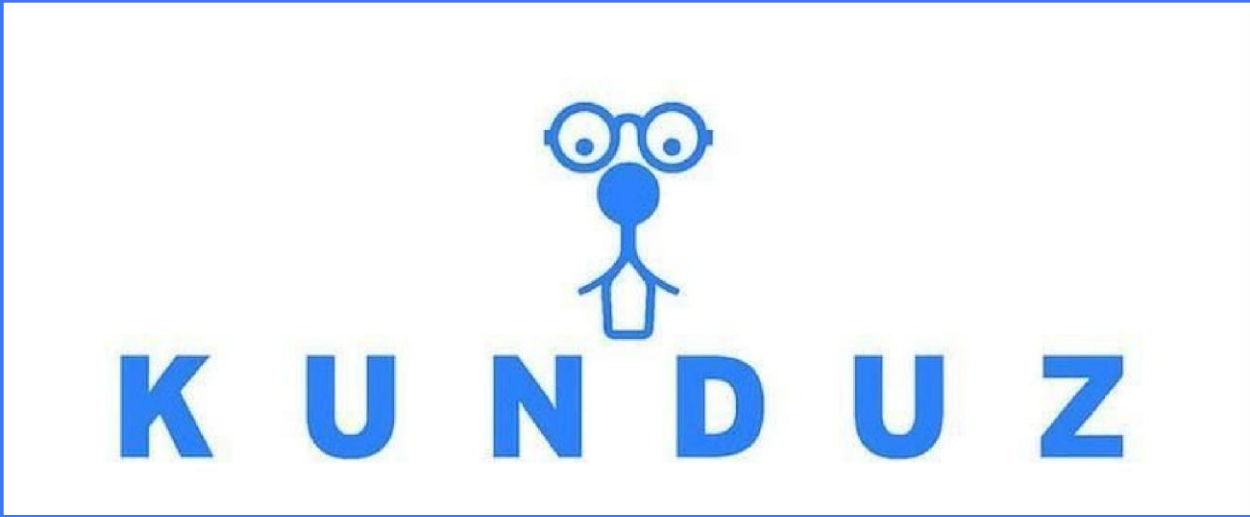


JEE Advanced 2020 Mathematics paper 2 solutions prepared by KUNDUZ tutors

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Question number	Question and Answers
1	

Q.1 For a complex number z, let $\mathrm{Re}(z)$ denote the real part of z. Let S be the set of all complex numbers z satisfying $z^4 - |z|^4 = 4\,i\,z^2$, where $i = \sqrt{-1}$. Then the minimum possible value of $|z_1 - z_2|^2$, where $z_1, z_2 \in S$ with $\mathrm{Re}(z_1) > 0$ and $\mathrm{Re}(z_2) < 0$, is ____

Q.1 let $z = a+ib$

$\Rightarrow z^4 - |z|^4 = 4iz^2$

$\because |z|^2 = z\bar{z} \Rightarrow z^4 - z^2(\bar{z})^2 = 4iz^2$

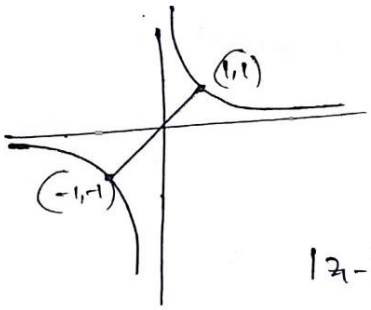
$\Rightarrow |z|^4 = z^2\bar{z}^2 \Rightarrow z^2(z^2 - \bar{z}^2) = 4iz^2$

$\because z^2 = a^2 - b^2 + 2iab$

$\bar{z}^2 = a^2 - b^2 - 2iab \Rightarrow z^2 - \bar{z}^2 = 4iab$

So, $z^2 = 0$ or $4iab = 4i$

$ab = 1 \rightarrow$ (hyperbola)



The line joining these two points have minimum length.

$|z_1 - z_2|^2 = (1+1)^2 + (1+1)^2$

$= 8$ Ans

2

Q.2 The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is **NOT** less than 0.95, is

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Q. 2

probability to hit the target successfully $= \frac{3}{4} = p$

$\Rightarrow q = 1 - p = \frac{1}{4}$.

