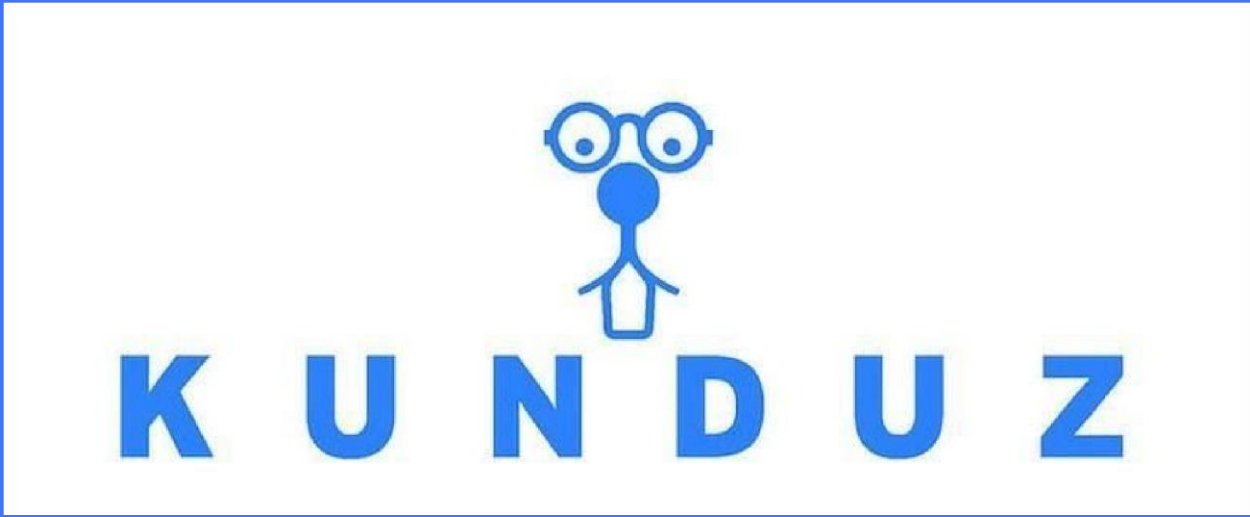


JEE Advanced 2020 Mathematics paper 1 solutions prepared by KUNDUZ tutors

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Question number	Question and Answers
1	

Q.1 Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of

$$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$$

is

(A) 0 (B) 8000 (C) 8080 (D) 16000

Q.1 a, b are roots of $x^2+20x-2020$

$\Rightarrow a+b = -20$ and $ab = -2020$

$(a+b)^2 = a^2+b^2+2ab$

$\Rightarrow a^2+b^2 = 400 + 4040 = 4440$

c, d are roots of $x^2-20x+2020$

$\Rightarrow c+d = 20$ and $cd = 2020$

$(c+d)^2 = c^2+d^2+2cd$

$\Rightarrow c^2+d^2 = 400 - 4040 = (-3640)$

$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$

$a(ac-c^2+ad-d^2) + b(bc-c^2+bd-d^2)$

$= a^2(c+d) - a(c^2+d^2) + b^2(c+d) - b(c^2+d^2)$

$= (a^2+b^2)(c+d) - (a+b)(c^2+d^2)$

$= (4440)(20) + 20(-3640)$

$= 20(800) = 16000$ option D

2

Q.2 If the function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=|x|(x-\sin x)$, then which of the following statements is **TRUE**?

(A) f is one-one, but **NOT** onto

(B) f is onto, but **NOT** one-one

(C) f is **BOTH** one-one and onto

(D) f is **NEITHER** one-one **NOR** onto

Q.2. $f(x)=\begin{cases} x(x-\sin x), & x\geq 0 \\ -x(x-\sin x), & x<0 \end{cases}$

$f(-x)=-x(-x+\sin x)=-(-x)(x-\sin x)$

$\Rightarrow f(-x)=-f(x)$. → odd function

$\boxed{f'(-x)=f'(x)}$ — ①

for one-one. $f(x)=x^2-x\sin x$, $x\geq 0$

$\Rightarrow f'(x)=2x-\sin x-x\cos x$

$=\underbrace{(x-\sin x)}_{\geq 0}+\underbrace{x(1-\cos x)}_{\geq 0}$.

So, $f'(x)\geq 0$ ⇒ from ①, $f'(-x)>0$

So, $f(x)$ is always increasing.

