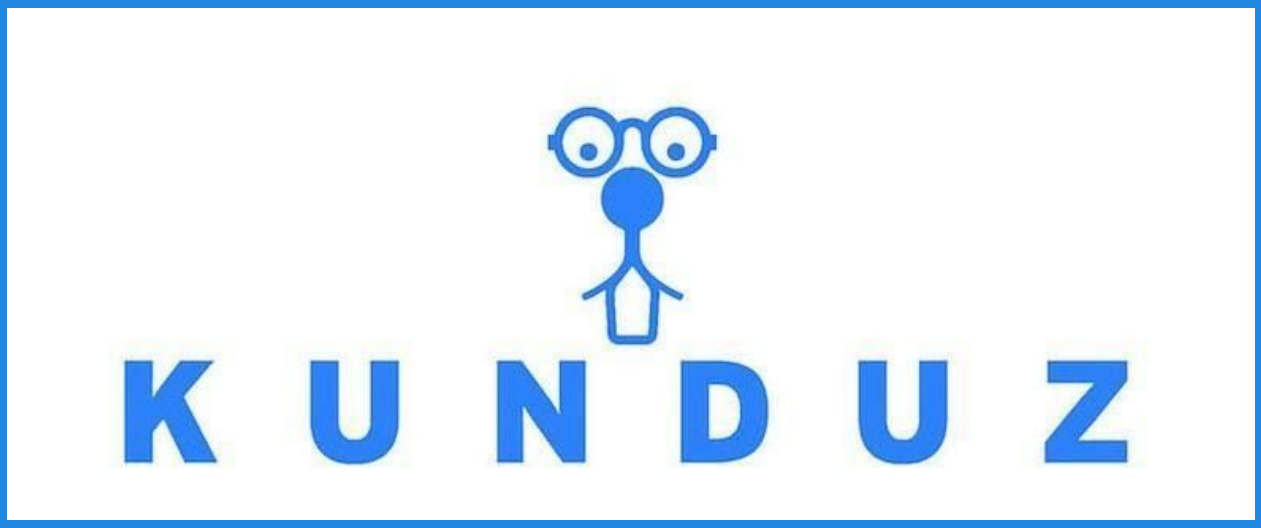
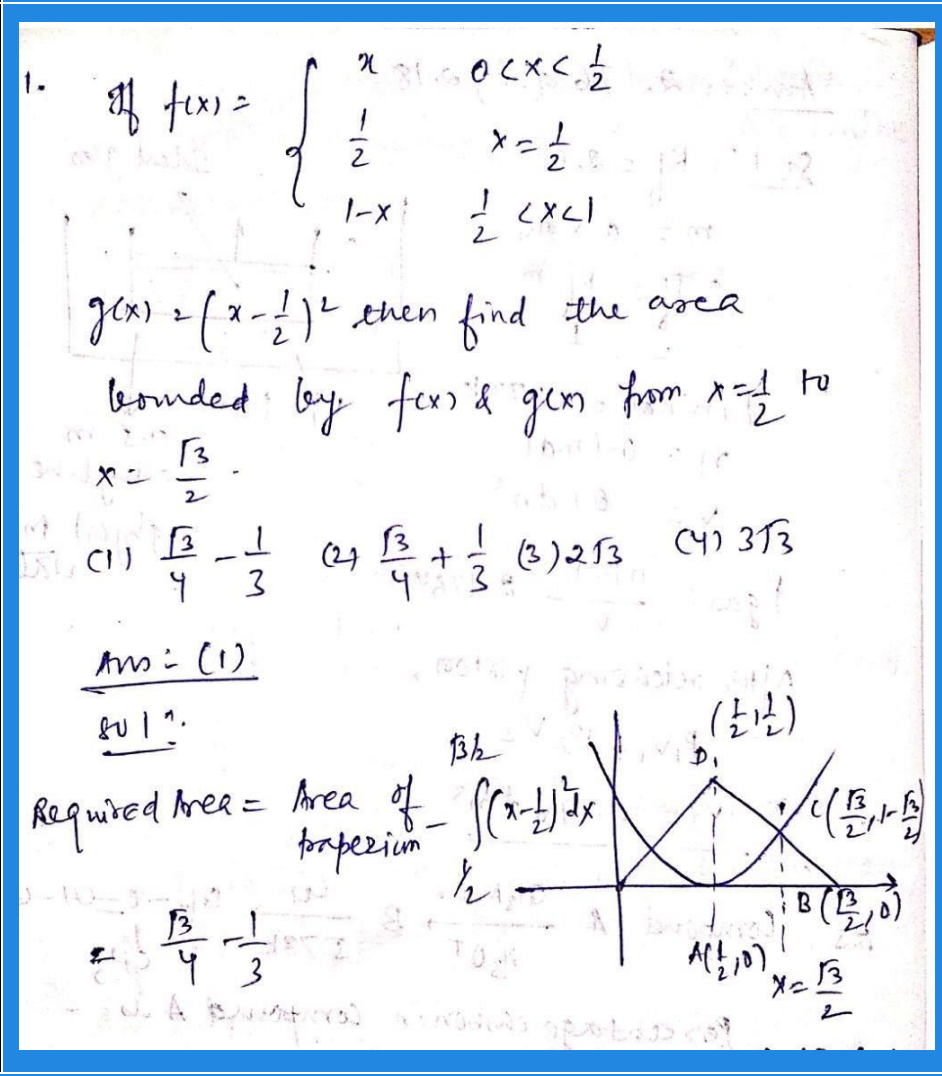