Now, the probability that the missile hits the target 3 times

$p(3) = {}^{n}C_3 \cdot p^3 \cdot q^{n-3}$

So, $p(3) + p(4) + \cdots\cdots \geq 0.95$

This can be written as,

$1 - [p(0) + p(1) + p(2)] \geq 0.95$

$\Rightarrow {}^{n}C_0 \left(\frac{1}{4}\right)^n + {}^{n}C_1 \frac{3}{4} \cdot \left(\frac{1}{4}\right)^{n-1} + {}^{n}C_2 \cdot \frac{9}{16} \cdot \left(\frac{1}{4}\right)^{n-2} \leq 0.05$

$\Rightarrow \left(\frac{1}{4}\right)^n \left(1 + 3n + \frac{9n^2 - 9n}{2}\right) \leq \frac{5}{100}$

$\Rightarrow 9n^2 - 3n + 2 \leq \frac{4^n}{10}$

There is no standard method to find n from this equation.

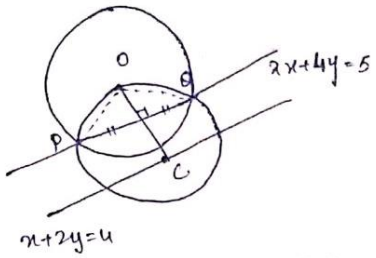
By trial and error, $n \geq 6$.

$\therefore$ least value $= \boxed{6}$ Ans.

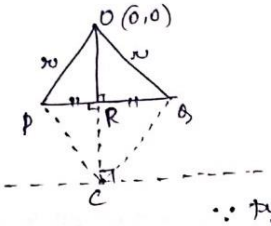
3

Q.3 Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is ____

Q.3



In ΔOPQ,



$\therefore$ Pythagoras theorem,

$$OR = \frac{|0+0-5|}{\sqrt{4+16}} = \frac{5}{2\sqrt{5}} \qquad r = \sqrt{PR^2 + OR^2}$$

$$OC = \frac{|0+0-4|}{\sqrt{1+4}} = \frac{4}{\sqrt{5}}$$

$CP = CO \Rightarrow CP = \frac{4}{\sqrt{5}}$ (Radius of circumcircle) and $CR = OC - OR$

$$CR = \frac{3}{2\sqrt{5}}$$

Using Pythagoras theorem,

$$PR = \sqrt{CP^2 - CR^2} = \sqrt{\frac{16}{5} - \frac{9}{4(5)}} = \sqrt{\frac{11}{4}}$$

Now, $r = \sqrt{\frac{5}{4} + \frac{11}{4}} = \sqrt{\frac{16}{4}} = \boxed{2}$ Ans.

4

Q.4 The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2×2 matrix such that the trace of A is 3 and the trace of A^3 is -18, then the value of the determinant of A is

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Q. 4 let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\Rightarrow$ tr $A = a + d = 3$

take, $|A - \lambda I| = 0$ when $\lambda = A$

$\Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$\Rightarrow ad + \lambda^2 - (a+d)\lambda - bc = 0$

$\Rightarrow A^2 - (a+d)A + ad - bc = 0$

So, $A^3 = (a+d)A^2 - (ad-bc)A$

$= (a+d)\left[(a+d)A - (ad-bc)\right] - (ad-bc)A$

$= (a+d)^2 A - (ad-bc)A - (a+d)(ad-bc)I$

$= \left[9 - (ad-bc)\right] A - 3(ad-bc)I$

$\Rightarrow$ tr $A^3 = (9 - (ad-bc))(a+d) - 3(ad-bc)(1+1)$

$-18 = 27 - 3 \det A - 6 \det A.$

$\left(\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc\right)$ $\Rightarrow \det A = 5$ Ans

5

Q.5 Let the functions $f:(-1,1)\to\mathbb{R}$ and $g:(-1,1)\to(-1,1)$ be defined by

$$f(x)=|2x-1|+|2x+1| \quad \text{and} \quad g(x)=x-[x],$$

where $[x]$ denotes the greatest integer less than or equal to x. Let $f\circ g:(-1,1)\to\mathbb{R}$ be the composite function defined by $(f\circ g)(x)=f(g(x))$. Suppose c is the number of points in the interval $(-1,1)$ at which $f\circ g$ is **NOT** continuous, and suppose d is the number of points in the interval $(-1,1)$ at which $f\circ g$ is **NOT** differentiable. Then the value of $c+d$ is ____

Q.5. $g(x)=x-[x]=\{x\}$

fractional part of x.