⇒ one-one function

$\lim_{x\to\infty} f(x)=\lim_{x\to\infty} x^2-x\sin x \longrightarrow \infty$ (infinity)

↓ always lies b/w $[-1,1]$

So, $\lim_{x\to-\infty} f(x)\longrightarrow(-\infty)$

⇒ Range of $f(x)=(-\infty,\infty)$

So, Range is co-domain onto

Therefore, $f(x)$ is both one-one and onto.

Option C

3

Q.3 Let the functions $f:\mathbb{R}\longrightarrow\mathbb{R}$ and $g:\mathbb{R}\longrightarrow\mathbb{R}$ be defined by

$$f(x)=e^{x-1}-e^{-|x-1|} \quad \text{and} \quad g(x)=\frac{1}{2}(e^{x-1}+e^{1-x}).$$

Then the area of the region in the first quadrant bounded by the curves $y=f(x)$, $y=g(x)$ and $x=0$ is

(A) $(2-\sqrt{3})+\frac{1}{2}(e-e^{-1})$ (B) $(2+\sqrt{3})+\frac{1}{2}(e-e^{-1})$

(C) $(2-\sqrt{3})+\frac{1}{2}(e+e^{-1})$ (D) $(2+\sqrt{3})+\frac{1}{2}(e+e^{-1})$

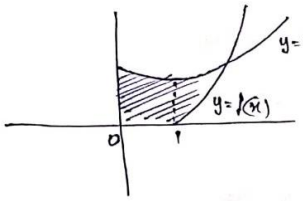
Q.3 $f(x)=e^{x-1}-e^{-|x-1|}$

$x-1<0 \Rightarrow f(x)=e^{x-1}-e^{x-1}=0$

$x-1\geq 0 \Rightarrow f(x)=e^{x-1}-e^{1-x}$

$g(x)=\frac{1}{2}(e^{x-1}+e^{1-x}) \Rightarrow g'(x)=\frac{1}{2}(e^{x-1}-e^{1-x})$

$\Rightarrow g'(x)>0$ for $x>1 \Rightarrow g(x)$ increases for $x>1$



$f(x)$ and $g(x)$ meets,

$\Rightarrow e^{x-1}-e^{1-x}=\frac{1}{2}(e^{x-1}+e^{1-x})$

$\Rightarrow e^{x-1}=3e^{1-x}$

Now, apply ln on both sides

$\Rightarrow \ln e^{x-1}=\ln 3+\ln e^{1-x}$

$\Rightarrow x-1=\ln 3+1-x \Rightarrow x=1+\frac{1}{2}\ln 3$

$$\text{Area}=\int_0^{1+\ln\sqrt{3}} g(x)\,dx-\int_1^{1+\ln\sqrt{3}} f(x)\,dx$$

$$=\int_0^{1+\ln\sqrt{3}} \frac{1}{2}(e^{x-1}+e^{1-x})\,dx-\int_1^{1+\ln\sqrt{3}}(e^{x-1}-e^{1-x})\,dx$$

$$=\frac{1}{2}\left[e^{x-1}-e^{1-x}\right]_0^{1+\ln\sqrt{3}}-\left[e^{x-1}+e^{1-x}\right]_1^{1+\ln\sqrt{3}}$$

$$=\frac{1}{2}\left(\sqrt{3}-\frac{1}{\sqrt{3}}-\frac{1}{e}+e\right)-\left(\sqrt{3}+\frac{1}{\sqrt{3}}-2\right)$$

$$=(2-\sqrt{3})+\frac{1}{2}(e-e^{-1})$$

Option A

4

Q.4 Let a, b and λ be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\lambda x$, and suppose the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the point P. If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$

Q.4 P is an end point of latus rectum.

$y^2 = 4\lambda x \Rightarrow P \equiv (\lambda, \pm 2\lambda)$

This point lies on the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{\lambda^2}{a^2} + \frac{4\lambda^2}{b^2} = 1 \Rightarrow \lambda^2 = \frac{1}{\left(\frac{1}{a^2} + \frac{4}{b^2}\right)}$

* $y^2 = 4\lambda x \Rightarrow 2y\frac{dy}{dx} = 4\lambda$

$\Rightarrow \left.\frac{dy}{dx}\right|_P = \pm 1 \rightarrow m_1$

* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} + \frac{2y\frac{dy}{dx}}{b^2} = 0$

$\Rightarrow \left.\frac{dy}{dx}\right|_P = -\frac{b^2}{a^2}\left(\frac{x}{y}\right) \rightarrow m_2$

$m_1 \times m_2 = -1$ (condition of perpendicularity)

$\Rightarrow \pm 1 \times +\frac{b^2}{a^2} \cdot \left(\pm\frac{1}{2}\right) = +1$

$\Rightarrow b^2 = 2a^2$

$e^2 = \frac{b^2 - a^2}{b^2} = \frac{a^2}{2a^2} \Rightarrow e = \frac{1}{\sqrt{2}}$ Option A

5

Q.5 Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently. Then the probability that the roots of the quadratic polynomial $x^2 - \alpha x + \beta$ are real and equal, is

(A) $\frac{40}{81}$ (B) $\frac{20}{81}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.5 $x^2 - \alpha x + \beta = 0$ should have real & equal.

