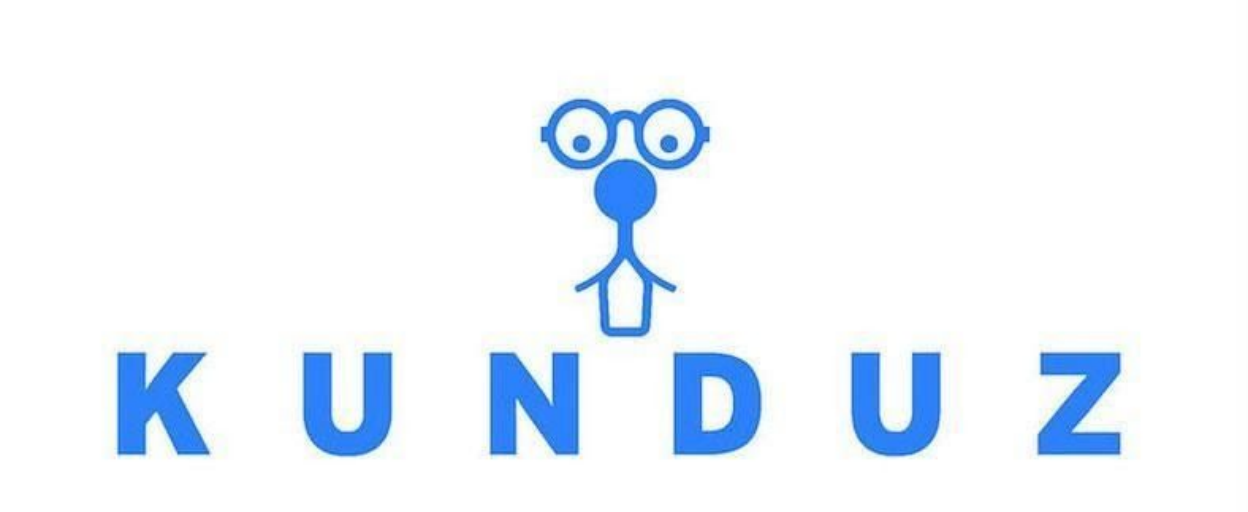
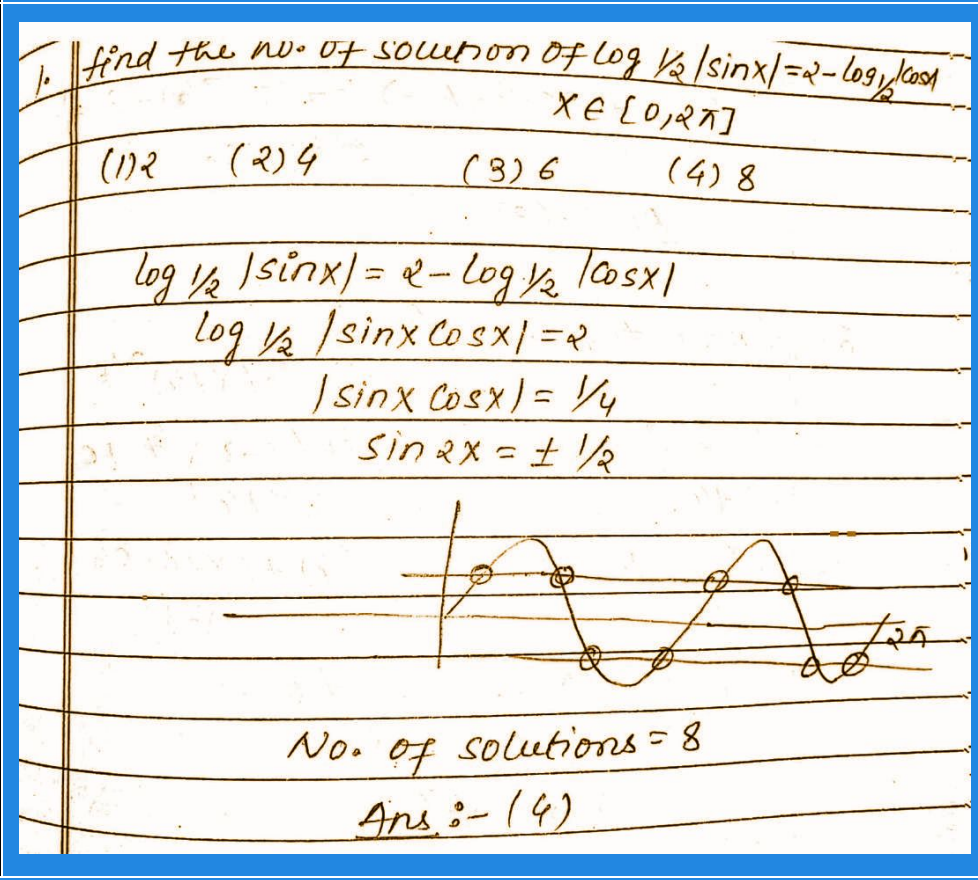