Jan 9 2020 shift 2 Mathematics prepared by KUNDUZ tutors

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Question no.	Questions and Answers
1	

1. If $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$ then find the area bounded by $f(x)$ & $g(x)$ from $x = \frac{1}{2}$ to $x = \frac{\sqrt{3}}{2}$.

(1) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (2) $\frac{\sqrt{3}}{4} + \frac{1}{3}$ (3) $2\sqrt{3}$ (4) $3\sqrt{3}$

Ans : (1)

Sol^n:

Required Area = Area of trapezium $- \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$

$= \frac{\sqrt{3}}{4} - \frac{1}{3}$

2

2. z is the complex no. such that $|Re(z)| + |Im(z)| = 4$ then $|z|$ can't be.

(1) $\sqrt{7}$ (2) $\sqrt{10}$ (3) $\sqrt{\frac{17}{2}}$ (4) $\sqrt{8}$

Ans: (1)

Soln: $z = x + iy$

$|x| + |y| = 4$

Minimum value of $|z| = 2\sqrt{2}$

Maximum value of $|z| = 4$

$|z| \in [\sqrt{8}, \sqrt{16}]$

So z can't be $\sqrt{7}$.

3

3. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ and $a - 2b + c = 1$ then

(1) $f(50) = 1$ (2) $f(-50) = -1$

(3) $f(50) = 501$ (4) $f(50) = -501$

Ans: (1)

Soln: Apply $R_1 = R_1 + R_3 - 2R_2$

$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix} \Rightarrow f(x) = 1 \Rightarrow f(50)$

4

4. Let a_n is a positive term of a GP and $\sum_{n=1}^{100} a_{2n+1} = 200$, $\sum_{n=1}^{100} a_{2n} = 100$

Then find $\sum_{n=1}^{200} a_n$

(1) 300 (2) 150 (3) 175 (4) 225

Ans: (2)

Sol n: Let GP is a, ar, ar^3

$$\sum_{n=1}^{100} a_{2n+1} = \frac{ar^2(r^{200}-1)}{(r^2-1)} = 200 \quad \text{---} ①$$

$$\sum_{n=1}^{100} a_{2n} = \frac{ar(r^{200}-1)}{(r^2-1)} = 100 \quad \text{---} ②$$

from ① & ② $\Rightarrow r = 2$,

$a_2 + a_3 + \cdots a_{201} = 300 \Rightarrow r(a_1 + \cdots a_{200}) = 300$

$\sum_{r=1}^{200} a_n = \frac{300}{r} = 150$

5

If $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, $y(1)=1$ and $y(x)=e$, then $x=$?

(1) $\frac{\sqrt{3}}{2}e$ (2) $\sqrt{3}e$ (3) $\sqrt{2}e$ (4) $\frac{e}{\sqrt{2}}$

Ans: (2)

Soln: Put $y=vx$,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx^2}{x^2+v^2x^2}$$

$$\int\left(\frac{1}{v^3}+\frac{1}{v}\right)dv = \int\frac{-1}{x}dx$$

$$-\frac{1}{2}\frac{1}{v^2} + \ln v = -\ln x + c$$

$$\frac{-x^2}{2y^2} = -\ln y + c$$

When $x=1$, $y=1 \Rightarrow c = -1/2$

$$x^2 = y^2(1+2\ln y)$$

$$x^2 = e^2 3$$

$$\boxed{x = \pm\sqrt{3}e}$$

$\therefore x = \sqrt{3}e$

6

6. Let probability distribution be.

x_i	1	2	3	4	5
P_i	k^2	$2k$	k	$2k$	$5k^2$

then the value of $P(X>2)$ is

(1) $\frac{7}{12}$ (2) $\frac{1}{36}$ (3) $\frac{1}{6}$ (4) $\frac{23}{36}$

Ans: (4)