$\Rightarrow b^2 - 4ac = 0$

$\alpha^2 - 4\beta = 0$

$\alpha^2 = 4\beta$

$\Rightarrow \alpha = 0, \beta = 0$ and $\alpha = 2, \beta = 1$ are the possible cases.

i) $\alpha = 0$ and $\beta = 0$

C_1 T T $\quad$ C_2 T T $\quad \Rightarrow P_1 = \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}$

$= \frac{4}{81}$

ii) $\alpha = 2$ and $\beta = 1$

C_1 H H $\quad$ C_2 T H or H T

$\frac{2}{3} \times \frac{2}{3} \quad \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3}$

$\Rightarrow P_2 = \frac{4}{9}\left(\frac{2}{9} + \frac{2}{9}\right) = \frac{16}{81}$

$\Rightarrow$ Total probability $= P_1 + P_2$

$= \frac{20}{81}$ Option B

6

Q.6 Consider all rectangles lying in the region

$$\left\{(x, y) \in \mathbb{R} \times \mathbb{R} : 0 \leq x \leq \frac{\pi}{2} \text{ and } 0 \leq y \leq 2\sin(2x)\right\}$$

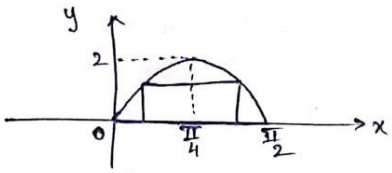
and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is

(A) $\frac{3\pi}{2}$ (B) π (C) $\frac{\pi}{2\sqrt{3}}$ (D) $\frac{\pi\sqrt{3}}{2}$

Q.6 $0 \le x \le \frac{\pi}{2}$

$\Rightarrow 0 \le 2x \le \pi.$

$0 \le \sin(2x) \le 1 \Rightarrow 0 \le y \le 2$



Given region is the region under this curve.

Let the rectangle length be a & breadth b.

(x, b) $(x+a, b)$ $(x, 0)$ $(x+a, 0)$ → lies on $y = 2\sin 2x$

$\Rightarrow b = 2\sin 2x = 2\sin 2(x+a)$

$\Rightarrow 2x = \pi - (2(x+a))$

$\Rightarrow 2x = \frac{\pi}{2} - a$

$x = \frac{\pi}{4} - \frac{a}{2}$

$\therefore b = 2\sin\left(\frac{\pi}{2} - a\right)$

$b = 2\cos a$

Perimeter $= 2a + 2b = 2a + 4\cos a$

$\Rightarrow \frac{dp}{da} = 2 - 4\sin a \Rightarrow 2 - 4\sin a = 0$ (maximise)

$\sin a = \frac{1}{2}$

Area $= ab = \frac{\pi}{6} \cdot \left(2\cos\frac{\pi}{6}\right) = \frac{\pi}{2\sqrt{3}}$ Option C

7

Q.7 Let the function $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^3-x^2+(x-1)\sin x$ and let $g:\mathbb{R}\to\mathbb{R}$ be an arbitrary function. Let $fg:\mathbb{R}\to\mathbb{R}$ be the product function defined by $(fg)(x)=f(x)g(x)$. Then which of the following statements is/are TRUE?

(A) If g is continuous at $x=1$, then fg is differentiable at $x=1$

(B) If fg is differentiable at $x=1$, then g is continuous at $x=1$

(C) If g is differentiable at $x=1$, then fg is differentiable at $x=1$

(D) If fg is differentiable at $x=1$, then g is differentiable at $x=1$

Q.7. $f(x) = x^2(x-1) + \sin x(x-1)$.

$\Rightarrow \lim_{x\to 1} f(x) = 0$.

Let $fg(x) = h(x) = f(x)\,g(x)$.