$\Rightarrow f(g(x))=|2\{x\}-1|+|2\{x\}+1|$

if $2\{x\}-1<0 \Rightarrow \{x\}<\frac{1}{2}$

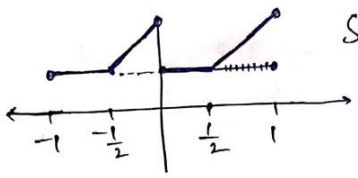
$\Rightarrow f(g(x))=1-2\{x\}+2\{x\}+1$

$=2$

if $2\{x\}-1\geq 0 \Rightarrow \{x\}\geq\frac{1}{2}$

$\Rightarrow f(g(x))=2\{x\}-1+2\{x\}+1$

$=4\{x\}$



So, $f(g(x))=2$; for, $x\in(-1,-\frac{1}{2})\cup[0,\frac{1}{2})$

$f(g(x))=4\{x\}$; for, $x\in[-\frac{1}{2},0)\cup[\frac{1}{2},1)$

So, $f(g(x))$ is discontinuous only at $x=0 \Rightarrow \boxed{c=1}$

$\therefore$ kink at $x=-\frac{1}{2},\frac{1}{2}$, so, not-differentiable at $x=-\frac{1}{2},0,\frac{1}{2}$. $\boxed{d=3}$

$\Rightarrow c+d=1+3=\boxed{4}$ Ans.

6

Q.6 The value of the limit

$$\lim_{x\to\frac{\pi}{2}} \frac{4\sqrt{2}(\sin 3x+\sin x)}{\left(2\sin 2x\sin\frac{3x}{2}+\cos\frac{5x}{2}\right)-\left(\sqrt{2}+\sqrt{2}\cos 2x+\cos\frac{3x}{2}\right)}$$

is ____

Q.6 $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{4\sqrt{2}\left(\sin 3x+\sin x\right)}{\left(2\sin 2x\sin\frac{3x}{2}+\cos\frac{5x}{2}\right)-\left(\sqrt{2}+\sqrt{2}\cos 2x+\cos\frac{3x}{2}\right)}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{4\sqrt{2}\left(2\sin 2x\cos x\right)}{\left(\cos\frac{5x}{2}-\cos\frac{3x}{2}\right)+2\sin 2x\sin\frac{3x}{2}-\sqrt{2}\left(1+\cos 2x\right)}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{4\sqrt{2}\cdot\left(2\sin 2x\cos x\right)}{-2\sin 2x\sin\frac{x}{2}+2\sin 2x\sin\frac{3x}{2}-2\sqrt{2}\cos^2 x}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{4\sqrt{2}\left(4\sin x\cos^2 x\right)}{2\sin 2x\left(\sin\frac{3x}{2}-\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{16\sqrt{2}\sin x\cos^2 x}{2\sin 2x\cdot\left(2\cos x\times\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{8\sqrt{2}\sin x}{\left(\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}\right)\cdot\left(2\sin\frac{x}{2}\right)-\sqrt{2}}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{8\sqrt{2}\sin x}{4\sin x\sin\frac{x}{2}-\sqrt{2}} = \dfrac{8\sqrt{2}}{2\sqrt{2}-\sqrt{2}}$

$= \boxed{8}$ Ans.

7

Q.7 Let b be a nonzero real number. Suppose $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(0) = 1$. If the derivative f' of f satisfies the equation

$$f'(x) = \frac{f(x)}{b^2 + x^2}$$

for all $x \in \mathbb{R}$, then which of the following statements is/are TRUE?

(A) If $b > 0$, then f is an increasing function

(B) If $b < 0$, then f is a decreasing function

(C) $f(x)f(-x) = 1$ for all $x \in \mathbb{R}$

(D) $f(x) - f(-x) = 0$ for all $x \in \mathbb{R}$

Q.7 $f'(x) = \frac{f(x)}{b^2+x^2}$

$\Rightarrow \int \frac{f'(x)}{f(x)} \cdot dx = \int \frac{1}{b^2+x^2} \cdot dx$

$\Rightarrow \ln|f(x)| = \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) + C$

when $x=0$, $f(0)=1$

$\Rightarrow 0 = \frac{1}{b}\tan^{-1} 0 + C$

$\boxed{C=0}$

$\Rightarrow f(x) = \pm e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)}$; $f(0)=1$

So, $f(x) = e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)} > 0 \quad \forall x$

$\Rightarrow f'(x) = \frac{f(x)}{x^2+b^2} > 0 \quad \forall x$ **option A**

f is always increasing.