Sol: $\Sigma P_i = 1 \Rightarrow 6k^2+5k=1$

$6k^2+5k-1=0$

$6k^2+6k-k-1=0$

$\therefore k=-1 \otimes k=1/6$

Rejected

$P(X>2) = k+2k+5k^2 = \frac{6+12+5}{36} = \frac{23}{36}$.

7

7. $\int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)} = \lambda \tan\theta + 2\log f(x) + C$

$(\lambda, f(x))$ is ——

(1) $(1, 1+\tan\theta)$ (2) $(1, 1-\tan\theta)$

(3) $(-1, 1+\tan\theta)$ (4) $(-1, 1-\tan\theta)$

Sol: $\int \frac{\sec^2\theta}{\sec 2\theta + \tan 2\theta} d\theta$

$= \int \frac{\sec^2\theta(1-\tan\theta)}{(1+\tan\theta)} d\theta$

$\tan\theta = t \quad \sec^2\theta\, d\theta = dt$

$= \int \frac{1-t}{1+t} dt = \int -1 + \frac{2}{1+t} dt$

$= -t + 2\log(1+t) + C$

$\lambda = -1 \quad f(x) = 1+\tan\theta$

8

8. If $p \to (p \wedge \sim q)$ is false, Truth value value of p & q will be_

(1) TT (2) TF (3) FT (4) FF

Ans : (1)

Soln:

p	q	~q	$p \wedge \sim q$	$p \to (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

9

Q. If $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$ then the value of x at which $f(x) = [x^2]\sin \pi x$ is discontinuous -

(1) $\sqrt{A+1}$ (2) $\sqrt{A+2}$ (3) $\sqrt{A}$ (4) $\sqrt{A+5}$

Ans: (1)

Sol":

$$\lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A$$

$$\lim 4 - x\left\{\frac{4}{x}\right\} = A \Rightarrow 4-0=A$$

check when,

(1) $x = \sqrt{A+1} \Rightarrow x = \sqrt{5}$ discontinuous

(2) $x = \sqrt{A+2} \Rightarrow x = 5$ discontinuous

(3) $x = \sqrt{A}$ $x = 2$ cont'nous

(4) $x = \sqrt{A+5}$ $x = 3$ continuous

10

10. Let one end of focal chord of parabola $y^2 = 8x$ is $\left(\frac{1}{2}, -2\right)$, then equation of tangent at other end of this focal chord is

(1) $x + 2y + 8 = 0$ (2) $x + 2y = 8$

(3) $x - 2y = 8$ (4) $x - 2y + 8 = 0$

Ans : 4

Sol^n : Let $\left(\frac{1}{2}, -2\right) = (2t^2, 4t) \Rightarrow t = \frac{-1}{2}$

Parameter of other end of chord is 2.

$\Rightarrow$ point is $(8, 8)$

Equation of tangent $8y - 4(x + 8) = 0$

$\boxed{2y - x = 8}$

11

11. Let $x+6y=8$ is tangent to standard ellipse whose minor axis is $\frac{4}{\sqrt{3}}$, then eccentricity of ellipse is

(1) $\sqrt{\frac{5}{6}}$ (2) $\sqrt{\frac{11}{12}}$ (3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{4}\sqrt{\frac{11}{12}}$

Ans: (2)

Soln: $2b=\frac{4}{\sqrt{3}} \Rightarrow b=\frac{2}{\sqrt{3}}$

Eqn of tangent $\Rightarrow y=mx\pm\sqrt{a^2m^2+b^2}$

comparing $\Rightarrow y=\frac{-x}{6}+\frac{4}{3}$

$m=\frac{-1}{6}$ $a^2m^2+b^2=\frac{16}{9}$

$\therefore \boxed{a^2=16}$

$\therefore e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{4}{3\times 16}}=\sqrt{\frac{11}{12}}$

12

12. If $f(x)$ and $g(x)$ are continuous functions, fog is identity function, $g'(b)=5$ & $g(b)=q$ then $f(a)$ is

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{5}$ (4) 5

Ans : (2)