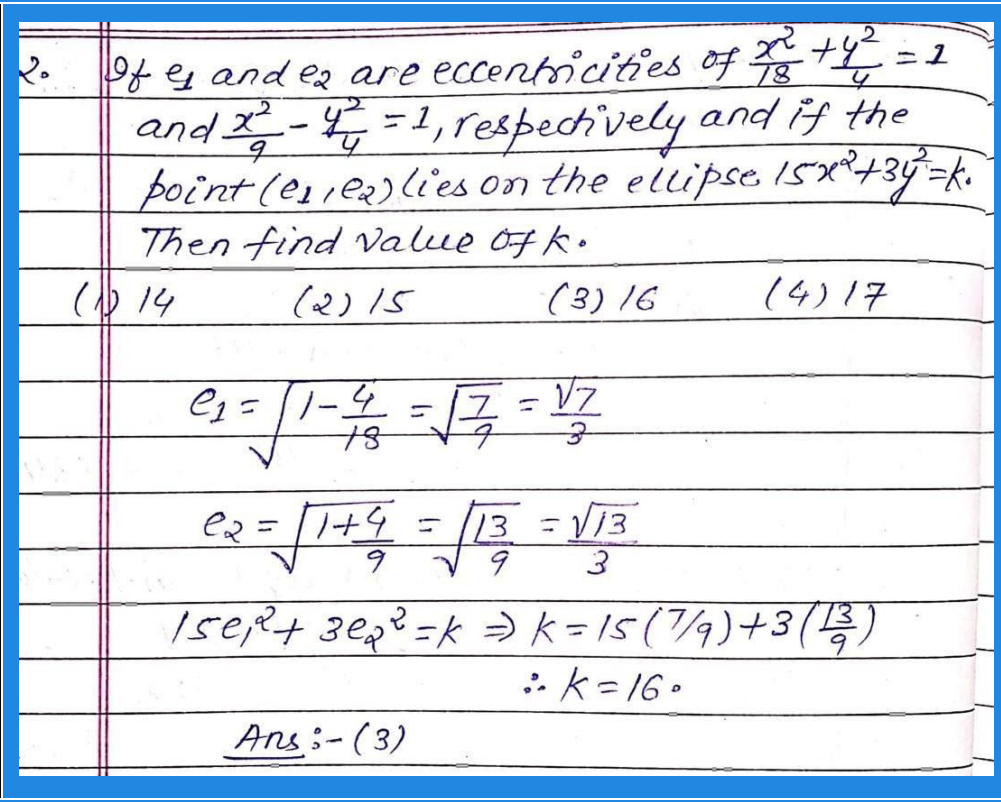