So, $f(x) \cdot f(-x) = e^{-\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)} \cdot e^{\frac{1}{b}\tan^{-1}\frac{x}{b}}$

$\Rightarrow f(x) \cdot f(-x) = 1$ **option C**

So, **A, C** are correct.

8

Q.8 Let a and b be positive real numbers such that $a > 1$ and $b < a$. Let P be a point in the first quadrant that lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Suppose the tangent to the hyperbola at P passes through the point $(1, 0)$, and suppose the normal to the hyperbola at P cuts off equal intercepts on the coordinate axes. Let Δ denote the area of the triangle formed by the tangent at P, the normal at P and the x-axis. If e denotes the eccentricity of the hyperbola, then which of the following statements is/are TRUE?

(A) $1 < e < \sqrt{2}$ (B) $\sqrt{2} < e < 2$ (C) $\Delta = a^4$ (D) $\Delta = b^4$

Q.8 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\quad x = a\sec\theta$, $y = b\tan\theta$

$\frac{2x}{a^2} - \frac{2y\frac{dy}{d\theta}}{b^2} = 1$

$\frac{dy}{d\theta} = \frac{b^2x}{a^2y} = \frac{b}{a\sin\theta}$

eq of tangent, $y - b\tan\theta = (x - a\sec\theta)\frac{b}{a\sin\theta}$

$\Rightarrow ay\sin\theta - bx = ab\tan\theta\sin\theta - ab\sec\theta$

Passes through $(1,0)$.

$\Rightarrow -b = \frac{ab\sin^2\theta}{\cos\theta} - \frac{ab}{\cos\theta}$

$\Rightarrow -\cos\theta = a(\sin^2\theta - 1) = -a\cos^2\theta$

$\Rightarrow \cos\theta = \frac{1}{a}$

eq of normal, $(y - b\tan\theta) = -\frac{a\sin\theta}{b}(x - a\sec\theta)$

$\Rightarrow by + ax\sin\theta = a^2\tan\theta + b^2\tan\theta$

Equal intercept $\Rightarrow b = a\sin\theta$

$\Rightarrow e^2 = \frac{a^2+b^2}{a^2}$

$\Rightarrow e^2 = \frac{2a^2-1}{a^2}$

$\Rightarrow e^2 = 2 - \frac{1}{a^2}$

$\Rightarrow e = \sqrt{2-\frac{1}{a^2}}$

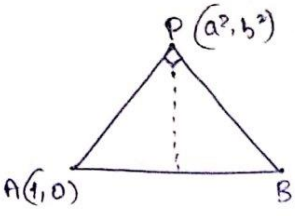
$\Rightarrow b = a\sqrt{\frac{a^2-1}{a^2}}$

$\Rightarrow b^2 = a^2 - 1$ — from (i)

$0 < \frac{1}{a^2} < 1$

$\Rightarrow 1 < 2 - \frac{1}{a^2} < 2$

$\Rightarrow \boxed{1 < e < \sqrt{2}}$



$x = a\sec\theta = a^2$

$y = b\tan\theta = b^2$

$\boxed{b^2 = a^2 - 1}$ from (i)

$\cos\theta = \frac{1}{a}$, $\sin\theta = \frac{\sqrt{a^2-1}}{a}$

$\Rightarrow \sin\theta = \frac{b}{a}$

$\Rightarrow \boxed{\tan\theta = b}$

Normal eq,

$by + ax\sin\theta = (a^2+b^2)\tan\theta$

$\Rightarrow by + bx = b(a^2+b^2)$

$\Rightarrow \boxed{x + y = a^2 + b^2}$

x-Intercept of Normal is $(a^2+b^2, 0)$

Area $= \frac{1}{2}(AP)(PB) = \frac{1}{2}\left(\sqrt{(a^2-1)^2+b^2}\right)\left(\sqrt{b^4+b^4}\right)$

$\Delta = \frac{1}{2}\left(\sqrt{2b^4}\right)\left(\sqrt{2b^4}\right)$

$\boxed{\Delta = b^4}$

So, **Option A, D** are correct

9

Q.9 Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be functions satisfying

$$f(x+y) = f(x) + f(y) + f(x)f(y) \quad \text{and} \quad f(x) = xg(x)$$

for all $x, y \in \mathbb{R}$. If $\lim_{x \to 0} g(x) = 1$, then which of the following statements is/are TRUE?