Soln: $f(g(x)) = x$

$f'(g(x))\, g'(x) = 1$

Put $x=b$

$f'(g(b)) \cdot g'(b) = 1$

$f'(a) = \frac{1}{5}$

13

13. If $7x+6y-2z=0$
$3x+4y+2z=0$
$x-2y-6z=0$ then which option is correct.

(1) No solution (2) only trivial solution
(3) Infinite solution of non-trivial type for $x=2z$
(4) Infinite non-trivial solution for $y=2z$

Ans = (3)

Soln: $\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$

$= -140+120+20 = 0$

so infinite non-trivial solution exists

Now eqn (1) + 3(eqn (3))

$10x-20z=0$ $\boxed{x=2z}$

14

14: Let $x = 2\sin\theta + \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$ find $\frac{d^2y}{dx^2}$ at $\theta = \pi$.

① $\frac{3}{8}$ ② $\frac{3}{2}$ ③ $\frac{5}{8}$ ④ $\frac{7}{8}$

Soln: $\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$

$\frac{dy}{dx} = \frac{\cos\theta - \cos 2\theta}{-\sin\theta + \sin 2\theta} = \cot\frac{3\theta}{2}$

$\frac{d^2y}{dx^2} = \frac{-3}{2}\operatorname{cosec}^2\frac{3\theta}{2}\frac{d\theta}{dx}$.

$\frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\operatorname{cosec}^2\frac{3\theta}{2}}{2(\cos\theta - \cos 2\theta)}$

$\left(\frac{d^2y}{dx^2}\right)_{\theta=\pi} = \frac{-3}{4(-1-1)} = \frac{3}{8}$

15

Ques. $f(x) : [0,5] \to R$, $F(x) = \int_0^x x^2 g(x)$, $f(1) = 3$

$g(x) = \int_1^x f(t)\,dt$ then correct choice is.

(1) F(x) has local minimum at $x=1$

(2) F(x) has local maximum at $x=1$

(3) F(x) has point of inflection at $x=1$

(4) f(x) has no critical point.

Ans : (1)

Sol^n : $F'(x) = x^2 g(x)$

$\Rightarrow F'(1) = 1 \cdot g(1) = 0$ ... (1)

Now, $F''(x) = 2x\,g(x) + x^2 g'(x)$.

$F''(x) = 2x\,g(x) + x^2 f(x)$

$F''(1) = 0 + 1 \times 3 = 3$ ..... (2) $\Rightarrow$ local minima at $x=1$

16

16. Let both roots of equation $ax^2-2bx+5=0$ are root of equation $x^2=2bx-10$ are α & β. find the value of $\alpha^2+\beta^2$=

(1) 20 (2) 25 (3) 15 (4) 30

Ans: (2)

Solⁿ: $2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$ & $\alpha^2 = \frac{5}{4}$

$\frac{b^2}{a^2} = \frac{5}{a}$

$\boxed{b^2 = 5a}$ ... (i)

$\alpha+\beta = 2b$ ——— (ii)

and $\alpha\beta = -10$ ... (iii)

$\alpha = \frac{b}{a}$ is also root of $x^2 = 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

by (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$

$20a^2 = 5a$

$a = \frac{1}{4}$ & $b = \frac{5}{4}$.

$\alpha^2 = 20$ & $\beta^2 = 5$

$\therefore \alpha^2+\beta^2 = 25$.

17

17. Let $A = \{x : |x| < 2\}$ and $B = \{x : |x-2| \geq 3\}$ then

(1) $A \cap B = [-2, -1]$ (2) $A \cup B = R - (2, 5)$

(3) $A - B = [-1, 2)$ (4) $A \cup B - A = R - (-2, 5)$

Ans: 4:

Sol^n: $A = \{x \mid x \in (-2, 2)\}$

$B = \{x \mid x \in (-\infty, -1] \cup [5, \infty)\}$.

$\therefore B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

18

18. Let $x = \sum_{n=0}^{\infty} (-1)^n (\tan\theta)^{2n}$ and

$y = \sum_{n=0}^{\infty} (\cos\theta)^{2n}$ where $\theta \in (0, \frac{\pi}{4})$

then,

(1) $x(y+1) = 1$ (2) $y(1-x) = 1$ (3) $y(x-1) = 1$

(4) $y(1+x) = 1$

Ans: (2)

Sol^n: $y = 1 + \cos^2\theta + \cos^4\theta + \cdots$

$y = \frac{1}{1-\cos^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$

$x = \frac{1}{1-(-\tan^2\theta)} = \frac{1}{\sec^2\theta}$

$\cos^2\theta = x$.