$\Rightarrow f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{f(1+h)}{h}$

Now, $h'(1) = \lim_{h\to 0} \frac{f(1+h)\,g(1+h) - f(1)\,g(1)}{h}$

$= \lim_{h\to 0} \frac{f(1+h)}{h}\cdot g(1+h)$

$\Rightarrow \lim_{h\to 0^+} \frac{f(1+h)}{h}\cdot g(1+h) = f'(1)\,g(1^+)$

$\lim_{h\to 0^-} \frac{f(1+h)}{h}\cdot g(1+h) = f'(1)\,g(1^-)$ — both should be equal

A) If g is continuous, $g(1^+) = g(1^-) = g(1)$.

So, $h'(1)$ exist.

B) If $h'(1)$ exists $\Rightarrow g(1^+) = g(1^-)$

This may or may not be equal to $g(1)$.

∴ So, we cannot say anything about $g(x)$.

C) If g is differentiable, $g(1^+) = g(1^-) = g(1)$

So, $h'(1)$ exists.

So, **Only A, C** are correct

8

Q.8 Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statements is/are ALWAYS TRUE?

(A) $M = I$ (B) $\det M = 1$ (C) $M^2 = I$ (D) $(\text{adj } M)^2 = I$

Q.8 we know that,

$$A^{-1} = \frac{1}{|A|} \text{adj } A.$$

Given, $M^{-1} = \text{adj}(\text{adj } M)$

$$\Rightarrow M^{-1} = |\text{adj}(M)| (\text{adj } M)^{-1}$$

$$\left[\because \text{adj } A = |A| A^{-1}, \text{ taking } A = \text{adj } M \right.$$
$$\left. \text{adj}(\text{adj } M) = |\text{adj } M|(\text{adj } M)^{-1} \right]$$

$$\Rightarrow M^{-1} = |\text{adj } M| \cdot (|M| M^{-1})^{-1}$$

$$\Rightarrow M^{-1} = |M|^2 \left(\frac{M}{|M|}\right) \quad \left[\because |\text{adj } A| = |A|^{n-1}, \ n \text{ is order}\right]$$

$$\Rightarrow M^{-1} = |M| M$$

$$\boxed{|M| M^2 = I} \Rightarrow |M|^3 = 1$$

$$\Rightarrow |M| = 1 \text{ is the only real root}$$

$$\Rightarrow (M^{-1})^2 = |M|$$

$$\Rightarrow \left(\frac{1}{|M|} \text{adj } M\right)^2 = |M| \quad * \text{ So, } M^2 = I$$

$$\Rightarrow (\text{adj } M)^2 = 1 \quad (\because |M| = 1)$$

$\Rightarrow$ Options B, C, D are always true.

9

Q.9 Let S be the set of all complex numbers z satisfying $|z^2 + z + 1| = 1$. Then which of the following statements is/are TRUE?

(A) $\left|z + \frac{1}{2}\right| \leq \frac{1}{2}$ for all $z \in S$

(B) $|z| \leq 2$ for all $z \in S$

(C) $\left|z + \frac{1}{2}\right| \geq \frac{1}{2}$ for all $z \in S$

(D) The set S has exactly four elements

Q 9. $|z^2 + z + 1| = 1 \Rightarrow z^2 + z + 1 = e^{i\theta}$ — (1)

$\left|z^2 + \left(\frac{1}{2}+\frac{1}{2}\right).z + \frac{1}{4} + \frac{3}{4}\right| = 1$

$\left|\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}\right| = 1.$

we know that,

$|a|^2 - |b|^2 \leq |a^2 + b^2| \leq |a|^2 + |b|^2$

$\Rightarrow \left|z + \frac{1}{2}\right|^2 - \frac{3}{4} \leq 1 \leq \left|\left(z + \frac{1}{2}\right)^2\right| + \frac{3}{4}$

$\Rightarrow \frac{1}{4} \leq \left|z + \frac{1}{2}\right|^2 \leq \frac{7}{4}$

$\Rightarrow \frac{1}{2} \leq \left|z + \frac{1}{2}\right| \leq \frac{\sqrt{7}}{2}$ Option C

from (1), we can get many numbers of roots for different 'θ' values

So, set 'S' has infinite elements.

$|z^2+z|-1 \leq |(z^2 + z) + 1| = 1$

$\Rightarrow |z^2 + z| \leq 2$

Now, $|z|^2 - |z| \leq |z^2 + z| \leq 2.$

$\Rightarrow |z|^2 - |z| - 2 \leq 0$

$\Rightarrow (|z| - 2)(|z| + 1) \leq 0 \Rightarrow |z| \leq 2$

$\Rightarrow$ Options B, C are correct

10

Q.10 Let x, y and z be positive real numbers. Suppose x, y and z are the lengths of the sides of a triangle opposite to its angles X, Y and Z, respectively. If

$$\tan\frac{X}{2}+\tan\frac{Z}{2}=\frac{2y}{x+y+z},$$

then which of the following statements is/are TRUE?