(A) f is differentiable at every $x \in \mathbb{R}$

(B) If $g(0) = 1$, then g is differentiable at every $x \in \mathbb{R}$

(C) The derivative $f'(1)$ is equal to 1

(D) The derivative $f'(0)$ is equal to 1

Q.9 $f(x+y) = f(x) + f(y) + f(x)f(y)$

taking $y = 0$.

$\Rightarrow f(x) = f(x) + f(0) + f(x)f(0)$

$\Rightarrow f(0)(1 + f(x)) = 0$ so, $f(0) = 0$

to check continuity for $f(x)$,

$$\lim_{h \to 0} f(x+h) = \lim_{h \to 0} \left(f(x) + f(h) + f(x)f(h)\right)$$

$$= f(x) + \lim_{h \to 0} f(h)(1 + f(x))$$

$$\lim_{h \to 0} f(x+h) = f(x) \quad \forall x$$ [option A]

So, f is continuous $\forall x$.

Now, differentiate w.r.t y.

$\Rightarrow f'(x+y) = f'(y) + f(x)f'(y)$

$f(x) = xg(x) \Rightarrow f'(x) = g(x) + xg'(x)$.

$f'(0) = g(0) + 0 = 1$ [option D]

Putting $y = 0$, $f'(x) = f'(0) + f(x)f'(0)$

$\Rightarrow \frac{f'(x)}{1 + f(x)} = 1$. (Integrate)

$\Rightarrow \ln(1 + f(x)) = x + C$

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$\Rightarrow f(x) = e^{x+c} - 1$, $x = 0$; $f(x) = 0$

$\Rightarrow e^c = 1$; $c = 0$

$\Rightarrow f'(x) = e^x = e$

$$g(x) = \frac{f(x)}{x} = \frac{e^x - 1}{x} \quad ; x \neq 0.$$

$\Rightarrow$ $g(x)$ & at $x = 0$, $g(x) = 1$

is differentiable every where

$g'(x)$ at $x = 0$

$$\Rightarrow g'(0) = \lim_{h \to 0} \frac{g(h) - g(0)}{h} = \lim_{h \to 0} \frac{\left(\frac{e^h - 1}{h}\right) - 1}{h}$$

$$e^h = 1 + \frac{h}{1!} + \frac{h^2}{2!} + \cdots$$

$$\frac{e^h - 1}{h} = 1 + \frac{h}{2!} + \frac{h^2}{3!} + \cdots$$

$$\Rightarrow g'(0) = \lim_{h \to 0} \frac{\cancel{1} + \frac{h}{2!} + \frac{h^2}{3!} \cdots \cancel{-1}}{h}$$

$$= \frac{1}{2}$$

So, g is diff. $\forall x$ if $g(0) = 1$ **option B**

$f(x) = e^x - 1$ $\therefore$ $f(x)$ is diff. $\forall x$

option A

10

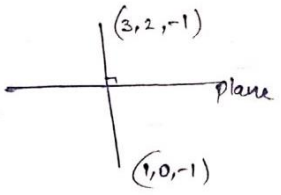
Q.10 Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0$ and $\alpha+\gamma=1$. Suppose the point $(3,2,-1)$ is the mirror image of the point $(1,0,-1)$ with respect to the plane $\alpha x+\beta y+\gamma z=\delta$. Then which of the following statements is/are TRUE?

(A) $\alpha+\beta=2$ (B) $\delta-\gamma=3$

(C) $\delta+\beta=4$ (D) $\alpha+\beta+\gamma=\delta$

Q.10 $(3,2,-1)$ is the mirror image of $(1,0,-1)$ w.r.t $\alpha x+\beta y+\gamma z=\delta$

& $\alpha+\gamma=1$ ... (1)



The line joining these two points should be perpendicular to the plane

$\Rightarrow \frac{\alpha}{3-1}=\frac{\beta}{2-0}=\frac{\gamma}{-1+1}=K$.

$\Rightarrow \alpha=2K,\ \beta=2K,\ \gamma=0$

from (1), $2K+0=1$

$K=\frac{1}{2}$ $\Rightarrow \boxed{\alpha=\beta=1}$

The midpoint of the two points should lie on the plane, $\left(\frac{3+1}{2}, \frac{2+0}{2}, \frac{-1-1}{2}\right)$

$\equiv (2,1,-1)$

$\Rightarrow 2\alpha+\beta-\gamma=\delta$

$\Rightarrow \delta=3$

A) $\alpha+\beta=1+1=2$ B) $\delta-\gamma=3-0=3$

C) $\delta+\beta=3+1=4$ D) $\alpha+\beta+\gamma=2\neq\delta$.