Jan 9 2020 shift 1 mathematics prepared by KUNDUZ tutors

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Question no.	Questions and Answers
1	

2



2. If e_1 and e_2 are eccentricities of $\frac{x^2}{18}+\frac{y^2}{4}=1$ and $\frac{x^2}{9}-\frac{y^2}{4}=1$, respectively and if the point (e_1, e_2) lies on the ellipse $15x^2+3y^2=k$. Then find value of k.

(1) 14 (2) 15 (3) 16 (4) 17

$$e_1=\sqrt{1-\frac{4}{18}}=\sqrt{\frac{7}{9}}=\frac{\sqrt{7}}{3}$$

$$e_2=\sqrt{1+\frac{4}{9}}=\sqrt{\frac{13}{9}}=\frac{\sqrt{13}}{3}$$

$$15e_1^2+3e_2^2=k \Rightarrow k=15(7/9)+3\left(\frac{13}{9}\right)$$

$$\therefore k=16.$$

Ans :- (3)

3

3. find integration $\int \frac{dx}{(x-3)^{6/7}\cdot(x+4)^{8/7}}$

(1) $\left(\frac{x-3}{x+4}\right)^{1/7}+c$ (2) $7\left(\frac{x-3}{x+4}\right)^{1/7}+c$

(3) $7\left(\frac{x-3}{x+4}\right)^{6/7}+c$ (4) $7\left(\frac{x+4}{x-3}\right)^{6/7}+c$

$$\int \left(\frac{x-3}{x+4}\right)^{-6/7}\cdot\frac{1}{(x+4)^2}\,dx$$

$$\frac{x-3}{x+4}=t^7$$

$$\frac{7}{(x+4)^2}\,dx=7t^6\,dt$$

$$\int t^{-6}\,t^6\,dt$$

$$=t+c$$

Ans :- (1)

4

4. If $\left|\frac{z-i}{z+2i}\right|=1$, $|z|=\frac{5}{2}$ then value of $|z+3i|$ is

(1) $7/2$ (2) $\sqrt{10}$ (3) $\sqrt{5}$ (4) $\sqrt{3}$

$$x^2+(y-1)^2=x^2+(y+2)^2$$

$$-2y+1=4y+4$$

$$\Rightarrow y=-1/2$$

$$x^2+y^2=25/4 \Rightarrow x^2=6$$

$$z=\pm\sqrt{6}-i/2$$

$$|z+3i|=7/2$$

Ans :- (1)

5

5. $2^{1/4} \cdot 4^{1/16} \cdot 8^{1/48} \cdots \infty =$

(1) $\sqrt{2}$ (2) 2 (3) $2^{1/4}$ (4) 1

$2^{1/4 + 2/16 + 3/48 + \cdots \infty}$

$\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots \infty$

$= 2^{\frac{1/4}{1-1/2}}$ $= 2^{1/2} = \sqrt{2}$

Ans :- (1)

6

6. Value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$ is

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $\frac{-1}{2}$

$\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$

$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right]$

$= \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}$

Ans :- (1)

7

7. Find Value of $\int_0^{2\pi} \frac{x \sin^8 x \, dx}{\sin^8 x + \cos^8 x}$

(1) π^2 (2) $2\pi^2$ (3) $3\pi^2$ (4) $4\pi^2$

$$\int_0^{\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx + \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x} dx$$

$$= 2\pi \int_0^{\pi/2} 1 \, dx$$

$$= 2\pi \times \frac{\pi}{2} = \pi^2$$

Ans :- (1)

8

8. if $f(x) = a + bx + cx^2$ where $a, b, c \in R$ then $\int_0^1 f(x)dx$ is

(1) $\frac{1}{3}\left(f(1) + f(0) + 2f\left(\frac{1}{2}\right)\right)$

(2) $\frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right)$

(3) $\frac{1}{6}\left(f(1) + f(0) - 4f\left(\frac{1}{2}\right)\right)$

(4) $\frac{1}{6}\left(f(1) - f(0) - 4f\left(\frac{1}{2}\right)\right)$

$$\int_0^1 (a + bx + cx^2)dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}\Big|_0^1$$

$$= a + b/2 + c/3$$

$f(1) = a + b + c$

$f(0) = a$

$f(1/2) = a + b/2 + c/4$

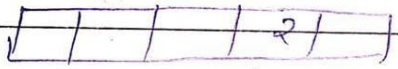
Putting value $= a + \frac{b}{2} + \frac{c}{3}$

Ans :- (2)

9

9. If number of 5 digit numbers which can be formed without repeating any digit while tenth place of all of the numbers must be 2 is $336k$, find value of k.