$\frac{1}{y} + x = 1$

$\boxed{y(1-x) = 1}$

19

Sol: Line must be intersecting

$(2s-1, 3s+3, 8s-1) = (2t-3, t-2, \lambda t+1)$

$\therefore t=-1, s=-2, \lambda=18$

Required distance equal to last point $(-3,-2,1)$

$$= \frac{-69+20-2+48}{\sqrt{529+100+4}}$$

$$= \frac{3}{\sqrt{633}} \Rightarrow \boxed{K=3}$$

19. Let the distance between the plane passing throng lines $\frac{x+1}{2} = \frac{y-3}{2} = \frac{z+1}{8} = s$ &

$\frac{x+3}{2} = \frac{y+2}{-1} = \frac{z-1}{\lambda}$ and plane $23x-10y-2z+48=0$ is $\frac{K}{\sqrt{633}}$ then find K.

(1) 1 (2) 2 (3) 3 (4) 4.

20

20. If ${}^{25}C_0 + 5\,{}^{25}C_1 + 9\,{}^{25}C_2 + \dots + 101\,{}^{25}C_{25} = 2^{25}k$

find k?

Ans: 51

Soln: $\sum_{r=0}^{25}(4r+1)\,{}^{25}C_r = 4\sum_{r=0}^{25} r\,{}^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r$

$= 4\sum_{r=1}^{25} r\,\frac{25}{r} \times {}^{24}C_{r-1} + 2^{25}$

$= 100 \times \sum_{r=1}^{25} {}^{24}C_{r-1} + 2^{25}$

$= 100 \times 2^{24} + 2^{25} = 51 \times 2^{25}$

$\boxed{k = 51}$

21

Ans: 36.

Soln: Two circles touch, $C_1C_2 = |r_1 \pm r_2|$

Distance between $C_2(3,0)$ and $C_1(0,4)$

is either $\sqrt{k}+1$ or $|\sqrt{k}-1|$ ... as $C_1C_2 = 5$

$\Rightarrow \sqrt{k}+1 = 5$ or $|\sqrt{k}-1| = 5$

$\therefore \boxed{k=16}$ or $\boxed{k=36}$

maximum value of k is 36.

21. Let circles $(x-0)^2+(y-4)^2=k$ and $(x-3)^2+(y-0)^2=1^2$ touches each other then find the maximum value of 'k'.

22

22. Let $|\bar{a}|=\sqrt{3}$, $|\bar{b}|=5$, $\bar{b}\cdot\bar{c}=10$ angle between $\bar{b}$ and $\bar{c}$ equal to $\frac{\pi}{3}$. If $\bar{a}$ is perpendicular to $\bar{b}\times\bar{c}$ then find the value of $|\bar{a}\times(\bar{b}\times\bar{c})|$

Ans: 30

Sol:- $\bar{b}\cdot\bar{c}=10 \Rightarrow |\bar{b}||\bar{c}|\cdot\cos\frac{\pi}{3}=10$

$5\cdot|\bar{c}|\cdot\frac{1}{2}=10 \Rightarrow \boxed{|\bar{c}|=4}$

Also, $\bar{a}\cdot(\bar{b}\times\bar{c})=0$

$|\bar{a}\times(\bar{b}\times\bar{c})|=|\bar{a}|\,|\bar{b}\times\bar{c}|\sin\frac{\pi}{2}$.

$\sqrt{3}\cdot|\bar{b}||\bar{c}|\sin\frac{\pi}{3}=\sqrt{3}\times5\times4\times\frac{\sqrt{3}}{2}=30$

23

23. Number of common terms in both sequence $3,7,11,\ldots 407$ and $8,9,16,\ldots 905$ is

Ans: 14.