ABC are correct

11

Q.11 Let a and b be positive real numbers. Suppose $\overrightarrow{PQ} = a\hat{i} + b\hat{j}$ and $\overrightarrow{PS} = a\hat{i} - b\hat{j}$ are adjacent sides of a parallelogram $PQRS$. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w} = \hat{i} + \hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$, respectively. If $|\vec{u}| + |\vec{v}| = |\vec{w}|$ and if the area of the parallelogram $PQRS$ is 8, then which of the following statements is/are TRUE?

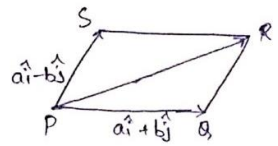
(A) $a + b = 4$

(B) $a - b = 2$

(C) The length of the diagonal PR of the parallelogram $PQRS$ is 4

(D) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

Q.11



* Area of PQRS $= |(a\hat{i}+b\hat{j}) \times (a\hat{i}-b\hat{j})|$

$\Rightarrow 2ab = 8$

$\boxed{ab = 4}$

$\vec{w} = i + j$

$\vec{u}$ → projection of $\vec{w}$ along $\overrightarrow{PQ}$

$$\Rightarrow \vec{u} = \left(\frac{\vec{w} \cdot \overrightarrow{PQ}}{|\vec{w}|\,|\overrightarrow{PQ}|}\right)\vec{w}$$

$$\Rightarrow \vec{u} = \frac{(i+j)\cdot(a\hat{i}+b\hat{j})}{\sqrt{2}\sqrt{a^2+b^2}}\,\vec{w}$$

$$\Rightarrow \vec{u} = \frac{a+b}{\sqrt{2(a^2+b^2)}}(\vec{w}) \Rightarrow |\vec{u}| = \frac{a+b}{\sqrt{a^2+b^2}}$$

Similarly, $\vec{v} = \frac{a-b}{\sqrt{2(a^2+b^2)}}(\vec{w}) \Rightarrow |\vec{v}| = \frac{|a-b|}{\sqrt{a^2+b^2}}$

$$|\vec{u}| + |\vec{v}| = |\vec{w}| \Rightarrow \frac{a+b+|a-b|}{\sqrt{a^2+b^2}} = \sqrt{2}$$

case i) $a > b$

$$\Rightarrow a+b+a-b = \sqrt{2a^2+2b^2}$$

$$\Rightarrow 4a^2 = 2a^2+2b^2 \Rightarrow a = b$$

case (ii) $a < b$ $\Rightarrow a+b-a+b = \sqrt{2a^2+2b^2}$

$$\Rightarrow 4b^2 = 2a^2+2b^2 \Rightarrow a = b$$

So, $ab = 4 \Rightarrow a = b = 2$ $\boxed{\text{Option A}}$

$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{PS} = 2a\hat{i} = 4\hat{i}$; $|\overrightarrow{PR}| = 4$ $\boxed{\text{opt. C}}$

12

Q.12 For nonnegative integers s and r, let

$$\binom{s}{r} = \begin{cases} \dfrac{s!}{r!\ (s-r)!} & \text{if } r \le s, \\ 0 & \text{if } r > s. \end{cases}$$

For positive integers m and n, let

$$g(m,n) = \sum_{p=0}^{m+n} \frac{f(m,n,p)}{\binom{n+p}{p}}$$

where for any nonnegative integer p,

$$f(m,n,p) = \sum_{i=0}^{p} \binom{m}{i}\binom{n+i}{p}\binom{p+n}{p-i}.$$

Then which of the following statements is/are TRUE?

(A) $g(m,n) = g(n,m)$ for all positive integers m, n

(B) $g(m,n+1) = g(m+1,n)$ for all positive integers m, n

(C) $g(2m,2n) = 2\,g(m,n)$ for all positive integers m, n

(D) $g(2m,2n) = (g(m,n))^2$ for all positive integers m, n

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Q.12

$$f(m,n,p) = \sum_{i=0}^{p} {}^{m}C_i \; {}^{n+i}C_p \; {}^{n+p}C_{p-i}$$

$$= \sum \frac{m!}{i!\,(m-i)!} \times \frac{(n+i)!}{p!\,(n+i-p)!} \times \frac{(n+p)!}{(n+i)!\,(p-i)!}$$

$$= \sum {}^{m}C_i \times \frac{(n+p)!}{p!\,n!} \times \frac{n!}{(p-i)!\,(n-(p-i))!}$$

$$= \sum {}^{m}C_i \times {}^{n+p}C_n \times {}^{n}C_{p-i}$$

$$= \sum (n+p C_n).\left({}^{m}C_i \times {}^{n}C_{p-i}\right) = {}^{n+p}C_n \cdot \left({}^{m+n}C_p\right)$$