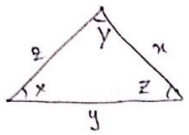
(A) $2Y = X + Z$

(B) $Y = X + Z$

(C) $\tan\frac{X}{2}=\frac{x}{y+z}$

(D) $x^2 + z^2 - y^2 = xz$

Q.10



$x+y+z = 2s$

From half angle formulas,

Given,

$\tan\frac{X}{2}+\tan\frac{Z}{2}=\frac{2y}{x+y+z}$

$\Rightarrow \frac{\Delta}{s(s-x)}+\frac{\Delta}{s(s-z)}=\frac{2y}{2s}$

$\Rightarrow \Delta\left(\frac{2s-x-z}{(s-x)(s-z)}\right)=y$

$\Rightarrow \frac{\Delta(y)}{(s-x)(s-z)}=y \Rightarrow \Delta=(s-x)(s-z)$

* $\tan\frac{X}{2}=\frac{\Delta}{s(s-x)}$

* $\tan\frac{Y}{2}=\frac{\Delta}{s(s-y)}$

* $\tan\frac{Z}{2}=\frac{\Delta}{s(s-z)}$

we know $\Delta=\sqrt{s(s-x)(s-y)(s-z)}=(s-x)(s-z)$

so, $\sqrt{s(s-y)}=\sqrt{(s-x)(s-z)}$ (square on both sides)

$\Rightarrow \frac{(x+z+y)(x+z-y)}{4}=\frac{(y+z-x)(y-z+x)}{4}$

$\Rightarrow (x+z)^2-y^2=y^2-(x-z)^2$

$\Rightarrow 2(x^2+z^2)=2y^2 \Rightarrow y^2=x^2+z^2$

(right angled Δle)

$\boxed{Y=90^\circ}$

& $\boxed{X+Z=90^\circ}$

* $\tan\frac{X}{2}=\frac{\Delta}{\frac{(x+y+z)(y+z-x)}{4}}$

$\Rightarrow \tan\frac{X}{2}=\frac{4\cdot\left(\frac{1}{2}\times z\right)}{(y+z)^2-x^2}=\frac{2xz}{2z^2+2yz}$

$\Rightarrow \tan\frac{X}{2}=\frac{x}{y+z}$

Option B, C

11

Q.11 Let L_1 and L_2 be the following straight lines.

$$L_1: \frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3} \quad \text{and} \quad L_2: \frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}.$$

Suppose the straight line

$$L: \frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE?

(A) $\alpha-\gamma=3$ (B) $l+m=2$ (C) $\alpha-\gamma=1$ (D) $l+m=0$

Q.11 $L_1: \frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3}$

$L_2: \frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}$

Both lines pass through $(1,0,1)$

Acute angle bisector cosines will be

$(1-3, -1-1, 3+1) \equiv (-2,-2,4)$

Eq, $\frac{x-1}{-1}=\frac{y}{-1}=\frac{z-1}{2}$

$L: \frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$

$\Rightarrow \frac{-1}{l}=\frac{-1}{m}=\frac{2}{-2}$

$\Rightarrow l=m=1$

L passes through $(1,0,1)$

$\Rightarrow \frac{1-\alpha}{1}=\frac{-1}{1}=\frac{1-\gamma}{-2} \Rightarrow \alpha=2$

& $\gamma=-1$

Now,

$\alpha-\gamma = 2+1=3$ Option A

$l+m = 1+1=2$ Option B

So, A, B are correct.

12

Q.12 Which of the following inequalities is/are TRUE?

(A) $\int_0^1 x\cos x\ dx \geq \frac{3}{8}$

(B) $\int_0^1 x\sin x\ dx \geq \frac{3}{10}$

(C) $\int_0^1 x^2\cos x\ dx \geq \frac{1}{2}$

(D) $\int_0^1 x^2\sin x\ dx \geq \frac{2}{9}$

Q.12 we can solve this type of question using expansions.

$\Rightarrow \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \geq 1 - \frac{x^2}{2!}$

and $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \geq x - \frac{x^3}{6}$

A) $\int_0^1 x\cos x\cdot dx \geq \int_0^1 x - \frac{x^3}{2}\cdot dx$

$\Rightarrow \int_0^1 x\cos x\cdot dx \geq \frac{1}{2} - \frac{1}{8} \geq \frac{4}{8}$.