(1) 8 (2) 7 (3) 6 (4) 5



Number of Numbers $= 8 \times 8 \times 7 \times 6$

$= 2688 = 336k$

$k = 8$

Ans :- (1)

10

10. $A(3,-1)$, $B(1,3)$, $C(2,4)$ are vertices of ΔABC. If D is centroid of ΔABC & P is point of intersection of lines $x+3y-1=0$ and $3x-y+1=0$ then which of the following points lies on line joining D and P

(1) $(-9,-7)$ (2) $(-9,-6)$ (3) $(9,6)$ (4) $(9,-6)$

$D(2,2)$

Point of intersection $P(-1/5, 2/5)$

equation of line DP

$8x-11y+6=0$.

So Ans :- (2).

11

11. If $f(x)$ is twice differentiable and continous function in $x \in [a,b]$ also $f'(x)>0$ & $f''(x)<0$ and $c \in (a,b)$ then $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

(1) $\frac{b-c}{c-a}$ (2) 1 (3) $\frac{a+b}{b-c}$ (4) $\frac{c-a}{b-c}$

Lets use LMVT for $x \in [a,c]$

$$\frac{f(c)-f(a)}{c-a} = f'(\alpha), \alpha \in (a,c)$$

also use LMVT for $x \in [c,b]$

$$\frac{f(b)-f(c)}{b-c} = f'\beta, \beta \in (c,b)$$

$\because f''(x)<0 \Rightarrow f'(x)$ is decreasing

$$f'(\alpha) > f'(\beta)$$

$$\frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$$\frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c}$$ ($\because f(x)$ is increasing).

12

12. If Plane

$x+4y-2z=1$

$x+7y-5z=\beta$

$x+5y+\alpha z=5$

intersect in a line ($R \times R \times R$) then $\alpha+\beta$ is equal to

(1) 0 (2) 10 (3) -10 (4) 2

$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$

$\Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$

Also $D_2 = 0 \Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0.$

$1(35 - 5\beta) - 15 + 1(4\beta - 7) = 0$

$\beta = 13$

$\alpha + \beta = 10$

Ans :- (2).

13

13) for observations x_i given $\sum_{i=1}^{10}(x_i-5)=10$ & $\sum_{i=1}^{10}(x_i-5)^2=40$

If mean & variance of observations $(x_1-3),(x_2-3)$ $\ldots(x_{10}-3)$ is λ & μ respectively then order pair (λ,μ) is

(1) (3,3) (2) (1,3) (3) (3,1) (4) (1,1)

$$\text{Mean} = \frac{\sum(x_i-5)}{10} = 1$$

$$\therefore \lambda = (x_i-5)+2 = 3$$

$$\mu = \text{Var}(x_i-5) = \frac{\sum(x_i-5)^2}{10} - \frac{\sum(x_i-5)}{10}$$

$$= 3$$

Ans :- (1)

14

14. On a bag there are 20 cards 10 names A and another 10 names B. Cards are drawn randomly one by one with replacement then find probability that second A comes before third B.

(1) 13/16 (2) 11/16 (3) 7/16 (4) 9/16

AA + ABA + BAA + ABBA + BBAA + BABA

$= 1/4 + 1/8 + 1/8 + 1/16 + 1/16 + 1/16$

$= \frac{11}{16}$

Ans :- (2)

15

15. The negation of $\sqrt{5}$ is an integer or 5 is an irrational number is

(1) $\sqrt{5}$ is an integer and 5 is not an irrational Number.

(2) $\sqrt{5}$ is not an integer and 5 is an irrational Number.