$\because$ ${}^{m}C_i \rightarrow$ co-efficient of x^i in $(1+x)^m$

${}^{n}C_{p-i} \rightarrow$ co-efficient of x^{p-i} in $(1+x)^n$.

so, $(1+x)^n \cdot (1+x)^m \rightarrow$ co-efficient of x^p

will be $\sum_{i=0}^{p} {}^{m}C_i \times {}^{n}C_{p-i} = {}^{m+n}C_p$

$$g(m,n) = \sum_{p=0}^{m+n} \frac{{}^{n+p}C_p \cdot \left({}^{m+n}C_p\right)}{{}^{n+p}C_p} = 2^{m+n}$$

$\Rightarrow$ $g(n,m) = 2^{n+m} = g(m,n)$. Option A

$\Rightarrow$ $g(m,n+1) = 2^{m+n+1} = g(n+1,m)$ Option B

$\Rightarrow$ $g(2m,2n) = 2^{2m+2n} = (2^2)^{m+n}$

$= (2^{m+n})^2 = [g(m,n)]^2$ Option D

13

Q.13 An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that **no** two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is ____

Q.13

From 1-15, we need to select any 4 dates such that they are not consecutive.

Remove 4 days from 15 = 15-4

Now, we place this 4 days in the gaps of the remaining days so that no 2 days are consecutive.

$\Rightarrow$ No. of gaps = 15-4+1 = 12

So, No. of ways = $^{12}C_4$

$= \frac{12 \times 11 \times 10 \times 9}{24} = 495$

14

Q.14 In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is ____

Q.14

_ _ _ _ (Arrange in 4!)

Each room should be accomodated with atleast 1 person

So, select any 4 persons from 6 = $^6C_4$

Now, for the other 2 we select any 2 rooms from 4 = $^4C_2$

While selecting, the cases repeat twice, because we can select 2 people in 1st selection or 2nd one

$\therefore$ No. of ways = $\frac{6!}{4!2!} \times \frac{4!}{2!2!} \times \frac{4!}{2!}$ → Arranging.

= 1080

15

Q.15 Two fair dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If p is the probability that this perfect square is an odd number, then the value of $14p$ is

---

Q.15 Perfect square : 4 or 9

4 → 1+3 → 2 ways
2+2 → 1 ways
) total 3

9 → 4+5 → 2 ways
3+6 → 2 ways
) total 4

$\Rightarrow P_S = \frac{3+4}{36} = \frac{7}{36}$

P_S(odd numbers) $= \frac{4}{36}$

Prime : 2, 3, 5, 7, 11

2 → 1+1 → 1 way

3 → 1+2 → 2 ways

5 → 1+4 → 2 ways
2+3 → 2 ways
) total 4

7 → 1+6 → 2 ways
2+5 → 2 ways
3+4 → 2 ways
) total 6

11 → 5+6 → 2 ways

$\Rightarrow P_P = \frac{1+2+4+6+2}{36} = \frac{15}{36}$

For not getting either prime or square

$= 1-\left(\frac{15}{36}+\frac{7}{36}\right) = \frac{14}{36}$

probability $= \frac{\frac{4}{36}+\frac{14}{36}\left(\frac{4}{36}\right)+\cdots}{\frac{7}{36}+\frac{14}{36}\cdot\frac{7}{36}+\cdots} = \frac{4}{7}$

$p = \frac{4}{7} \Rightarrow 14p = 14\times\frac{4}{7} = \boxed{8}$ Ans.

16

Q.16 Let the function $f:[0,1] \to \mathbb{R}$ be defined by

$$f(x) = \frac{4^x}{4^x + 2}.$$

Then the value of

$$f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \cdots + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right)$$

is ____

Q.16 $f(x) = \dfrac{4^x}{4^x+2}$

$f(1-x) = \dfrac{4 \cdot 4^{-x}}{4 \cdot 4^{-x} + 2} = \dfrac{\frac{2}{4^x}}{\frac{2}{4^x} + 1}$

$\Rightarrow f(1-x) = \dfrac{2}{2 + 4^x}$

So, $f(x) + f(1-x) = \dfrac{4^x + 2}{4^x + 2} = 1$

$f\left(\frac{1}{40}\right) + f\left(\frac{39}{40}\right) = 1$

$f\left(\frac{2}{40}\right) + f\left(\frac{38}{40}\right) = 1$

$\vdots$

$f\left(\frac{19}{40}\right) + f\left(\frac{21}{40}\right) = 1$

} 19 terms

adding all these gives, 19.