B) $\int_0^1 x\sin x\ dx \geq \int_0^1 x^2 - \frac{x^4}{6}\cdot dx$

$\Rightarrow \int_0^1 x\sin x\ dx \geq \frac{1}{3} - \frac{1}{30} \geq \frac{3}{10}$

C) $\int_0^1 x^2\cos x \geq \int_0^1 x^2 - \frac{x^4}{2}\cdot dx$

$\Rightarrow \int_0^1 x^2\cos x \geq \frac{1}{3} - \frac{1}{10} \geq \frac{7}{30}$

D) $\int_0^1 x^2\sin x \geq \int_0^1 x^3 - \frac{x^5}{6}\cdot dx \geq \frac{1}{4} - \frac{1}{36}$

$\Rightarrow \int_0^1 x^2\sin x \geq \frac{2}{9}$

Now C) $\int_0^1 x^2\cos x\cdot dx < \int_0^1 x^2 dx < \frac{1}{3}$

lies b/w $\left[\frac{7}{30}, \frac{1}{3}\right)$

So, ABD are correct

13

Q.13 Let m be the minimum possible value of $\log_3(3^{y_1}+3^{y_2}+3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1+y_2+y_3=9$. Let M be the maximum possible value of $(\log_3 x_1+\log_3 x_2+\log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1+x_2+x_3=9$. Then the value of $\log_2(m^3)+\log_3(M^2)$ is _____

Q.13 $3^{y_1}, 3^{y_2}, 3^{y_3}$ [Given, $y_1+y_2+y_3=9$]

A.M $\geq$ G.M

$\Rightarrow \dfrac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq \left(3^{y_1+y_2+y_3}\right)^{1/3}$

apply $\log_3$ both sides

$\Rightarrow \log_3 \dfrac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq \log_3 3.(3^9)^{1/3}$

$\Rightarrow m = \log_3 3^4 = \underline{4} = 2^2$

for x_1, x_2, x_3 [Given, $x_1+x_2+x_3=9$]

Apply A.M $\geq$ G.M

$\Rightarrow \dfrac{x_1+x_2+x_3}{3} \geq (x_1x_2x_3)^{1/3}$

apply $\log_3$ both sides

$\Rightarrow \log_3 \dfrac{9}{3} \geq \dfrac{1}{3}\left(\log_3 x_1x_2x_3\right)$

$\Rightarrow (\log_3 x_1+\log_3 x_2+\log_3 x_3) \leq 3$

$\Rightarrow M=3$

Now, $\log_2 m^3+\log_3 M^2 = \log_2 2^6+\log_3 3^2$

$\Rightarrow \log_2 m^3+\log_3 M^2 = 6+2 = 8$ Ans

14

Q.14 Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, \ldots$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality

$$2(a_1 + a_2 + \cdots + a_n) = b_1 + b_2 + \cdots + b_n$$

holds for some positive integer n, is _____

Q.14 $a_1, a_2, a_3 \ldots \ldots a_n \rightarrow$ positive Integers

In A.p with $d = 2$; $a_n = a_1 + (n-1)d$

$\Rightarrow$ $\text{Sum}_1 = \frac{n}{2}(2a_1 + (n-1)2)$

$b_1, b_2, b_3 \ldots \ldots b_n \rightarrow$ positive Integers

In G.p with $r = 2$; $b_n = b_1 r^{n-1}$

$\Rightarrow$ $\text{Sum}_2 = \frac{b_1(r^n - 1)}{r-1} = b_1(2^n - 1)$

$a_1 = b_1 = c$ (Given)

$2(a_1 + a_2 + a_3 + \ldots a_n) = (b_1 + b_2 + b_3 + \ldots b_n)$

$\Rightarrow$ $2 \times \frac{n}{2}[2c + 2(n-1)] = c(2^n - 1)$

$\Rightarrow$ $2n(n-1) = c(2^n - 1 - 2n)$

$\Rightarrow$ $c = \frac{2n(n-1)}{2^n - 1 - 2n} \geq 1$

$\Rightarrow$ $2n^2 - 2n \geq 2^n - 1 - 2n$

$\Rightarrow$ $2^n \leq 2n^2 + 1$ (Check n values)

$\Rightarrow$ $n = 1, 2, 3, 4, 5, 6$ are the only possible values.