(3) $\sqrt{5}$ is not an integer or 5 is not an irrational Number

(4) $\sqrt{5}$ is not an integer and 5 is not an irrational Number

$\sqrt{5}$ is not an Integer & 5 is not an irrational Number

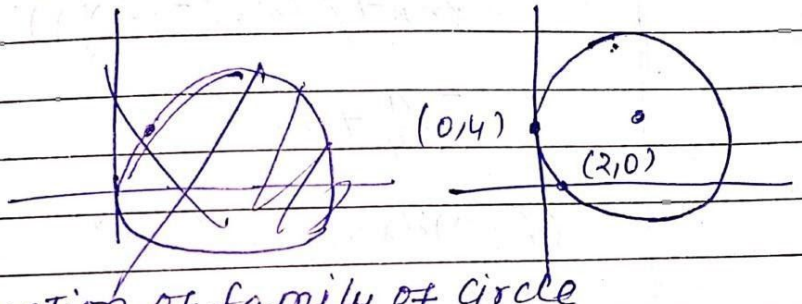
Ans :- (4).

16

16. If a circle touches y-axis at (0,4) & passes through (2,0) then which of the following can not be the tangent to the circle

(1) $3x+4y-6=0$ (2) $3x+4y-24=0$

(3) $4x-3y-17=0$ (4) $4x+3y-6=0$



Equation of family of circle

$(x-0)^2+(y-4)^2+\lambda x=0$

$4+16+2\lambda=0 \Rightarrow \lambda=-10$

$x^2+y^2-10x-8y+16=0$

$R=\sqrt{25+16-16}=5$

Check the options:

Ans:- (1)

17

17. If $f(x) = \tan^{-1}(\sec x + \tan x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $f(0) = 0$ then the value of $f(1)$ is

(1) $\frac{\pi+1}{4}$ (2) $\frac{\pi-1}{4}$ (3) $\frac{\pi+1}{2}$ (4) 0

$f'(x) = \tan^{-1}(\sec x + \tan x)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$

$= \frac{\pi}{4} + \frac{x}{2}$

$(f'(x))\,dx = \frac{\pi}{4} + x/2\,dx$

$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$

$f(0) = c = 0 \Rightarrow f(x) = \frac{\pi}{4}x + x^2/4$

$f(1) = \frac{\pi+1}{4}$ Ans :- (1).

18

18. A sphere of 10 cm radius has a uniform thickness of ice around it. Ice is melting at rate 50 cm^3/min when thickness is 5 cm then rate of change of thickness.

(1) $1/36\pi$ (2) $1/18\pi$ (3) $1/9\pi$ (4) $1/12\pi$

Let thickness = x cm

Total volume = $4/3\,\pi(10+x)^3$

$\frac{dv}{dt} = 4\pi(10+x)^2 \frac{dx}{dt}$

At x = 5 cm

$\frac{dx}{dt} = \frac{1}{18\pi}$ cm/min Ans :- (2).

19

19. find no. of Real roots of equation

$e^{4x} + e^{3x} - 4e^{2x} + e^{x} + 1 = 0$.