$\Rightarrow f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + \cdots f\left(\frac{20}{40}\right) + \cdots f\left(\frac{39}{40}\right) - f\left(\frac{20}{40}\right)$

$\Rightarrow$ The sum gives 19 Ans.

17

Q.17 Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that its derivative f' is continuous and $f(\pi)=-6$. If $F:[0,\pi]\to\mathbb{R}$ is defined by $F(x)=\int_0^x f(t)dt$, and if

$$\int_0^{\pi}\left(f'(x)+F(x)\right)\cos x\,dx = 2,$$

then the value of $f(0)$ is ____

Q.17 $F(x) = \int_0^x f(t)\,dt$ & $f(\pi) = -6$

$\Rightarrow F'(x) = f(x)$

$$\int_0^{\pi}\left(f'(x)\cos x + F(x)\cos x\right)\cdot dx = 2$$

Let $I = \int_0^{\pi} \underset{II}{F(x)}\,\underset{I}{\cos x}\cdot dx$

apply Integration by parts

$I = F(x)\sin x\Big|_0^{\pi} - \int_0^{\pi} F'(x)\sin x\,dx$

$\Rightarrow I = -\int_0^{\pi} \underset{II}{f(x)}\,\underset{I}{\sin x}\cdot dx.$

apply IBP again

$\Rightarrow I = +f(x)\cos x\Big|_0^{\pi} - \int_0^{\pi} f'(x)\cos x\cdot dx$

$\Rightarrow I + \int_0^{\pi} f(x)\cos x\cdot dx = f(\pi)(\cos\pi) - f(0)$

$\Rightarrow \int_0^{\pi}\left(F(x)+f'(x)\right)\cos x\cdot dx = 6 - f(0)$

$\Rightarrow f(0) = 6-2 = \boxed{4}$ Ans.

18

Q.18 Let the function $f:(0,\pi)\to\mathbb{R}$ be defined by

$$f(\theta)=(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^4.$$

Suppose the function f has a local minimum at θ precisely when $\theta\in\{\lambda_1\pi,\ldots,\lambda_r\pi\}$, where $0<\lambda_1<\cdots<\lambda_r<1$. Then the value of $\lambda_1+\cdots+\lambda_r$ is ____

Q.18

$$f(\theta)=(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^4$$

$$\Rightarrow f(\theta)=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)^2$$

$$\Rightarrow f(\theta)=1+\sin 2\theta+(1-\sin 2\theta)^2$$

$$\Rightarrow f(\theta)=\sin^2 2\theta-\sin 2\theta+2.$$

Now, $f'(\theta)=(2\sin 2\theta\cos 2\theta)2-2\cos 2\theta$

$$\Rightarrow f'(\theta)=2\cos 2\theta(2\sin 2\theta-1)$$

$f'(\theta)=0 \Rightarrow \cos 2\theta=0$ or $\sin 2\theta=\frac{1}{2}$

$\theta=\frac{\pi}{4},\frac{3\pi}{4}$ $\qquad$ $\theta=\frac{\pi}{12},\frac{5\pi}{12}$

$$f'(\theta)=2\sin 4\theta-2\cos 2\theta$$

$$f''(\theta)=8\cos 4\theta+4\sin 2\theta$$

$f''\left(\frac{\pi}{4}\right)=8\cos\pi+4\sin\frac{\pi}{2}=-4$ (maxima)

$f''\left(\frac{3\pi}{4}\right)=8\cos 3\pi+4\sin\frac{3\pi}{2}=-12$ (maxima)

$f''\left(\frac{\pi}{12}\right)=8\cos\frac{\pi}{3}+4\sin\frac{\pi}{6}=6$ (minima)

$f''\left(\frac{5\pi}{12}\right)=8\cos\frac{5\pi}{3}+4\sin\frac{5\pi}{6}=6$ (minima)

minimum at $\theta=\frac{\pi}{12},\frac{5\pi}{12}$.

$\Rightarrow \lambda_1+\lambda_2=\frac{1}{12}+\frac{5}{12}=\boxed{\frac{1}{2}}$ Ans.

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