$c = 0, \frac{4}{-1}, \frac{12}{1}, \frac{24}{7}, \frac{40}{21}, \frac{60}{51}$

(0) (✗) (✓) (✗) (✗) (✗)

$\Rightarrow$ only one value is possible for c.

Ans. 1

15

Q.15 Let $f:[0,2] \to \mathbb{R}$ be the function defined by

$$f(x) = (3 - \sin(2\pi x))\sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right).$$

If $\alpha, \beta \in [0,2]$ are such that $\{x \in [0,2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is ____

Q.15 $f(x) = \left(3 - \sin 2\pi x\right)\sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$

$f(x) \geq 0.$

$\Rightarrow \left(3 - \sin\left[\frac{\pi}{2} + 2\left(\pi x - \frac{\pi}{4}\right)\right]\right)\sin\left(\pi x - \frac{\pi}{4}\right) \geq \sin\left(\pi + 3\left(\pi x - \frac{\pi}{4}\right)\right)$

$\Rightarrow \left[3 - \cos\left(2\left(\pi x - \frac{\pi}{4}\right)\right)\right]\sin\left(\pi x - \frac{\pi}{4}\right) \geq -\sin\left(3\left(\pi x - \frac{\pi}{4}\right)\right)$

$x \in [0,2] \Rightarrow \pi x \in [0, 2\pi]$

$\pi x - \frac{\pi}{4} \in \left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$

Let be $\theta \in \left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$

$\Rightarrow (3 - \cos 2\theta)\sin\theta \geq -\sin 3\theta$

$\Rightarrow 2\sin\theta + 2\sin^3\theta \geq 4\sin^3\theta - 3\sin\theta$

$\Rightarrow 5\sin\theta \geq 2\sin^3\theta$

$\Rightarrow \sin\theta\left(5 - 2\sin^2\theta\right) \geq 0.$

$0 \leq \sin^2\theta \leq 1 \Rightarrow 0 \leq 2\sin^2\theta \leq 2$

$\Rightarrow 3 \leq 5 - 2\sin^2\theta \leq 5$

So, $\sin\theta \geq 0$; $\theta \in [0, \pi]$

$x = \frac{\theta + \frac{\pi}{4}}{\pi} \Rightarrow x \in \left[\frac{1}{4}, \frac{5}{4}\right]$

$\beta = \frac{5}{4}$ and $\alpha = \frac{1}{4}$, $\boxed{\alpha - \beta = 1}$ Ans.

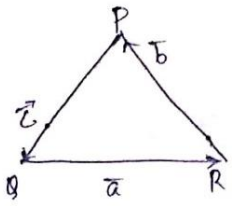
16

Q.16 In a triangle PQR, let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If

$$|\vec{a}| = 3, \quad |\vec{b}| = 4 \quad \text{and} \quad \frac{\vec{a}\cdot(\vec{c}-\vec{b})}{\vec{c}\cdot(\vec{a}-\vec{b})} = \frac{|\vec{a}|}{|\vec{a}|+|\vec{b}|},$$

then the value of $|\vec{a}\times\vec{b}|^2$ is _____

Q.16



Since this is a closed figure,

$\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{c} = -\vec{a}-\vec{b}$

$|\vec{a}| = 3, \quad |\vec{b}| = 4$

$$\frac{\vec{a}\cdot(\vec{c}-\vec{b})}{\vec{c}\cdot(\vec{a}-\vec{b})} = \frac{\vec{a}\cdot(-2\vec{b}-\vec{a})}{(-\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})} = \frac{3}{3+4}$$

$$\Rightarrow \frac{2\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{a}}{\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{b}} = \frac{3}{7}$$

$\because \vec{a}\cdot\vec{a} = |\vec{a}|^2 = 9$

$\vec{b}\cdot\vec{b} = |\vec{b}| = 16$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\Rightarrow \frac{2\vec{a}\cdot\vec{b}+9}{9-16} = \frac{3}{7}$$

$\Rightarrow 2(12\cos\theta)+9 = -3$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$|\vec{a}\times\vec{b}|^2 = |a|^2|b|^2\sin^2\theta = 9\times16\times\frac{3}{4}$

$\Rightarrow |\vec{a}\times\vec{b}|^2 = 108$ Ans

17

Q.17 For a polynomial $g(x)$ with real coefficients, let m_g denote the number of distinct real roots of $g(x)$. Suppose S is the set of polynomials with real coefficients defined by

$$S = \{(x^2-1)^2(a_0 + a_1x + a_2x^2 + a_3x^3) : a_0, a_1, a_2, a_3 \in \mathbb{R}\}.$$

For a polynomial f, let f' and f'' denote its first and second order derivatives, respectively. Then the minimum possible value of $(m_{f'} + m_{f''})$, where $f \in S$, is _____

Q.17

$f(x) = (x^2-1)^2 (a_0 + a_1x + a_2x^2 + a_3x^3)$

$f(x) = 0$ at $x = 1, -1$.