Sol^n: First common term $=23$

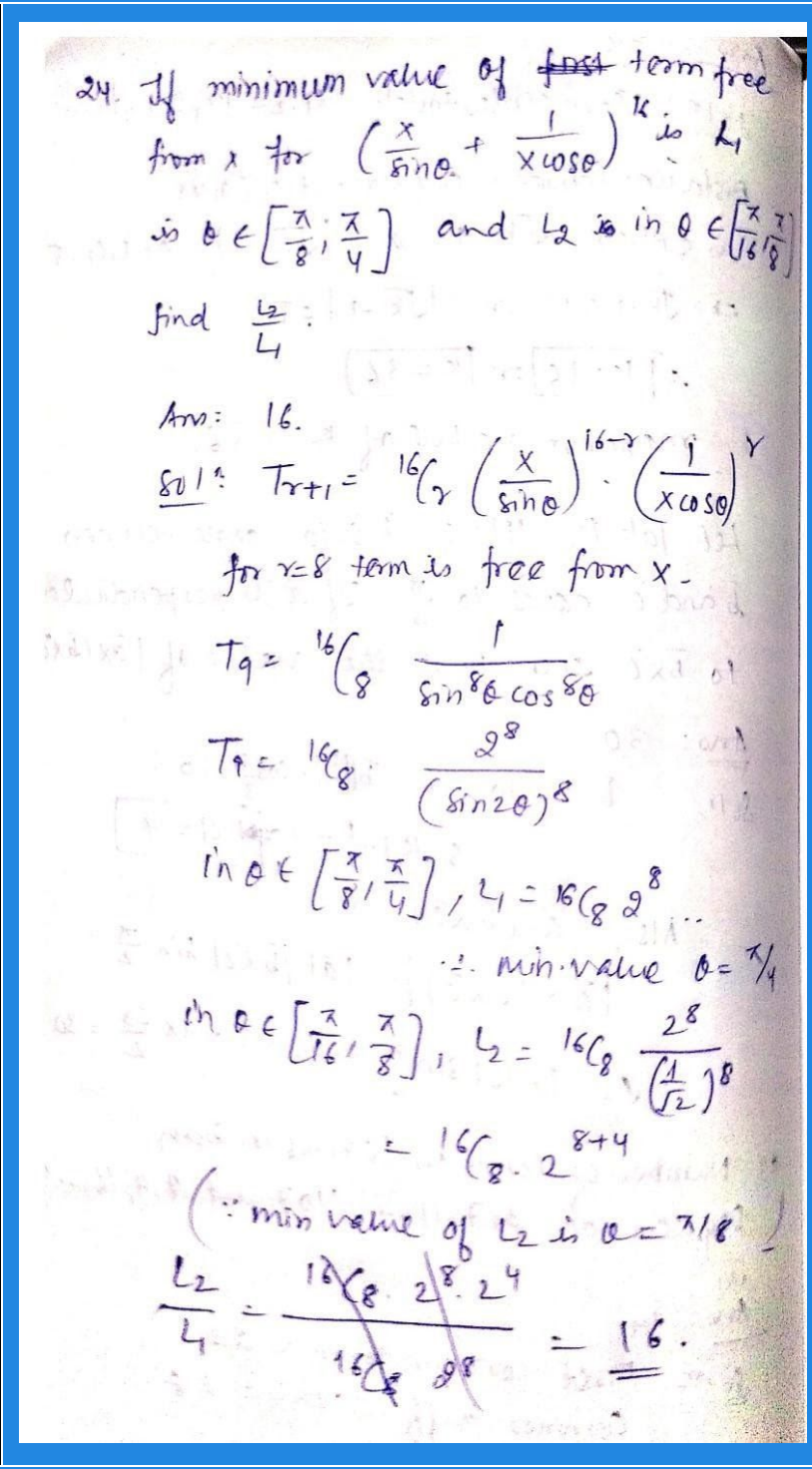
Common diff $=7\times4=28$

last term ≤ 407

$23+(n-1)\times28\le407$

$n\le14.71 \Rightarrow \boxed{n=14}$

24



24. If minimum value of ~~[illegible]~~ term free from x for $\left(\frac{x}{\sin\theta} + \frac{1}{x\cos\theta}\right)^{16}$ is L_1 is $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$ and L_2 is in $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$

find $\frac{L_2}{L_1}$:

Ans: 16.

Sol: $T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta}\right)^{16-r} \left(\frac{1}{x\cos\theta}\right)^{r}$

for $r=8$ term is free from x.

$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta \cos^8\theta}$

$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$

In $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$, $L_1 = {}^{16}C_8 2^8$

$\therefore$ min. value $\theta = \pi/4$

In $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, $L_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8}$

$= {}^{16}C_8 \cdot 2^{8+4}$

($\because$ min value of L_2 is $\theta = \pi/8$)

$\frac{L_2}{L_1} = \frac{{}^{16}C_8 \cdot 2^8 \cdot 2^4}{{}^{16}C_8 \cdot 2^8} = 16.$

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