(1) 1 (2) 2 (3) 3 (4) 4

Let $e^x = t \in (0, \infty)$

Given equation

$t^4 + t^3 - 4 + 1/t + 1/t^2 = 0$

$(t^2 + 1/t^2) + (t + 1/t) - 4 = 0$

Put $t + 1/t = \alpha$

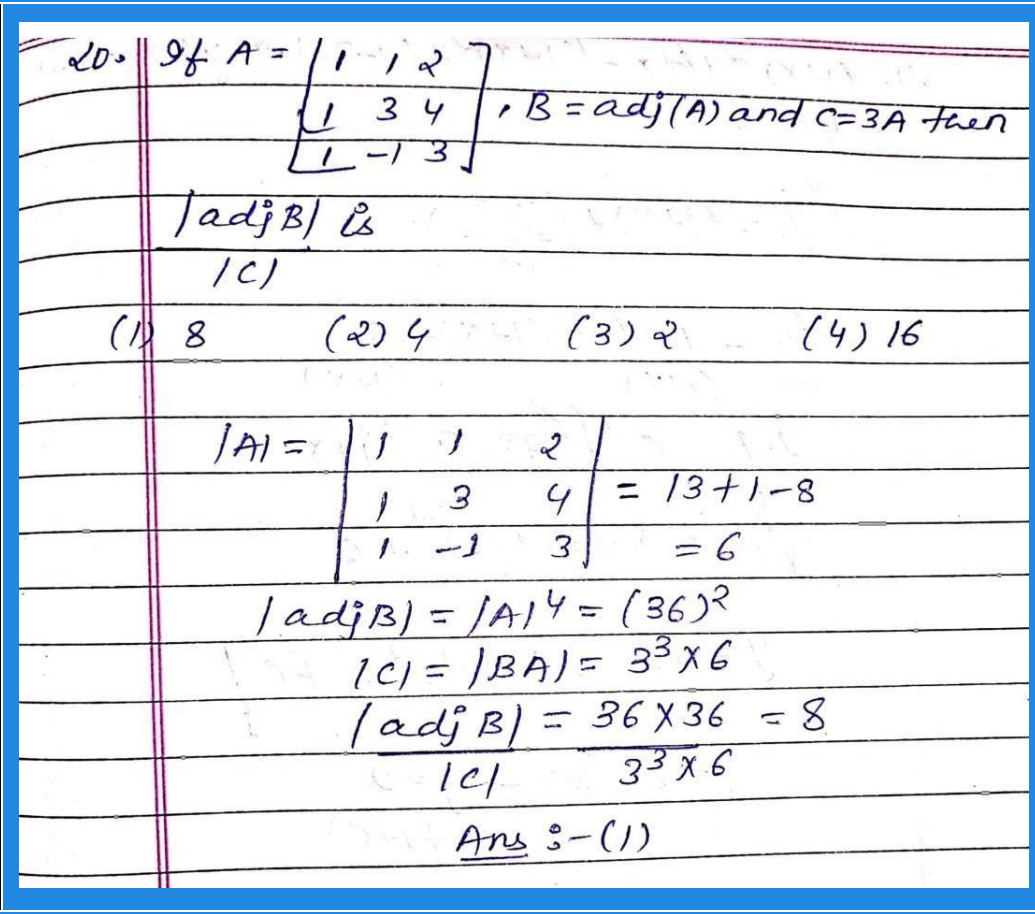
$\alpha^2 + \alpha - 6 = 0$

$\alpha = -3, 2 \quad \alpha = 2$

So, $x = 0$

Ans :- (1)

20



20. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = adj(A)$ and $C = 3A$ then $\frac{|adj B|}{|C|}$ is

(1) 8 (2) 4 (3) 2 (4) 16

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 13 + 1 - 8 = 6$$

$$|adj B| = |A|^4 = (36)^2$$

$$|C| = |BA| = 3^3 \times 6$$

$$\frac{|adj B|}{|C|} = \frac{36 \times 36}{3^3 \times 6} = 8$$

Ans :- (1)

21

21. $(1+x)\,dy/dx = ((1+x)^2+(y-3))$, if $y(2)=0$ then $y(3)=?$

$$\frac{dy}{dx} = (1+x) + \left(\frac{y-3}{1+x}\right)$$

$$\frac{dy}{dx} - \frac{1}{(1+x)}\,y = (1+x) - \frac{3}{(1+x)}$$

$$I.F = e^{-\int \frac{dx}{1+x}} = 1/1+x$$

$$\frac{y}{1+x} = x + 3(1+x)^{-1} + c$$

$$y = (1+x)\left[x + \frac{3}{1+x} + c\right]$$

$\therefore$ at $x=2, y=0$

$$0 = 3(2+1+c)$$

$$c = -3$$

$\therefore$ at $x=3$ $y=3$

22

22. $$f(x) = \begin{cases} \dfrac{\sin(a+2)x + \sin x}{x} & ; x<0 \\ b & ; x=0 \\ \dfrac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}} & ; x>0 \end{cases}$$