Now, $x=1$ and -1 are repeating roots, so

$f'(1)$ and $f'(-1) = 0$.

By Rolle's theorem, there will be $x=c$

where $f'(c) = 0$ and $c \in [-1, 1]$

So, $f'(x)$ has 3 roots. → (minimum)

Again by Rolles theorem,

$f''(k) = 0$ for $k \in [-1, c]$

and $f''(h) = 0$ for $h \in [c, 1]$

So, $f''(x)$ has 2 roots. → (minimum)

$\Rightarrow m_{f'} + m_{f''} = 3 + 2 = \boxed{5}$ Ans.

18

Q.18 Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit

$$\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$$

is equal to a nonzero real number, is _____

Q. 18 $\lim_{x\to 0^+} \dfrac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$

as $x \to 0^+$, $(1-x)^{1/x} \to 1^\infty$

can be written as $e^{\frac{x-x-1}{x}} = e^{-1}$

So, it is in the form $\frac{0}{0}$.

apply l'hopitals rule.

let $y = (1-x)^{\frac{1}{x}}$; $\ln y = \frac{1}{x}\ln(1-x)$

$$\Rightarrow \frac{dy/dx}{y} = -\frac{\ln(1-x)}{x^2} + \left(\frac{-1}{x(1-x)}\right)$$

$$= \lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}}\left[-\frac{\ln(1-x)}{x^2} + \left(\frac{-1}{x(1-x)}\right)\right]}{a\cdot x^{a-1}}$$

$$= \lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}}\left(-\frac{\ln(1-x)}{x} - \frac{1}{1-x}\right)}{a\,x^a}$$

$\ln(1-x) = x - \frac{x^2}{2} - \frac{x^3}{3} - - - -$

$\Rightarrow -\frac{\ln(1-x)}{x} = 1 + \frac{x}{2} + \frac{x^2}{3} - - -$

$$= \lim_{x\to 0^+} (1-x)^{\frac{1}{x}} \left(\frac{-x}{1-x} + \frac{x}{2} + \frac{x^2}{3} + \dots\right) \Big/ a\,x^a.$$

$(1-x)^{\frac{1}{x}} \to e^{-1}$

So, $\boxed{a=1}$ to get a non-zero number.

Ans.

(limit $= -\frac{1}{2e}$)

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Other way.

Q.18 $$\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$$

write $1-x$ as $e^{\ln(1-x)}$

$$= \lim_{x\to 0^+} \frac{e^{\frac{\ln(1-x)}{x}} - e^{-1}}{x^a}$$

we know that, $\lim_{x\to 0^+} \frac{\ln(1-x)}{x} = -1$

so,

$$= \frac{1}{e} \lim_{x\to 0^+} \left[\frac{e^{1+\frac{\ln(1-x)}{x}} - 1}{1+\frac{\ln(1-x)}{x}}\right] \times \left(\frac{1+\frac{\ln(1-x)}{x}}{x^a}\right)$$

$$= \frac{1}{e} \underbrace{\lim_{k\to 0} \left(\frac{e^k - 1}{k}\right)}_{=1} \cdot \lim_{x\to 0^+} \left(\frac{1+\frac{\ln(1-x)}{x}}{x^a}\right)$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$$

$$\Rightarrow \frac{\ln(1-x)}{x} = -1 - \left(\frac{x}{2}\right) - \frac{x^2}{3} - \cdots$$

$$= \frac{1}{e} \lim_{x\to 0^+} \left(\frac{-x - \frac{x}{2} - \frac{x^2}{3} - \cdots}{x^a}\right)$$

$$= \frac{1}{e} \lim_{x\to 0^+} \left(\frac{-\frac{1}{2} - \frac{x}{3} - \cdots}{x^{a-1}}\right)$$

So, when $\boxed{a=1}$, limit $= \frac{-1}{2e}$ (non-zero)

Ans.

Enrolling yourself for coaching classes has long been considered the first step towards cracking JEE. However, this trend is starting to slip. Recent studies show that, for the students who cleared the JEE Main/Advance, the number of students who had joined a coaching class is lower than the number of students who managed the feat with self-study.

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