beta = \frac{1}{3}$

$\tan 2\beta = \frac{2\times\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$

$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan 2\beta)}{1-\tan\alpha\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}}$

$= \frac{\frac{4+21}{28}}{\frac{25}{28}} = \frac{1}{}$

22



22. The number of four letter words that can be made from the letters of word "EXAMINATION" is

Ans: 2454

Sol: EXAMINATION, 2N, 2A, 2I, E, X, M, T, O

Case I: All are different ${}^{8}P_4 = \frac{8!}{4!} = 1680$

Case II: 2 same, 2 differ ${}^{3}C_1 \cdot {}^{7}C_2 \cdot \frac{4!}{2!} = 756$

Case III: 2 same, 2 same so, ${}^{3}C_2 \cdot \frac{4!}{2!2!} = 3.6 = 18$

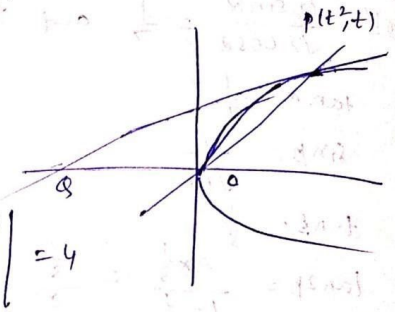
Total = 1680 + 756 + 18 = 2454

23

23. Let the line $y=mx$ intersects the curve $y^2=x$ at P and tangent to $y^2=x$ at P intersect x-axis at Q, if area $(\Delta OPQ)=4$, find m $(m>0)$

Ans: 0.5

Sol^n:



$2ty = x+t^2$

$Q(t^2, 0)$.

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{vmatrix} = 4$$

$|t|^3 = 8$

$t = \pm 2 \quad (t>0)$

$m = \frac{1}{2}$

24

24. $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to

Ans: 504

Sol^n: $\frac{1}{4}\sum_{n=1}^{7}(2n^3+3n^2+n)$

$= \frac{1}{4}\left[2\cdot\left(\frac{7\cdot 8}{2}\right)^2 + 3\cdot\left(\frac{7\cdot 8\cdot 15}{6}\right) + \frac{7\cdot 8}{2}\right]$

$= \frac{1}{4}\left[2\times 49\times 16 + 15\times 28 + 28\right]$

$= \frac{1}{4}\left[1568+420+28\right] = 504$

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