function is continous at $x=0$ find $a+2b$

$LHL = a+3$

$f(0) = b$

$$RHL = \lim_{h\to 0}\left(\frac{(1+3h)^{1/3}-1}{h}\right) = 1$$

$\therefore a = -2$

$b = 1$ $\quad a+2b = 0$

23

23. find the coefficient of x^4 in $(1+x+x^2)^{10}$

General term $= \frac{10!}{\alpha!\beta!\gamma!} x^{\beta+2\gamma}$

Total $= \frac{10!}{6!4!0!} + \frac{10!}{7!2!1!} + \frac{10!}{8!0!2!}$

$= 210 + 360 + 45$

$= 615$

24

24. If $\vec{P}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$

$\vec{Q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$

$\vec{R}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$

and $\vec{P},\vec{Q},\vec{R}$ are Coplanar Vectors and $3(\vec{P}\cdot\vec{Q})^2-\lambda|\vec{R}\times\vec{Q}|^2=0$ then Value of λ is

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$\Rightarrow a+1+a+a=0$

$a=-1/3$

$\vec{P}=\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}-\frac{1}{3}\hat{k}$

$\vec{Q}=\frac{1}{3}(-\hat{i}+2\hat{j}-\hat{k})$

$\vec{R}=\frac{1}{3}(-\hat{i}-\hat{j}+2\hat{k})$

$\vec{P}\cdot\vec{Q}=\frac{1}{9}(-2-2+1)=-\frac{1}{3}$

$$\vec{R}\times\vec{Q}=\frac{1}{9}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$$

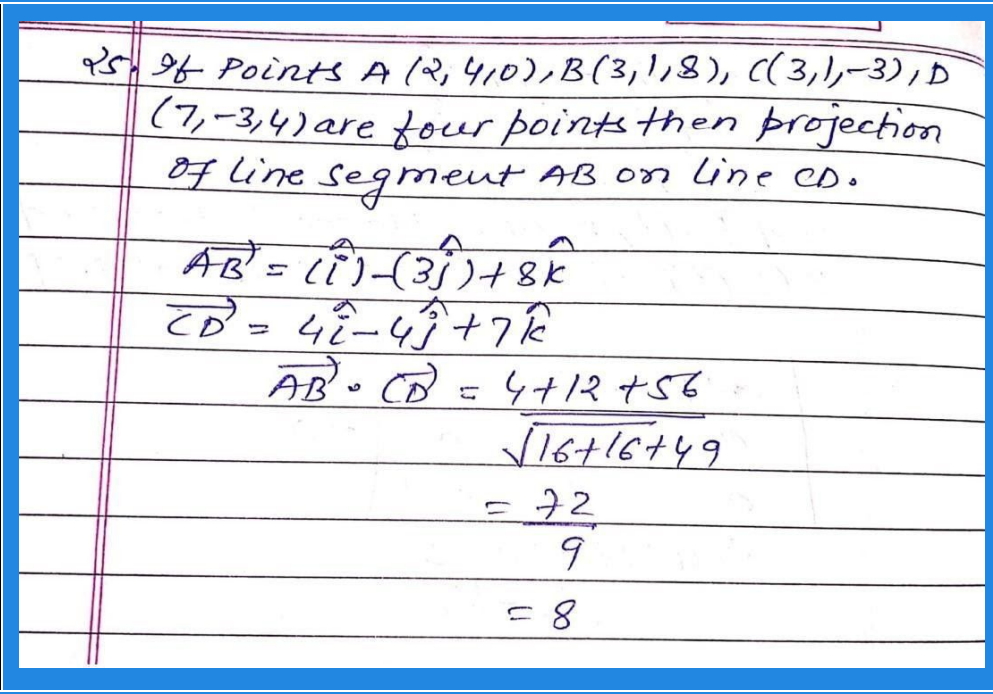
$=\frac{1}{9}(\hat{i}(4-1)-\hat{j}(-2-1)+\hat{k}(1+2))$

$=\frac{\hat{i}+\hat{j}+\hat{k}}{3}$

$|\vec{R}\times\vec{Q}|=\frac{1}{\sqrt{3}}$ $\quad |\vec{R}\times\vec{Q}|^2=\frac{1}{3}$

$\frac{3}{9}-\frac{\lambda}{3}=0 \Rightarrow \lambda=1$

25



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